APPROACHING EEG PATHOLOGICAL SPIKES IN TERMS OF SOLITONS

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ABSTRACT

A delicate balance between dissipative and nonlinear forces allows traveling waves termed solitons to preserve their shape and energy for long distances without steepen and flatten out. Solitons are so widespread that can generate both destructive waves in oceans' surface and noise-free message propagation in silica optic fibers. They are naturally observed or artificially produced in countless physical systems at very different coarse-grained scales, from solar winds to Bose-Einstein condensates. We hypothesize that some of the electric oscillations detectable by scalp electroencephalography (EEG) could be assessed in terms of solitons. A nervous spike must fulfill strict mathematical and physical requirements to be termed a soliton. They include the proper physical parameters like wave height, horizontal distance and unchanging shape, the appropriate nonlinear wave equations' solutions, the correct superposition between sinusoidal and non-sinusoidal waves. After a thorough analytical comparison with the EEG traces available in literature, we argue that solitons bear striking similarities with the electric activity recorded from medical conditions like epilepsies and encephalopathies. Emerging from the noisy background of the normal electric activity, high-amplitude, low frequency EEG soliton-like pathological waves with relatively uniform morphology and duration can be observed, characterized by repeated, stereotyped patterns propagating on the hemispheric surface of the brain over relatively large distances. Apart from the implications for the study of cognitive activities in the healthy brain, the theoretical possibility to treat pathological brain oscillations in terms of solitons has powerful operational implications, suggesting new therapeutical options to counteract their detrimental effects.

KEYWORDS: electrodes; critically ill patients; Juvenile absence epilepsy; nonlinear Schrödinger equation; Korteweg-de Vries equation; Peregrine soliton.

INTRODUCTION

Solitons are single, spatially localized waves that travel without changing their shape and size (Chabchoub et al., 2021). Due to a delicate balance between dissipative effects and nonlinear dynamics, solitons are coherent structures propagating in space and time at a constant velocity without attenuation, distortion or energy loss (Poznanski et al., 2017; Sharma et al., 2019). At least in an idealized framework, the stability and stationarity of soliton's analytical solutions is confirmed against non-integrable perturbations (Ye et al., 2020), so that frictional effects like viscosity do not cause solitons to decay over time (Minardi et al., 2010). The energy of solitons is carried adiabatically through the medium, i.e., without transferring heat or mass (Redor et al., 2019). Since the energy transfer to shorter wavelength regions is difficult, solitons reach a state of thermodynamic equilibrium that allows them to evolve for a long time without damping (Krylov and Yan'kov, 1980; Diebel et al., 2016). Solitons are naturally observed or artificially produced in different macro-, meso- and microscopic settings, playing key roles in the context of out of equilibrium nonlinear systems. They can be found in space plasmas, solar winds, planetary magnetospheres (Lakhina et al., 2021), marine and fluvial surface gravity waves, telegraph lines (Kayum et al, 2020), high-speed silica fiber-optic telecommunication, optical coherence tomography, photorefractive crystals (Bouchet et al., 2022), micromechanical systems (Manley et al., 2009), magnetic matter, superconductors (Kachulin et al., 2020), Bose-Einstein condensates (Strecker et al., 2002), etc. Their occurrence in ionic crystals, crystalline solids and nematic liquid crystals (Li et al., 2018) suggests a role for solitons in the physical properties of solids at high temperatures (Manley et al., 2009). It has been conjectured that soliton-like mechanisms could underlie also biological phenomena like electron/proton transport along proteins' helices (Ciblis and Cosic, 1997; Sinkala and Zachariah, 2006) as well as cardiac action potentials (Heimburg 2022).
Still, attempts have been provided to describe the microscopic nervous activity of the central nervous system in terms of solitons (Heimburg and Jackson, 2005). In has been conjectured that the standard Hodgkin-Huxley model of the neurons' action potential could be piezo electrically coupled with synchronized soliton pressure pulses generated inside the lipids of biological membranes (Andersen et al., 2009) and influenced by thermodynamic variables like voltage, temperature, pressure, chemical potentials (Heimburg 2022). According to this model, the traveling solitons are localized in the membrane, do not spread out in the surrounding medium, do not release heat and can modify the local membrane density and thickness (Heimburg and Jackson, 2006; Johnson and Winlow, 2018).
Here we take a different turn, suggesting another potential approach to assess the nervous electric spikes in the context of solitary waves. Looking at the central nervous system's nervous activity at the macroscopic scale, we argue that some of the brain electric oscillations detected by electroencephalography (EEG) could be mathematically and biophysically

approached in terms of solitons' motion. We contend that the electric oscillations spikes attained by scalp EEG monitoring techniques resemble in many respects solitary waves. Since an electric wave must fulfill strict mathematical and physical requirements in order to be termed a soliton, we will analytically examine the manifold physical features of solitary waves and will look for their feasible counterparts in the cortical electric activity measurable on the scalp.

