Unfringing Interference of Cross-Polarized Slits

W. Michael Feuerstein

I project a diagonally-polarized (D) monochromatic complex plane-wave[(1)] electric field onto cross-polarized slits. (1) What is the instantaneous behavior of the electric field versus diffraction angle in the far field[(2)]? (2) The time-parametric vector sum of crossed-polarized slits' electric fields in a fixed plane about normal to the slits' rays traces Lissajous curves[(3)] particular to the diffraction[(4)] angle. (3) These uniform-brightness, diffraction-angle-dependent lines, circles, and ellipses constructively constitute unfringing interference of transverse field undulations[(5)] summed in parallel and falsify the first Fresnel-Arago Law[(6,7)] since interference (the 3-d vector sum)[(1)], somewhat ironically, predicts the outcome. This result essentially retro-extends Fresnel optics (transverse vibrations) upon the Young's Double-Slit via the (parallel and direct) lineages of Maxwell, Heaviside, and Poynting, supporting Young's original assertion of light as a wave[(8.9)].

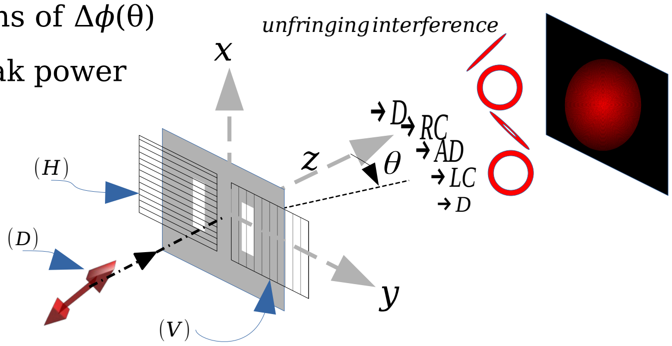
In the diffraction plane beyond linearly crossed-polarized optical slits, assuming a plane-wave E-field or wavefunction[(11)], the parametric vector sum, neglecting the diffraction envelope, is[(10)]:

$$\psi=\begin{pmatrix} e^{i\phi_1} & 0 \\ 0 & e^{i\phi_2} \end{pmatrix}\begin{bmatrix} E_{0x} \\ E_{0y} \end{bmatrix} e^{-i\omega t} \quad \text{so that} \quad \Re(\psi)=\begin{bmatrix} E_{0x}\cos(-\omega t) \\ E_{0y}\cos(-\omega t+\Delta\phi) \end{bmatrix} \qquad (1)$$

(where ϕ_1, $\phi_2 \equiv \phi_1(\theta,+d),\phi_2(\theta,-d)$, $\Delta\phi = \phi_2 - \phi_1$, $\omega = 2\pi\lambda/c$, θ is the diffraction angle, and d is the slit spacing/2). This vector sum traces Lissajous patterns[(3)] which are cyclic versus θ. The time-average of

$$\vec{S}=\frac{1}{\mu_0}(\Re\vec{E})\times(\Re\vec{B})=\hat{k}\sqrt{\frac{\epsilon_0}{\mu_0}}\left[E_{0x}^2\cos^2(\omega t)+E_{0y}^2\cos^2(\omega t-\Delta\phi)\right] \qquad (2)$$

(the instantaneous real irradiance[(12, 13)]), is independent of $\Delta\phi(\theta)$. (1) cycles in polarization and (2) in pulsed versus constant irradiance as functions of $\Delta\phi(\theta)$ (e.g., $\Delta\phi$'s of 0 and π contain D/A[nti]D with 2x-peak power pulses at 2ν, and $\Delta\phi$'s of $\pi/2$ and $3\pi/2$ contain L[eft]C[ircular-polarization]/R[ight]C with 1x-constant power.) For $E_{0y} = E_{0z}$, (fixed θ) sums constructing RC/LC change in direction only and sums constructing D/AD change in magnitude only with t. Comparing the present to parallel-polarized[(8.9)] (fringing) slits gives the two-slit



geometry information encoded in polarization (and time-domain irradiance) in the former and in brightness (fringes) in the latter.

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