Addition of Velocities: An Adjusted Equation

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Abstract

Using the ideas of the constancy of speed of light and time dilation, a revised equation of addition of velocities is obtained, which coincides with Einstein's similar equation only in one-dimensional cases. The necessity of abandoning the idea of a ruler as a tool for instant measuring of distances, which is tantamount to instant transfer of information, is discussed. In the absence of a ruler, electromagnetic waves remain the only tools for measuring distances, and a quantum, the smallest portion of an electromagnetic emission, becomes the smallest portion (i.e. the smallest unit of measurement and the size of uncertainty) of space-time as well.

The theory of relativity, in order to meet its fundamental principles, makes some approximations of the physical reality (such as substituting spherical electromagnetic (EM) waves with plane ones or assuming infinite distance between the observer and the object of observation). This may lead to incorrect results. So, the results obtained by the theory, especially if they involve finite distances and do not belong to one-dimensional cases, require additional proofs [1]. In this paper, a revision of the rule of addition of velocities is made, where only the ideas of constancy of the light speed in free space and time dilation (which are deemed as absolutely true) are used from the theory of relativity.

Without prejudice to the generality, let us assume that, at the starting moment of time, three synchronous clocks with zero readings coincide in some point O. The velocity of clock_2 relative to clock_1 is $v = \beta c$, where c is the speed of light in free space. The velocity of clock_3 relative to clock_1 is $v_1 = \beta_1 c$, and relative to clock_2 - $v_2 = \beta_2 c$. The angle between the velocities of the moving clocks in the frame of clock_1 is θ.

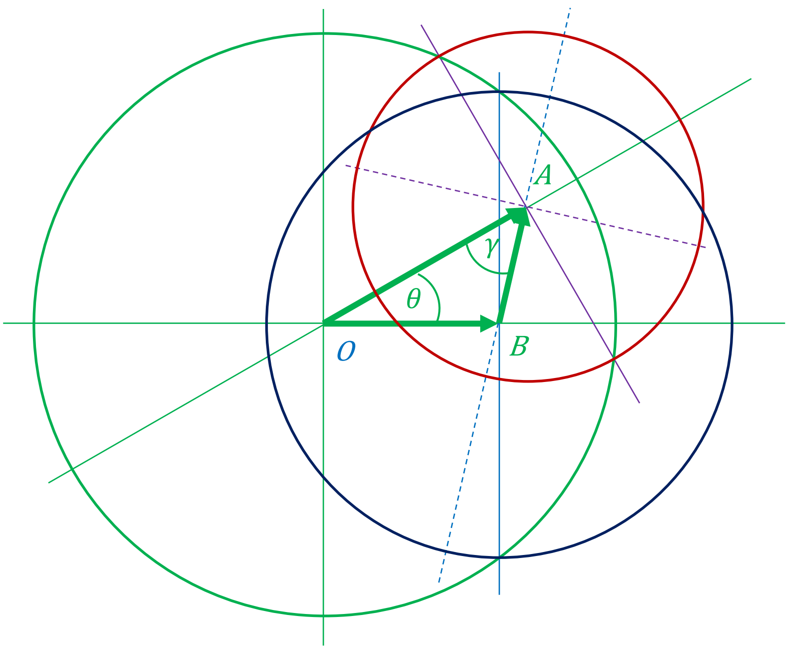

Fig.1

Fig.1 shows the picture at some moment of time *t* in the frame of clock_1. Due to time dilation, the reading of clock_2 in point *B* is $t_2 = t\sqrt{1-\beta^2}$, and the reading of clock_3 in point *A* is $t_1 = t\sqrt{1-\beta_1^2}$. The radii of the circles with the clocks as centers are proportional to the readings of those clocks, i.e., the times passed in the frames of each clock. Let's choose as the coefficient of proportionality the speed of light in free space, which is the same in all frames. The distance between clock_1 and clock_2 is $|OB| = \beta ct$, and between clock_1 and clock_3 - $|OA| = \beta_1 ct$. Thus, in Fig.1, the circles with centres in *O* and *A*, as well as the circles with centres in *O* and *B* represent the diagrams of space-time as defined in [1] and [2].