NONLINEAR MEDIA AND SOLITONS

Solitons are naturally robust to perturbations as they travel with little dissipation over long timeframes (Rowley et al., 2022). The peculiar coherence of these highly localized wave packets can be attributed to the fact that the tendency for a wave pulse to disperse is exactly balanced by nonlinear effects occurring in the propagation medium (Minardi et al., 2010). While the linear and nonlinear effects acting alone would cause the wave to decay, in the right combination they lead to waves that persist stable and compact as they propagate, periodically returning to the initial state.
Numerous natural and artificial phenomena are characterized by the slight nonlinear dynamics required to generate solitons. Looking for a model of soliton-like EEG oscillations, we have at first to look for the possible presence of nonlinear dynamics in the central nervous system.

Nervous nonlinear medium. Solitons require a dissipative physical medium characterized by a certain degree of nonlinearity. In touch with this claim, nervous nonlinear dynamics have been ubiquitously detected in both experimental settings and ecologically valid conditions (Hernandez et al., 2023; Poikonen et al., 2023). Using brain network analysis, data-driven approaches and machine learning (Luo et al., 2022), various types of nonlinear dynamic behavior have been found in the wave series extracted from EEG data sets (Hernandez et al., 2023). Nonlinear dynamics in the central nervous system consist of phase transitions, branching processes, metastability, limit cycles, self-oscillations, non-stationarity, neuronal avalanches, chaotic behavior, collective phenomena with emergence of order (Meisel et al., 2013; Mahgoub and Qaraqe, 2023). Increasing evidence suggests that cortical neuronal networks operate near a critical state, where balanced activity patterns support optimal information processing, scale-free correlations and emergence of adaptive collective behavior (Meisel et al., 2013; Tiago et al., 2020).
Therefore, it can be stated that the nonlinear dynamics required by a nervous soliton-like model are widely diffused in the brain.

EQUATIONS FOR SOLITONS

In mathematical terms, solitons stand for one of the countless possible solutions for different types of nonlinear wave equations (Redor et al., 2019). In particular, the Korteweg-de Vries equation (KdVE), originally developed to describe weakly nonlinear wave propagation in shallow water, admits analytic solutions representing solitons. KdVE equation can be written as:

$$\partial_t \phi + \partial_x^3 \phi - 6\,\phi\,\partial_x \phi = 0$$

Where and are partial derivatives of x and t, *Φ* stands for the height displacement of the water surface from its equilibrium height. KdVE implies an equilibrium between dispersion $\partial_x^3 \phi$ and advection $\phi\,\partial_x \phi$, the latter standing for the particles transport provided by bulk motion and velocity of the fluid. KdVE can describe solitons systems in which the particles are driven by fluid pressure (Alquran et al., 2021) as well as soliton turbulence in ocean waves (Costa et al., 2014). The Benjamin–Bona–Mahony equation (BBME) is a KdVE variant for modeling long surface gravity waves of small amplitude, propagating uni-directionally in 1+1 dimensions. Yet, a particle-based fluid trajectory description of multi-soliton interactions can be achieved by the Kadomtsev-Petviashvili equation (KPE), i.e., a two-dimensional version of KdVE.

Apart from KdVE, BBME and KPE, other wave equations display soliton solutions. The most prominent is the one-dimensional nonlinear Schrödinger equation (NLSE), an example of integrable systems of nonlinear partial differential equations owning an infinite set of conservation laws. NLSE stands for a universal tool to model the dynamical evolution in dispersive media of weakly nonlinear, varying packets of quasi-monochromatic waves (Karjanto 2019). Among the manifold versions, the linearly damped and driven NLSE can be expressed in the dimensionless form:

$$\mathrm{i}u_t + \tfrac{1}{2}u_{xx} + |u|^2 u = f - \mathrm{i}\gamma u$$

Where $f = f(x,t)$ is the driving (or forcing) of the system, $-\mathrm{i}\gamma u$ is linear damping of strength $\gamma > 0$ (Fotopoulos et al., 2020). Depending on the chosen physical setting, different NLSE formulations provide different results according to the values of the dispersive nonlinear coefficients (Karjanto 2019). NLSE has a wide range of applications, including, e.g.,

the slow evolution of waves amplitude in shallow water of uniform depth, the irregular wave trains propagating in deep water, superconductivity devices, nonlinear optics, etc (Xia et al., 2021). Concerning Bose-Einstein condensates, the soliton's behavior can be described by a variant of the NLSE referred to as the Gross-Pitaevskii equation (GPE) (Rubino et al., 2012; Karjanto 2019). In this case, the velocities of single particles are determined by the phase of the wave-function. **Figure 1** illustrates a few examples of soliton solutions to various nonlinear wave equations.
Therefore, every attempt to look for hints of nervous solitons requires electric solitary waves to obey explicit nonlinear wave equations.

