Topological Theory of Hopf Bundle and Mass

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February 29, 2024

Abstract

Why a particle has the specific rest mass it does is an open question. To address this problem an alternative theory of mass is put forward. Mass is the intersection of a Hopf bundle and 3-space. The masses of six lighter hyperons and electron are derived as functions of the proton and neutron masses. Nine free parameters are thereby reduced to two. The most significant outcome is the derivation of the electron mass.

Keywords: hyperon, electron, hypersphere, Hopf, Higgs, mass splitting

In the standard model the Higgs field imparts mass to a small number of fundamental particles. In the crowd analogy the field acts like a mob impeding the progress of a celebrity across a room.[1] If we dig a little bit deeper, particles that exhibit Lie group symmetry at higher energy gain mass when spontaneous symmetry breaking couples with the Higgs field. However, the Higgs field only applies to simpler particles, which includes quarks, leptons and some bosons. The bulk of Hadron mass is due to quark confinement. Whether a particle is a conglomeration or not, theory and math eventually give out. Unable to say why a particle has the precise mass that it does our best theories rely on observation. It is for this reason particle rest mass is an open question. To address this problem an alternative theory of mass is put forward that rethinks why a particle *notices* a force. In this theory a particle resists an external force in order to preserve symmetry. If an analogy helps, a massive particle is socially awkward and struggles to make their way through a party crowd because they are reluctant to say hello and shake hands.

The topological theory brings together a Hopf bundle and a three dimensional field (3-space). The geometry of a Hopf Bundle is well established in the literature.[2, 3, 4] A Hopf bundle maps a 3-sphere to a 2-sphere. The 3-sphere is the set of four dimensional points S^3. The 2-sphere is a two dimensional surface described by the set of three dimensional points S^2. A Hopf fibration continuously maps S^3 to S^2. This is done with Hopf maps. A Hopf map, $h : S^3 \rightarrow S^2$, is a surjective function that maps a subset of S^3 points to a single S^2 point. An individual Hopf map describes a circle (Hopf circle). As the mapping is continuous there is an infinite number of maps for each point in S^3. This entails an infinite bundle of circles connect an S2 point to every point in S3. The total space is transitive. Added to the conventional description of a Hopf bundle is the physical interpretation that says Hopf bundle topology is partly responsible for particle mass.

A 'Hopf-particle', as we shall call it, interacts with ambient 3-space. The 3-space is a field with a ground state like the Higgs field. In 3-space a force is a vector. A force vector is one dimensional at point of contact P with the Hopf-particle. P is also a point on a bundle of Hopf circles. This raises the question of the differing topologies of a circle and point. Continuous retraction of the circle is impossible. Only by *cutting* the circle may the circle retract to a point. The discontinuity prevents smooth transmission of an external force at P. If a circle does not break, the force must *jump* topologies. To turn this observation into a theory of mass the topological hitch is interpreted as physical resistance to the external force. On this view, if P had some other topology that deform retracts to a point then the particle would be massless. The actual topology however, such that the bundle of Hopf circles at P is related to every point in S^3, means the size of the 3-sphere is the measure of the particle's resistance to an external force. These thoughts motivate our first five equations. The first tells us mass is determined by the size of the 3-sphere.

$$M = 2\pi^2 r^3. \tag{1}$$

Eq. (1) determines radius r. For example, if the mass of the proton is 938.272 MeV/c^2 then $r \approx 3.622$ MeV.

The volume of a 2-sphere is the space the Hopf-particle occupies in the ambient 3-space. This is the volume of an ordinary ball. If every point in the ball is the set B^3, there is a function that maps every point in S^3 to B^3. As the total space is transitive the mass associated with the ball is the resistance to a force registered by S^3. However, in 3-space the Hopf-particle still has an ordinary ball volume.

$$V = \frac{2M}{3\pi} = \frac{4\pi}{3}r^3. \tag{2}$$

At Eq. (2), radius r is derived from Eq. (1). In the case of the proton $V \approx 199.108$ MeV. The ball described at Eq. (2) is the ball volume formed in the intersection of a Hopf-particle with 3-space. The 3-sphere's extra fourth dimension does not contribute to the 3-space ball volume; it is a dark dimension in the sense it is not a direction the 3-space version of the ball is able to move in, for the same reason it is only an ordinary ball that appears in ambient 3-space.

