Unification of the hiperoperators theory and the serial operators theory

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0. Abstract:
In this paper I combined all the hiperoperations theory with the theory of series of functions developing new notation when it was necessary.

1. Introduction and notation that we are going to use:
To develop a fusion between series and hiperoperators we need first to introduce some notation. I will use sometimes a classic types of writing and innovate when it will be necessary. Lets start with the definitions of some operations and their symbols depending the operation value k.

$$a[1]b = a + b \tag{1}$$

$$a[2]b = a \cdot b \tag{2}$$

Using Knuth notation we can assign to the third the following,

$$a[3]b = a^b = a \uparrow b \tag{3}$$

Now starting with tetration, the first concept of hiperoperations we have:

$$a[4]b = a \uparrow\uparrow b = a \sqcap b. \tag{4}$$

Continuing with pentation or 5-ation:

$$a[5]b = a \uparrow\uparrow\uparrow b = a \bigtriangleup b. \tag{5}$$

We can now assign equivalent symbols to the negative operations, following a mirror thinking.

$$a[-1]b = a - b \tag{6}$$

$$a[-2]b = a \div b \tag{7}$$

$$a[-3]b = \sqrt[b]{a} = a \downarrow b \tag{8}$$

Next, with anti-tetration,

$$a[-4]b = a \downarrow\downarrow b = a \sqcup b. \tag{9}$$

And ending with anti- pentation:

$$a[-5]b = a \downarrow\downarrow\downarrow b = a \bigtriangledown b. \tag{10}$$

In k=0 we will get a set as result so:

$$a[0]b = (a, b) \tag{11}$$

In general, we can assign a k-ation to:

$$a[k]b \tag{12}$$

And this will be true $\forall k \in Z$.

2. An application of the hiperoperations to series

Lets start with the most common notation of the definition of serial sum.

$$\begin{matrix} n \\ \Sigma \\ i=1 \end{matrix} \quad a_i = a_1 + a_2 + ... + a_{n-1} + a_n \tag{13}$$

But I prefer the next notation which is equivalent to the previous but it also goes over functions, so we will in the future use this notation, so the summatory function will be:

$$\begin{matrix} b \\ \Sigma \\ n=a \end{matrix} \quad f(n) = f(a) + f(a+1) + ... + f(b-1) + f(b). \tag{14}$$

Now we can apply the same concept to the famous product notation or productory,

$$\begin{matrix} b \\ \Pi \\ n=a \end{matrix} \quad f(n) = f(a) \cdot f(a+1) \cdot ... \cdot f(b-1) \cdot f(b) \tag{15}$$

Following with my own notation for continuous exponents, producing a serial exponent function, as we can see using 3-ation we stablish a parallelism using Knuth notation, to get exponentory :

$$\underset{n=a}{\overset{b}{\Theta}} f(n) = f(a) \uparrow f(a+1) \uparrow ... \uparrow f(b-1) \uparrow f(b) \tag{16}$$

In my way to innovate I apply my knowledge of series to invent the next symbol for the use tetration in series (tetratory):

$$\underset{n=a}{\overset{b}{T}} f(n) = f(a) \sqcap f(a+1) \sqcap ... \sqcap f(b-1) \sqcap f(b) \tag{17}$$

And the next symbology to the pentation in series (pentatory).

$$\underset{n=a}{\overset{b}{\Psi}} f(n) = f(a) \vartriangle f(a+1) \vartriangle ... \vartriangle f(b-1) \vartriangle f(b) \tag{18}$$

The opposite functions of these serial operators, namely negative serial operators start with restory.

$$\underset{n=a}{\overset{b}{P}} f(n) = -f(a) - f(a+1) - ... - f(b-1) - f(b) \tag{19}$$

Following with division on series or divisory,

$$\underset{n=a}{\overset{b}{\Delta}} f(n) = f(a) \div f(a+1) \div ... \div f(b-1) \div f(b). \tag{20}$$

Rootory can be expanded in a simple way as:

$$\underset{n=a}{\overset{b}{Z}} f(n) = f(a) \downarrow f(a+1) \downarrow ... \downarrow f(b-1) \downarrow f(b) \tag{21}$$

And this means that we should do a series of roots, so other way to express that is:

$$\underset{n=a}{\overset{b}{Z}} f(n) = \sqrt[f(b)]{ANS} \sqrt[f(b-1)]{ANS} ... \sqrt[f(a+1)]{f(a)} \tag{22}$$

Where ANS means the answer of previous root. Continuing with negative serial operators we have anti-tetratory:

$$\underset{n=a}{\overset{b}{\Gamma}} \quad f(n) = f(a) \sqcup f(a+1) \sqcup ... \sqcup f(b-1) \sqcup f(b) \tag{23}$$

And anti-pentatory,

$$\underset{n=a}{\overset{b}{\Phi}} \quad f(n) = f(a) \bigtriangledown f(a+1) \bigtriangledown ... \bigtriangledown f(b-1) \bigtriangledown f(b) \tag{24}$$

In general, we can assign a k-ation serial operator:

$$\underset{n=a;k\in Z}{\overset{b}{\Omega}} \quad f(n) = f(a)[k]f(a+1)[k]...[k]f(b-1)[k]f(b) \tag{25}$$

So, for example for k=1:

$$\underset{n=a;k=1}{\overset{b}{\Omega}} \quad f(n) = f(a)[1]f(a+1)[1]...[1]f(b-1)[1]f(b) \tag{26}$$

Which is equal to sumatory.

3. Conclusions and the future

As we could see this is the fusion of two different theories and is enough closed to be a solid supertheory. In the future it will be interesting to do an approach to the properties of the different functions.

4. References

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