From $\Delta(OBA)$ the distance between clock_2 and clock_3 in the frame of clock_1 is:

$$|BA| = \sqrt{\beta_1^2 + \beta^2 - 2\beta_1\beta\cos\theta} \cdot ct = \beta_x ct \qquad (1)$$

It is obvious that, $\beta_x \neq \beta_2$ (i.e. the circles with centres in *B* and *A* do not make a diagram of space-time). But one can make a guess what the expression of β_2 should be like. Due to the constancy of the light velocity in free space, when $\beta = 1$ or $\beta_1 = 1$, then $\beta_2 = 1$. It is easy to notice that these conditions are met by the equation:

$$\beta_2 = \frac{\sqrt{\beta_1^2 + \beta^2 - 2\beta_1\beta\cos\theta}}{\sqrt{1 + \beta_1^2\beta^2 - 2\beta_1\beta\cos\theta}}. \qquad (2)$$

Here is Einstein's equation of addition of velocities, which, for the sake of comparison, can be written in the following form:

$$\beta_2 = \frac{\sqrt{\beta_1^2 + \beta^2 - 2\,\beta_1\beta\cos\theta - \beta_1^2\beta^2\sin^2\theta}}{\sqrt{1 + \beta_1^2\beta^2 - 2\beta_1\beta\cos\theta - \beta_1^2\beta^2\sin^2\theta}}. \qquad (2')$$

It is obvious, that Eqs. (2) and (2′) cannot both be true (except for one-dimensional cases, of course). So, let us see whether the guess made above was correct.

In the frame of clock_2, Eq. (1) becomes:

$$|BA|' = \beta_x' ct',$$

where $|BA|'$ is the value of $|BA|$, and β_x' is the value of β_x in that frame; $t'$is the time interval *t* in the frame of clock_1 measured from the frame of clock_2. Time dilation gives: $t' = \frac{t}{\sqrt{1-\beta^2}}$.

Let's go into the frame of clock_2 at the moment when clock_3 shows $t_1 = t\sqrt{1-\beta_1^2}$. Due to time dilation, the reading of clock_2 at that moment is:

$$t_2' = \frac{t\sqrt{1-\beta_1^2}}{\sqrt{1-\beta_2^2}}.$$

On the other hand, in this frame the distance covered by clock_3 relative clock_2 during this period of time is $|BA|' = \beta_2 ct_2'$. Thus, in the frame of clock_2:

$$\beta_2 \frac{\sqrt{1-\beta_1^2}}{\sqrt{1-\beta_2^2}} = \beta_x' \frac{1}{\sqrt{1-\beta^2}}. \qquad (3)$$

A similar reasoning in the frame of clock_3 at the moment when clock_2 shows $t_2 = t\sqrt{1-\beta^2}$, gives:

$$\beta_2 \frac{\sqrt{1-\beta^2}}{\sqrt{1-\beta_2^2}} = \beta_x'' \frac{1}{\sqrt{1-\beta_1^2}} \ , \tag{4}$$

where β_x'' is the value of β_x in the frame of clock_3.

It is clear from Eqs. (3) and (4) that the value of β_x does not depend on the frame. That means:

$$\beta_x' = \beta_x'' = \beta_x = \sqrt{\beta_1^2 + \beta^2 - 2\beta_1\beta\cos\theta}\,.$$

Thus,

$$\beta_2 \frac{\sqrt{1-\beta_1^2}}{\sqrt{1-\beta_2^2}} = \frac{\sqrt{\beta_1^2 + \beta^2 - 2\beta_1\beta\cos\theta}}{\sqrt{1-\beta^2}}\,, \tag{5}$$

which gives Eq. (2).

The difference between Eqs. (2) and (2′) is due to the fact that Eq. (2′), besides the ideas of light-speed constancy and time dilation, has to meet some other (disputable) requirements of the Lorentz transformation as well.