$$\rho = \frac{M}{V} = \frac{3\pi}{2}. \tag{3}$$

Eq. (3) means the ball is hyper-dense. We call the excess mass 'hypermass'. The extra dark dimension is not completely dark in the sense it contributes to Hopf-particle mass. Hyper-mass is the evidence of an extra dimension. Hypermass (H) has the following Hopf/Hypermass signature (H-signature).

$$H = M - V. \tag{4}$$

Hopf-particle mass also has the H-signature:

$$M = (H)(\frac{\rho}{\rho - 1}). \tag{5}$$

Mass splitting formula which contain either V, H, or ρ are H-signatures. A H-signature indicates a massive particle is a Hopf-particle. H-signatures found in the rest mass data suggests lighter hyperons are hyper-massive Hopf-particles.

For what follows the 2018 CODATA recommended values are used for the proton and neutron masses (ignoring the standard deviation).[5]

$$\begin{aligned} M_p &= 938.272\ 088\ 16\ \pm 0.000\ 000\ 29\ \mathrm{MeV}/c^2. \\ M_n &= 939.565\ 420\ 52\ \pm 0.000\ 000\ 54\ \mathrm{MeV}/c^2. \end{aligned} \tag{6}$$

All other masses derived in this paper are a function of M_p and M_n. For instance, the light Σ (Sigma) masses are revealed in the following functions.

$$M_{\Sigma^+} = (2M_p - M_n)(\frac{\rho}{\rho - 1}) \approx 1189.3712. \tag{7}$$

$$M_{\Sigma^0} = (M_n)(\frac{\rho}{\rho - 1}) \approx 1192.6546. \tag{8}$$

$$M_{\Sigma^-} = (4M_n - 3M_p)(\frac{\rho}{\rho - 1}) \approx 1197.5797. \tag{9}$$

Eq. (8) allows us to say that in an energetic event a Σ^0 hyperon is created when there is sufficient energy to form a hypermass equivalent to the mass of the neutron. The asymmetry of Eqs. (7, 9) reveal the charged Σ^+ and Σ^- have complex hypermasses; the cause of the asymmetry is not presently understood.

Eqs. (7, 8) are within one standard deviation of the Particle Data Group (PDG) recommended rest energies. [6] Eq. (8) is particularly close to Wang 1192.65 $\pm$0.020.[7] However, Eq. (9) is over four standard deviations shy of the PDG value. The present PDG fit for M_{Σ^-} draws on three results. Schmidt (1197.43) [8], and Gurev (1197.417) [9] are too low to be the value derived here, though Eq. (9) is within one standard deviation of Gall (1197.532 $\pm$0.057).[10] The H-signatures for the Ξ (Xi) pair introduce a useful complication that decides whether Eqs. (7, 8, 9) are correct.

$$M_{\Xi^0} = (M_{\Sigma^0})(\frac{\rho}{\rho - 1}) - V_p \approx 1314.8104. \tag{10}$$

$$(M_{\Sigma^-})(\frac{\rho}{\rho - 1}) - V_p \approx 1321.0622. \tag{11}$$

Eq. (10) is within one standard deviation of the PDG fit and looks to be a direct hit for Fanti (1314.82 $\pm$0.06)[11], but a problem looms. When the basic pattern of Eq. (10) is repeated at Eq. (11) the result (1321.0622) is over nine standard deviations adrift of the PDG fit for Ξ^-. The present PDG recommended value (1321.71 Mev) is a fit for a 2006 study of a large 1992-1995 data sample.[12] Realistically, the 2006 result makes a future nine standard deviation downward adjustment unlikely. Accepting Eq. (11) will not do, we are about to see why [12] is accurate.

If M_{Σ^-} is close to 1321.71 a fudge ≈ 0.51 is needed to adjust the Ξ^- value upward. The electron mass ≈ 0.511 MeV is an obvious candidate. For the moment we call the additional weighting value 'W'. I.E.

$$M_{\Xi^-} = (M_{\Sigma^-} + W)(\frac{\rho}{\rho - 1}) - V_p. \tag{12}$$

At face value W appears ad hoc, but there is a firm reason for thinking otherwise. There are a few more equations to walk through before we can see why. First, we give the formula for the Ω^- (Omega) mass.

$$M_{\Omega^-} = \left(\frac{3M_{\Xi^0} + 2M_{\Xi^-}}{5}\right)\left(\frac{\rho}{\rho - 1}\right). \tag{13}$$