Solitons in the electrical domain and in EEG spikes. Solitons may be produced in the context of the electrical domain, both at the macro-, meso- and the micro-levels of observation (Ricketts and Ham, 2010; Roy and Misra, 2022). The occurrence of electrostatic solitons is ubiquitous in macroscopic contexts such as two-fluid astrophysical plasmas, planetary magnetospheres, lunar wake (Sharma et al., 2019; Roy and Misra, 2022). In solar wind, the interplanetary magnetic fields provide favorable conditions for solitons formation (Shah et al., 2020). In space plasmas, nonlinear electrostatic solitary waves can be interpreted in terms of ion- and electron- acoustic solitons propagating parallel to the ambient magnetic field (Lakhina et al., 2021). Concerning the micro-levels of observation, electromagnetic fields and saturable nonlinear dielectrics may influence solitary waves' shape and behavior (Bradley and De Angelis, 1996). Self-reinforcing magnetic solitons have been generated in Bose-Einstein condensates from atoms with different spins (Chai et al., 2020). In nematic and cholesteric liquid crystals, external electric fields drive collective reorientation of nematic molecules. This process generates solitons representing self-trapped "bullets" that move with high speed, perpendicularly to the electric field and to the initial alignment direction (Li et al., 2018; Shen and Dierking, 2020). The above-mentioned nonlinear wave equations have been widely used to cope with electric as well as external uniform magnetic fields, inside the discrete nonlinear electrical lattices used for electrical transmission (Yamgoué et al., 2018; Kieffer and Loss, 2020).
Once achieved that solitons can be generated in an electromagnetic milieu, the question is: could they occur in the hemispheric electric field detected by scalp EEG? The fact that NLSE may govern soliton propagation on spherical surfaces of electromagnetic fields traveling in coiled optical fibers (Körpınara et al., 2020) provides an indirect hint that solitons can be uncovered inside spherical electric fields (Lima et al., 2018). Further, it has been shown that electric or magnetic monopole pairs can interact with Coulombic fields equipped with the SO(3) rotations typical of the spherical surfaces (Wabnig et al., 2022).
Therefore, the nervous electric activity detected on the scalp could be assessed by using nonlinear wave equations with solitons solutions.

THE REMARKABLE PHYSICAL FEATURES OF SOLITONS

Solitons' solutions can be given analytically by using mathematical devices such as e.g., first-order Fourier analysis and statistical analysis (Chabchoub et al., 2013 Redor et al., 2019). Different solutions of KdVE, BBME, NLSE, KPE and GPE generate different types of propagating solitons characterized by distinct physical features and various wave shapes. The physical parameters to assess the presence and the dynamics of solitons are manifold. They include surface elevation, wave height, phase velocity (i.e., the rate at which the wave phase propagates), wavelength, spatiotemporal evolution of the normalized force as a function of a fixed or infinite horizontal distance, breathing period and maximum intensity, trajectories, velocities, acceleration, medium depth, material parameters (Poznanski et al., 2017; Xiao et al., 2021). In fluid media like seas or channels, also surface tension, density, gravity, channel height, water depth, carrier frequency and amplitude of the background wave (Chabchoub et al., 2013) conspire to generate the solitons' features.
Therefore, looking for the theoretical occurrence of solitons in EEG traces, the above-mentioned physical parameters must be kept into account.

Different shapes for different solitary waves. One of the most distinguished features of solitons is the shape, so different from the usual sine waves (**Figures 1A-G**). Solitary waves include W-shaped, bright, dark, kink-dark, singular, kink antikink solitons (Yamgoué et al., 2018; Al-Kalbani et al., 2023; Liu et al., 2024), cnoidal waves (**Figure 1A**), abnormally large waves (also termed "freak" or "rogue" waves) (Didenkulova 2021), "thick" or "top-table" solitons, algebraic solitons, solitons of different polarities (Didenkulova 2021), multi-solitons with doubly-localized peaks (Chabchoub et al., 2021), etc. Unlike "bright" solitons, which locally amplify the surrounding medium, "dark" solitons stand for a local decrease in the wave amplitude able to cause a depression (Chabchoub et al., 2013). Dark solitons on the surface of water might influence extreme events like tsunamis, such that an almost flat ocean surface may evolve into a singular, destructive rogue wave with high amplitude, as energy is conveyed in the central zone from the surrounding areas (Chabchoub et al., 2013) (**Figure 1B**). Contrary to the usual solitons that keep their profile unchanged during propagation, the Peregrine soliton displays a double spatio-temporal localization characterized by progressive, standardized and cyclic changes in shape (Peregrine 1983) (**Figure 1D**). Apart from the ones described above, other parameters may influence the large variety of solitons' shapes. These parameters include hyperbolic function solutions (Al-Kalbani et al., 2023), elliptic parameters for cnoidal waves, variable-coefficient functions, modulation instability gain (Liu et al., 2024), the Sine–Gordon expansion-method (Yamgoué et al., 2018), etc. Furthermore, periodic traveling-wave solutions may give

rise to “trains” of many solitons (**Figure 1F**). This occurs in solar wind: whereas weak shock waves produce only single soliton pulses, strong shocks produce trains of solitons (Shah et al., 2020) (**Figure 1G**).
Therefore, looking for the possible occurrence of solitary waves in EEG traces, it is mandatory to carefully evaluate whether the shape of the electric spikes matches the features of solitons.