In favour of Eq. (2) speaks also its symmetry in regard to reciprocal values of the velocities. For instance, the substitution of $\beta_1$with $\frac{1}{\beta_1}$ gives the value of $\frac{1}{\beta_2}$:

$$\frac{1}{\beta_2} = \frac{\sqrt{\left(\frac{1}{\beta_1}\right)^2 + \beta^2 - 2\left(\frac{1}{\beta_1}\right)\beta\cos\theta}}{\sqrt{1 + \left(\frac{1}{\beta_1}\right)^2\beta^2 - 2\left(\frac{1}{\beta_1}\right)\beta\cos\theta}}\,.$$

That happens for a reason: according to de Broglie hypothesis, $\frac{1}{\beta_1}$ is the phase velocity of the "phase wave" (also called matter wave or wavefunction) corresponding to clock_1, while β_1 is the group velocity of the same wave. It is understandable that the transformation of both velocities in different inertial frames occurs by the same equation: a particle and its phase wave are two aspects of the same thing. In the case of Eq. (2′) all that is true in only one-dimensional cases.

It may be supposed that, Eq. (2), although derived for subluminal velocities, is usable for any superluminal velocities as well. It is a curious fact that the sum of two superluminal velocities is the same as that of their reciprocal subluminal velocities:

$$\beta = \frac{\sqrt{\left(\frac{1}{\beta_1}\right)^2 + \left(\frac{1}{\beta_2}\right)^2 + 2\left(\frac{1}{\beta_1}\right)\left(\frac{1}{\beta_2}\right)\cos\theta}}{\sqrt{1 + \left(\frac{1}{\beta_1}\right)^2\left(\frac{1}{\beta_2}\right)^2 + 2\left(\frac{1}{\beta_1}\right)\left(\frac{1}{\beta_2}\right)\cos\theta}} = \frac{\sqrt{\beta_1^2 + \beta_2^2 + 2\beta_1\beta_2\cos\theta}}{\sqrt{1 + \beta_1^2\beta_2^2 + 2\beta_1\beta_2\cos\theta}}\,.$$

Let us get an analogue of $\Delta(OBA)$ in the frame of clock_2.

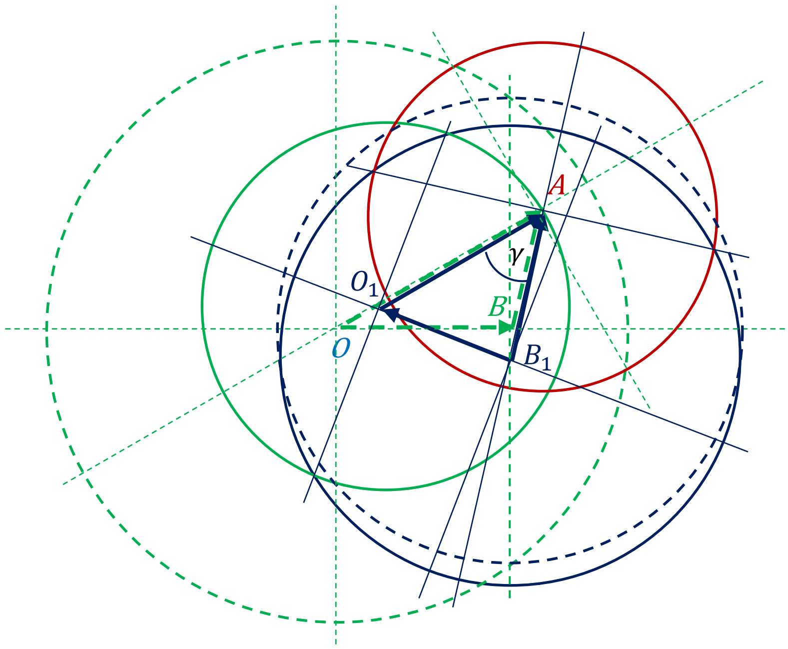

Fig.2

As Eq. (5) shows, in the frame of clock_2 that is at rest in point B_1, the side of the triangle corresponding to the velocity β_2 is $|BA|' = \frac{|BA|}{\sqrt{1-\beta^2}} = |B_1A|$ (Fig.2). Similarly, the side corresponding to the velocity β_1 is $|OA|\sqrt{1-\beta^2} = |O_1A|$. It is not hard to prove that the angle between the velocities β_1 and β_2 is the same in both frames: $\angle B_1AO_1 = \angle OAB = \gamma$. Thus, for the moment of time when clock_3 shows $t\sqrt{1-\beta_1^2}$, $\Delta(OBA)$ of the frame of clock_1 transforms into $\Delta(B_1O_1A)$ in the frame of clock_2.