Given Eqs. (8, 9, 10, 12, 13), and using Eq. 2 and Eq. 13 to also find V_{Ω^-}, we derive the following equivalences.

$$\left(\frac{(M_{\Sigma^0})(M_{\Xi^-}) - (M_{\Sigma^0})(M_{\Xi^0})}{M_{\Sigma^-} - M_{\Sigma^0}} - M_{\Xi^0} - V_{\Omega^-}\right)\left(\frac{\rho - 1}{\rho}\right) = 1. \tag{14}$$

$$\left(\frac{(M_{\Sigma^-})(M_{\Xi^-}) - (M_{\Sigma^-})(M_{\Xi^0})}{M_{\Sigma^-} - M_{\Sigma^0}} - M_{\Xi^-} - V_{\Omega^-}\right)\left(\frac{\rho - 1}{\rho}\right) = 1. \tag{15}$$

When Eqs. (14, 15) = 1 then $W \approx 0.510\ 998\ 961\ 080$. This compares to 2018 CODATA value 0.510 998 9500 ±0.000 000 0015.[5] An adjustment within one standard deviation to M_p and M_n at Eq. (6) allows the numerical value for W to come within one standard deviation of the CODATA value. [5] From this we conclude $W = M_e$ MeV. If so, then the mass value for Σ^- at Eq. (9) is correct and the values for Ξ^- and Ω^- are also within one standard deviation of the PDG recommendation. I.E.

$$M_{\Xi^-} = (M_{\Sigma^-} + M_e)(\frac{\rho}{\rho - 1}) \approx 1321.7109. \tag{16}$$

$$M_{\Omega^-} \approx 1672.4824 \text{ (Eq. 13)}. \tag{17}$$

Before we are able to conclude, there is a problem that needs to be resolved. Eqs. (14, 15) are only equivalent to 1 when masses are given in MeV. In eV, Eqs. (14, 15) = 1,000,000; or in Kg they equal M_e Kg · MeV^{-1}. It seems the formulae only work when denominated in the arbitrary unit MeV. By itself this makes no sense and we are forced to search for an alternative system of units. We find the answer lies in an obsolete cgs unit of magnetomotive force, the Gilbert (Gb).[13] As the unit of current in an electric circuit is the Volt, the Gilbert is a unit of magnetic flux in a magnetic circuit. The

SI units for magnetomotive force are Ampere (A) and turn (tr). Turns are the winding number of an electromagnetic coil. The winding number is the number of times the coil wraps around a point. In SI units a Gilbert is equal to:

$$1\ Gb = \frac{10}{4\pi}\ A \cdot tr. \tag{18}$$

The magnetic permeability μ_0 (mu zero) is proportional to the energy stored in a magnetic field.

$$\mu_0 \approx (4\pi)(10^{-7})\ N \cdot A^{-2}. \tag{19}$$

The revaluation of SI units in 2019 means μ is no longer an exact value. However, it is sufficiently close to the number $4\pi \times 10^{-7}$ for the difference to be of no importance here. Magnetic permeability is related to electric permittivity ε_0 (epsilon nought) by the following equivalence.

$$\varepsilon_0 = \frac{1}{\mu_0 c^2}. \tag{20}$$

ε_0 is proportional to the energy stored in an electric field. We divide a Gilbert by ε_0 and use Eq. 16 to simplify and parse dimensions.

$$\frac{1\ Gb}{\varepsilon_0} \approx (10^{-6})(c^2)\ N \cdot A^{-1} \cdot tr. \tag{21}$$

The arrangement of units of Eq. 17 converts mass denominated in eV / c^2 into an equivalent rest energy described in Newton-Volt-turns. I.E.

$$\left(\frac{1\ eV}{c^2}\right)\left(\frac{1\ Gb}{\varepsilon_0}\right) \approx 10^{-6}\ N \cdot V \cdot tr. \tag{22}$$

By combining the mass energy of an electric field in electron volts and the mass energy of a magnetic field in Gilberts we find the total mass energy of a particle. If we assume Gb = 1 and measure mass in electron volts then Eq. (22) gives a mass value numerically indistinguishable from MeV.

The discrepant topologies of point and circle offer an economical theory of mass, but not one that plays well with the standard model. The smattering of results presented here are a long way from a thorough-going theory, while the many questions left open make it easy to discount a challenge to the standard model. Nonetheless, the Σ, Ξ, Ω and electron masses are derived as functions of the proton and neutron. It is the first time this has been done.

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