Solitons in healthy EEG traces? The occurrence of solitons in EEG traces from healthy individuals has been conjectured. Sen (2022) generated action potential-like solitons resembling EEG-like surface waves on a synthetic brain connectome, in response to surface deformations produced by local pinch. Yet, part of the electric activity of the healthy human brain is characterized by nervous electric spikes that are reminiscent of solitons. For instance, the neuronal activity in the medial entorhinal cortex displays peculiar ultraslow periodic oscillations from tens of seconds to minutes (Gonzalo Cogno et al., 2024). Further, electrocorticographic studies suggest that alpha and theta oscillations generate traveling waves propagating in a posterior-to-anterior direction at ∼0.25–0.75 m/s (Zhang et al., 2018; Sato 2022). The last, but not the least, electrical recordings and spectral analysis from many brain regions exhibit neural oscillations at multiple spatial scales that, like solitons, are non-sinusoidal (Cole and Voytek, 2017).
Therefore, theoretical mathematical findings as well as implicit cues from neurophysiological studies suggest the possibility that at least part of the normal electric EEG activity might involve solitons.

Solitons in pathological EEG traces. Going through the neuroscientific literature, it is easy to notice that quite a few EEG electric spikes detectable in disease states look like solitons (Handa et al., 2021). Emerging from the noisy background of the normal electric activity, high-amplitude, low frequency EEG solitary waves of different shapes can be observed in various human diseases. A few cases are illustrated in **Figures 1H-M**. Distinct medical conditions, including critically ill patients, encephalitis and epilepsy, are characterized by EEG traces with features that remind solitons trains, i.e., repeated patterns or stereotyped activity with or without interchange interval, at approximately regular rate or intervals (Gélisse et al., 2023). Just as solitons, pathological EEG waveforms may display relatively uniform morphology and duration. Like solitons, pathological EEG oscillations can travel on the hemispheric surface of the brain over relatively large distances and, like solitons, neither tend to flatten out, nor steepen and topple over. Further, some pathological EEG waves display double spatio-temporal localization with cyclic changes in shape just as Peregrine solitons.

Summarizing, the occurrence in EEG traces of features and shapes corresponding to various types of solitary waves suggests that some pathological brain electric activities might represent the biological counterpart of physical solitons.