In Fig.1 there are three distances between the clocks: $|OA|$, $|OB|$, and $|BA|$. The first two are "ordinary" distances: their division by the time passed in the frame of clock_1 gives the velocities of the respective clocks relative clock_1, and so they fit into the space-time diagrams (that are nothing more than geometric pictures of time dilation.) The third one is not an "ordinary" distance in those respects. The reason of differences between these types of distances is obvious: for the "ordinary" distances (at least) one clock is at rest in the observer's frame, and for the "unordinary" distance both clocks are moving. This can be checked easily by the observer's transfer into the frame of one of the moving clocks, as for instance is shown in Fig.2, where $|B_1O_1|$ and $|B_1A|$ are "ordinary" distances and $|O_1A|$ is an "unordinary" one in the frame of clock_2.

Actually, $|BA|$ is not a distance between the clocks, but a distance between two points of the frame of clock_1 in which those clocks are present at some instant of time *t*. (When using cosine rule for measuring distance $|BA|$, the observer succeeds only in measuring the distance between those points and not between moving points.)

Classical physics knows two ways of direct measuring of a distance between two points: using a ruler or a radar. With a ruler it is possible to measure the distance instantly, a radar requires some time. The theory of relativity does not allow the existence of an absolutely rigid body, which could be used as a ruler, since it would make possible instant transfer of information between its ends.

But, while discarding the idea of an absolutely rigid body, the theory of relativity leaves untouched the idea of a ruler, which assumes the existence of an instant distance or "a distance at a specific moment of time". The distance between two particles is considered as the distance between their positions in a given frame at the same time. For instance, the space-time interval between the events of arrivals of clock_2 at point B and of clock_3 at point A in the frame of clock_1 is the square of the instant distance between those points or the distance between two points of an absolutely rigid ruler. Also, the statement of the Lorentz transformation that the lengths must remain the same in normal to the velocity directions requires the existence of absolutely rigid rulers in the moving and resting frames or possibility of instant transfer of information between the end points of those lengths.

The absence of a ruler deprives physical sense to the idea of instant distance and it becomes an abstract or mathematical idea which is useful for geometric presentation, like (the lengths of) a radius vector, for instance.

Using a radar, the time necessary for a round trip of an EM signal to an object and back is counted. Its half value multiplied by the light velocity (which is only a choice of the unit of measurement) gives the value of the distance from the radar to the object. It is assumed that the distance from the radar to the object is exactly equal to the distance from the object to the radar. The measurement of a distance may be imaginary but theoretically possible one.

When speaking of the real distance between two points, one of them must be considered at rest with a radar (or source of EM waves) attached to it, which is the same as the transfer of the observer into the frame of that point. Any real solid body consists of numerous particles the distances between which determine the total lengths of the body. Each one of those particles may be considered an imaginary radar. Any measurement using a real solid body as a ruler may be imagined as a measurement using a radar in measuring the distance between the end points of the body.

Distinction must be made between a fixed distance between two points at rest and an instant distance. The latter requires no time for its measurement, and the former requires checking that the distance remains the same, which is a new measurement (or, rather, the measurement of a new distance) with a radar. The possibility of the existence of the constant distance between two fixed points does not violate the principle of impossibility of instant transfer of information.

Any physical distance is created or exists in time that cannot be less than the time necessary for its covering by light in free space (under "time of existence" the time necessary for light for covering that distance in free space is meant). Instead of the phrase "distance between two points at a specific moment of time" it may be used "past distance or future (i.e. expected) distance between two points at a specific moment of time". "Past distance" is a fact, "future distance" is a prediction, which may never come true.

When speaking of a distance between two points the first thing to do is to define which point is considered at rest. Next things are to assume from which point and at what instant of time is an EM wave issued (or reflected).