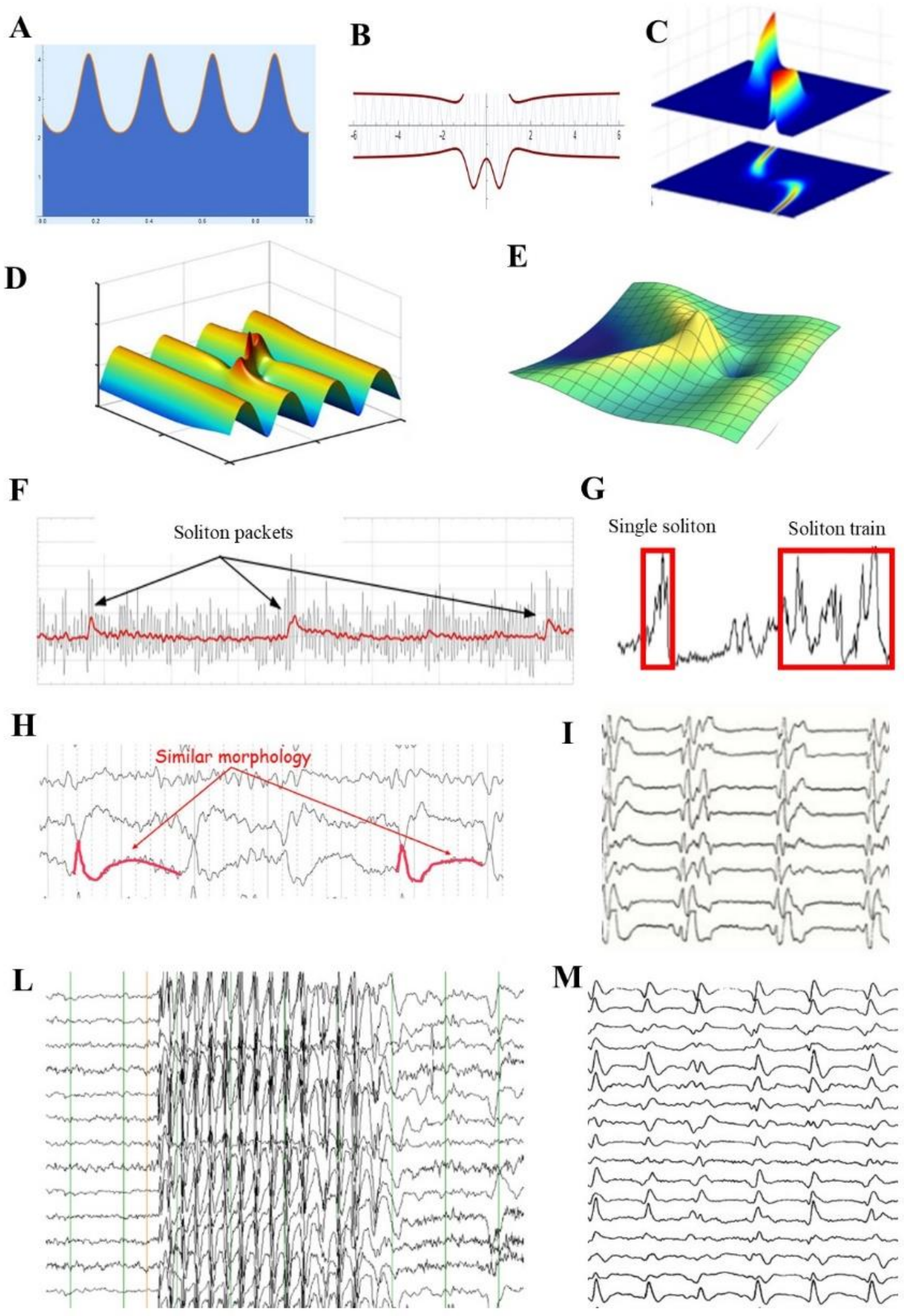

Figure 1. Similarities between solitons (**Figures 1A-F**) and pathological EEG patterns (**Figures G-J1**)
Figures 1A-E. Features of different types of solitons. **Figure 1A**. A cnoidal wave is a periodic traveling-wave solution of KdVE. It refers to surface waves whose wavelength is large compared to the water depth. Modified from https://demonstrations.wolfram.com/CnoidalWavesFromKortewegDeVriesEquation/ **Figure 1B**. Formation of a rogue wave in the deep ocean, modeled using LNSE. Modified from https://demonstrations.wolfram.com/RogueOceanWaves/ **Figure 1C**. Top figure: example of dynamical behavior of an optical soliton with spatio-temporal dispersion, based on variable-coefficient NLSE. Bottom Figure: Time evolution of the soliton position x(t) interpreted in terms of wave

streamlines obeying NLSE. Modified from Liu et al. (2024). **Figure 1D**. Peregrine soliton solution based on a periodic-wave background generated by a NLSE variant. Modified from Ye et al. (2020). **Figure 1E**. Soliton solution of the integrable focusing NLSE with spectral data consisting of a pair of conjugate poles of order 2n. Modified from Bilman and Buckingham (2019).
Figures 1F-G. Features of solitons trains. **Figure 1F**. Soliton trains (packets) in shallow water wind waves. The wave train (black) is achieved by low pass filtering of the data (red). Modified from Costa et al. (2014). **Figure 1G**. Single soliton and soliton trains detected for a month in the solar wind's pressure. Modified from Shah et al. (2020).
Figures 1H-M. Pathological EEG patterns resembling trains of solitons. **Figure 2H**. Herpesvirus-6 limbic encephalitis. Modified from Gélisse et al. (2023). **Figure 2I**. Typical "metronomic" delta burst pattern with almost flat amplitude during burst intervals, recorded during advanced stages of subacute sclerosing panencephalitis. Modified from Gadoth (2011). **Figure 2L**. Juvenile absence epilepsy. Modified from: https://www.epilepsy.com/what-is-epilepsy/syndromes/juvenile-absence-epilepsy **Figure 2M**. Creutzfeldt-Jacob disease. Modified from: https://www.eurocjd.ed.ac.uk/images/typical-eeg-sporadic-cjd

DYNAMICS OF SOLITARY WAVES

Solitons travel long distances with no need of leading or trailing waves. The soliton's wave crest can be much higher than the surrounding waves (Costa et al., 2014). Since the velocity depends on the wave's amplitude and height, a soliton is slower than its carrier waves. In addition, the velocity of solitons can be increased by an acceleration induced by an external water flow. When the distance along the surface approaches zero, a sharp rise of the wave crest can be detected above the rest of the deep fluid medium. The deeper the water, the more negative the nonlinearity parameter and the higher the possibility to generate rogue solitons, characterized by large-amplitude oscillations with height two times larger than the wave train (Xia et al., 2021). When the soliton disappears, a defocusing process takes place in which the instantaneous frequencies in front become lower than those behind it (Xia et al., 2021).

Interactions between solitons. Time reversible interactions among multiple solitons, carriers and media occur in both natural and artificial settings. When a carrier wave enters the soliton, its amplitude drops to zero; when it exits, its amplitude returns to the previous value (Chabchoub et al., 2013). Two colliding solitons penetrate through each other, exchange their amplitude and velocity and emerge fully intact as the exact pulses that entered the collision (Poznanski et al., 2017) (**Figure 2A**). At first, the two waves would align to reproduce temporarily the initial sine wave, before separating again and repeating the same cycle. The higher amplitude wave is narrower and faster than the wave with the minor amplitude. Heart-and bell-shaped-cusp optical solitons can be produced by two waves superposition (see the Figures in Alquran et al., 2021). Though 90% of the soliton collisions occur between pairs of solitons, the remnant collisions involve multiple solitons (Didenkulova 2021). Multi-spot soliton packets bound together with the appropriate relative phase can be achieved in saturable Kerr nonlinear media by simply shifting the refractive index with harmonic transverse modulations (Kartashov et al., 2004).
Therefore, theoretical multiple soliton-like waves detectable in EEG traces are required to preserve their shape after collision.

Dynamics of solitary waves. Viable examples of interactions between solitons can be found in pathological EEG traces. For instance,

a) **Figure 2C** illustrates the coexistence of generalized normal and focal pathological discharges that are spatially separated.
b) **Figure 2D** illustrates the coexistence of generalized and focal epileptiform discharges that are spatially mixed.

Hence, it is theoretically possible that EEG waves could be produced by the transient superposition of diverse solitary waves from different brain areas. In touch with this claim, models of soliton conduction in microscopic branchlets with polarized microstructure suggest that linear superposition of two oppositely directed traveling waves might occur (Poznanski et al., 2017). The resulting wave made of superimposed solitons can be resolved at late times into separate single solitons, making it possible to uncover the cortical source of every one of the solitary oscillations (Xia et al. 2021). Therefore, clues from the physical literature and from pathological EEG traces let us argue that superposition of solitons might occur in the electrical fields produced by the cortical activity.