For instance, during the displacement of clock_2 in Fig.1, the distance $|OB|$ is created between clock_1 and clock_2 in the time interval of $\Delta t = \frac{|OB|}{v}$. That is not the distance at the instant of time *t* (there is no such thing), but the past distance from resting point *O* to moving point *B* that existed in time interval $\Delta t_1 = \frac{|OB|}{c} = \beta\Delta t$ immediately before *t* (it would take Δt_1time if its measurement were carried out) or the future distance from moving point *B* to resting point *O*, that will exist in the same time interval immediately after *t* if its measurement is carried out. The future distance from resting point *O* to moving point *B* at the same instant of time *t* is expected to be $|OB_1| = \frac{|OB|}{1-\beta}$. Its measurement would take $\Delta t_2 = \frac{\beta\Delta t}{1-\beta}$ time if conducted immediately after *t*. The phrase: "the distance from *O* to *B* is always exactly equal to the distance from *B* to *O*" now means: in the frame of point *O*, at any instant of time, the past distance from *O* to *B* is exactly equal to the future distance from *B* to *O* (which means the traveling of an EM wave from point *O* to point *B* takes the same time as would the traveling of the reflected wave take from point *B* back to point *O*.)

In Fig. 3, a rod of real solid material, *AB*, is moving in normal to its length direction with constant velocity *v*. *A* and *B* are the end (outmost) atoms of the rod. |*AB*| is the length of the rod at rest, i.e. the distance between its outmost atoms in the frame of the rod. This length exists in time $\Delta t = \frac{|AB|}{c}$. After the period of time $\Delta t' = \frac{\Delta t}{\sqrt{1-\beta^2}}$ the rod is displaced to $A'B'$ position. $|AA'| = v\Delta t'$. At this moment of time, in the frame at rest the length of the rod is $|AB'| = \frac{|AB|}{\sqrt{1-\beta^2}}$, and it is equal to the past distance between the end points of the rod. In the moving frame the length of the rod (the past distance between its outmost atoms) is $|A'B'| = |AB|$. The future distance between the end points of the rod at this moment of time is $|A'B''| = |B'A''| = |AB'|$in the resting frame, and $|A'B'| = |B'A'| = |AB|$ in the moving frame.

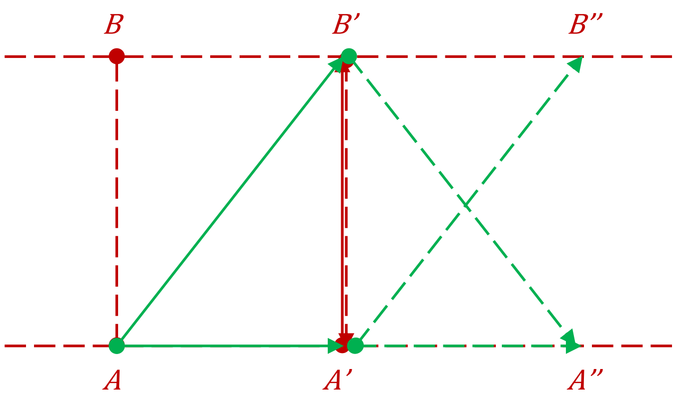

Fig.3

Any measurement using EM waves, in contrast to an imaginary ruler, implies some inaccuracy or uncertainty, the theoretical limit of which is determined by the lengths and period of the EM wave used in the process of measuring.

The existence of a ruler not only contradicts the principle of finite speed of information transfer but also makes the classical physics irreconcilable with quantization principle. By abandoning the idea of a ruler, which leaves a radar (or EM waves) the only tool for measuring distances, a quantum, being the smallest portion of an EM emission, mechanically becomes the smallest portion i.e. the smallest unit of measurement, as well as the size of uncertainty, of space-time as well.

References:

[1] Bregadze, T., Chachua S. “Diagram of space-time. Quantum equation of the Doppler shift.” viXra:2006.0060v2. 2023.
[2] Bregadze, T. "A property of sphere and relativity." No. EXT-2004-038. 2004.