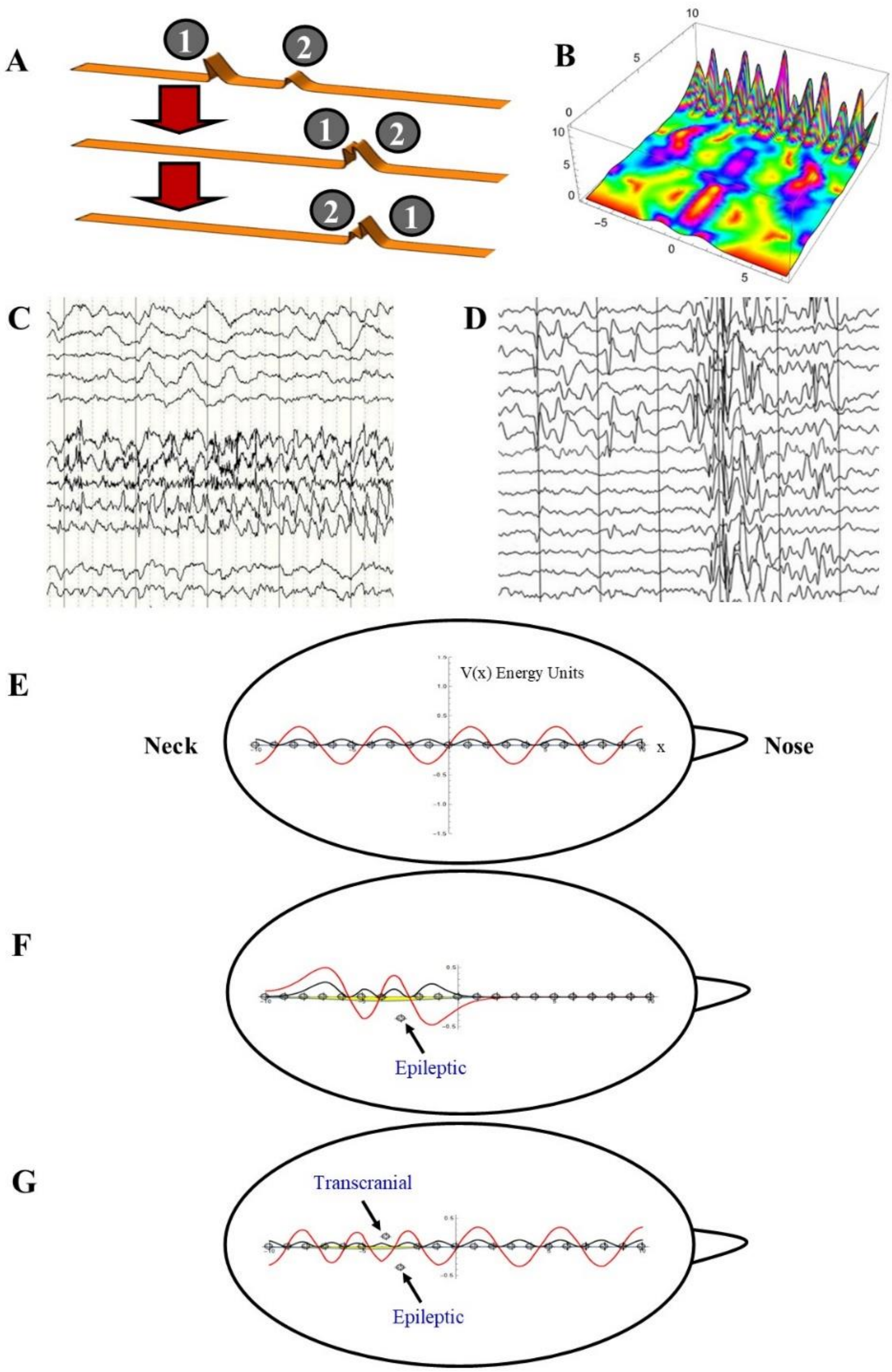

Figure 2A. Weakly nonlinear two-solitons propagation in shallow water described by KdVE. Modified from https://demonstrations.wolfram.com/SolitonsFromTheKortewegDeVriesEquation/
Figure 2B. NLSE-derived soliton profile perturbed by a periodic potential. Modified from https://demonstrations.wolfram.com/SolutionOfANonlinearSchroedingerEquation/
Figures 2C-D. Pathological EEG patterns resembling groups of solitons and solitons trains. **Figure 2C**. EEG snapshot showing a right temporal focal seizure. Modified from Chari et al. (2019). **Figure 2D**. Coexistence of focal and generalized epileptiform discharges in a 7-year-old boy with benign childhood epilepsy with occipital paroxysms (Fernández-Torre 2010).
Figures 2E-G. Schrödinger wavefunctions in a continuously varying potential field. The red oscillations stand for the wavefunctions, the black ones for the probability density. The plot of the Schrödinger equation is embedded in a head

contour. The plot represents an EEG wave spreading across the scalp electrodes between the neck and the nose. Solutions of the one-dimensional Schrödinger equation with quantum number 10 are provided in a potential-energy field V(x) which can be varied by moving a series of locators (the little circles). Even small changes in position of one or more locators modify the potential energy field, generating waves with fully different height, frequency, shape, periodicity. In the example, when the elementary periodic wave (**Fig. 2E**) is disrupted by a change in position of one locator (**Fig. 2F**), it can be restored by changing the position of another locator (**Fig. 2G**). In the context of a human disease like epilepsy, this means that the healthy oscillation (**Fig. 2E**) disrupted by epileptic foci (**Fig. 2F**) could be restored by the proper transcranial current (**Fig. 2G**). Modified from: https://demonstrations.wolfram.com/SchroedingerWavefunctionsInAContinuouslyVaryingPotential/

SOLITONS' GENERATION

Due to their inability to form spontaneously from noisy and/or dissipative systems, solitons' generation relies on the occurrence of external perturbations (Rowley et al., 2022). Artificial solitons can be engineered via different techniques in a wide range of media. A steady train of water waves with a well-defined shape can be produced in a shallow water regime by driving a piston at one end of the tank. The solitary waves may either ricochet back and forth, or to be absorbed at the other end with a porous material acting as an artificial beach (Chabchoub et al., 2013; Redor et al., 2019). In contrast to homogeneous media in which stable multi-peaked solitons do not exist, localized "lumps" of light can be produced in slightly nonlinear diffractive media such as glass, photonic lattices and optical fibers (Kartashov et al., 2004; Rubino et al., 2012) by manipulating the light in miniaturized semiconductor chips (Genevet et al., 2008). In nonlinear silica optical fiber systems, the spreading due to dispersion and the intensity-dependent refraction due to nonlinearity are extremely balanced to generate solitons traveling in the waveguide for long distances (Liu et al., 2024). In this case, the production of compact solitons beams is driven by femtosecond pulses oriented towards the transverse dimension (Minardi et al., 2010), allowing ultrafast switching rates of several terahertz. Yet, solitons made of repulsively interacting atoms have been produced in microcavities and Bose-Einstein condensates (Fritsch et al., 2022). Every soliton generated at each boundary of the box propagates in a uniform background density, colliding with one another in a way that can be controlled (Halperin and Bohn, 2020).
Therefore, a model that predicts soliton propagation in the mammalian central nervous system must carefully describe how solitary waves could be produced in the electromagnetic milieu produced by the neural tissue.

Generation of neuronal solitons. Once achieved in the previous Chapters that surface solitons might emerge from (and may interact with) electromagnetic fields like the ones detected on the scalp, a question arises: how could solitary waves be physically produced in the nervous tissue? Different theoretical possibilities might be considered:

a) The nervous solitons detectable on the scalp are electric waves produced by the underlying neuronal tissue. In this case, solitons are produced INSIDE the electric fields.
b) The nervous solitons detectable on the scalp are pressure waves that only later become electric currents. In this case, solitons are produced OUTSIDE the electric fields. This can be accomplished in several ways, including the piezoelectric effect, e.g., a simple electromechanical interaction between the mechanical stress and the electrical fields.

Another distinction could be made concerning the anatomical source of these theoretical nervous solitons:

c) The nervous solitons can be naturally produced in the healthy brain.
d) The nervous solitons can be produced in the pathological brain, e.g., by the spared tissue adjacent to injured areas.

In sum, further studies are needed to elucidate the possible mechanisms underlying solitons generation in the central nervous system.

CONCLUSIONS

We suggested that the central nervous system's nervous macroscopic activity could be assessed in terms of solitons. Particularly, we argued that the brain electric oscillations detected by scalp EEG monitoring could be described as solitary waves. Going through many examples, we claimed that solitons resemble in many respects the cortical electric spikes.

The main questions are: what does the potential relationship between solitons and cortical electric field bring on the table? What are the beneficial outcomes, operational qualities and methodological advantages? Solitons can generate both destructive waves in the ocean and noise-free messages in optic fibers. They have the potential to achieve powerful and reliable information-processing in different fields, including telecommunications networks in electronics, optical

computing and signal control (Gaur et al., 2024), as well as practical applications in ultra-sharp pulse/edge generation and high-speed metrology (Ricketts and Ham, 2010). Solitons stand for bits of information that are robust to perturbations and noise, highly stable over long timeframes and able to recover spontaneously even after complete disruption (Rowley et al., 2022). Solitons in Bose-Einstein condensates could function as qubits with long lifetimes of the order of a few seconds (Shaukat et al., 2017). While isolated solitons can only encode single bits, two or more coupled solitons could boost transmission capacity by encoding more than one bit of information at a time (Krupa et al., 2017). Also, charged soliton pulses could be used as a substrate for memory storage, since they carry details of the input deformation parameters (Sen 2022). Further, multidimensional solitons in nematic and cholesteric liquid crystals can be used as vehicles for two-dimensional delivery of micro-cargos (Shen and Dierking, 2020). Therefore, the theoretical occurrence of EEG soliton-like waves in the brain might provide benefits by a physiological and evolutionary standpoint.
The functional contributions of solitary waves to the brain activity in healthy individuals could be manifold. Solitons would allow electric noise-free transport of messages without dissipation. It has been suggested that soliton pulses and surface waveforms might cooperate to identify incoming signals, help focus attention and retrieve memory (Sen 2022). Traveling waves could be behaviorally relevant in supporting brain connectivity and correlation between task events (Zhang et al., 2018). Ultraslow sequences of solitary traveling oscillations might couple circuits across extended time scales, serving as a template for new sequence formation during navigation (Gonzalo Cogno et al., 2024). Since the mutual attraction of multiple solitons traveling in optical fibers can generate bound states (Krupa et al., 2017), we speculate that solitary waves could help phase synchronization of cortical networks (Sato 2022). Solitons' production could compensate maladjustments of one variable by fine-tuning another one, providing a theoretical explanation for the effects of anesthetics (Heimburg and Jackson, 2006; Andersen et al., 2009; Heimburg 2022). Further, cortical traveling waves might contribute to hierarchical organization of each brain region by unidirectional communication between arbitrarily paired areas (Sato 2022).
We argue that potential detrimental solitons correlated with medical conditions might be countered and neutralized in various ways. Since solitons are produced by a delicate balance between nonlinear and dissipative forces like viscosity, a pharmacologically induced modification of the brain viscosity might lead to derangement of pathological solitons. Further, solitons display peculiar physical features like non-sinusoidal waves that could be theoretically manipulated by focal electrical brain stimulation induced by external magnetic fields, namely, by transcranial magnetic stimulation (TMS) (Chung et al. 2019; Kohli and Casson, 2019). In pathological states, the ordinary EEG oscillations deviate from the norm. In this case, our model of nervous solitons based on Schrodinger-like equations predicts that pathological spikes could be correlated with changes in the anatomical/functional location of the electric sources. **Figures 2E-G** provides a proof-of-concept example of brain EEG waves treated in terms of the Schrodinger equation. In the healthy brain, the energy sources are arranged in a regular order along the straight line connecting two scalp electrodes (**Figure 2E**). This generates a regular wave with well-defined periodicity and rhythm. When a disease occurs, e.g., an epileptic focus, one of the energy sources turns off its location on the straight line (**Figure 2F**). This divergence perturbs the wave profile: even slight changes in the location of energy sources can vary the potential, leading to waves with fully different shapes. To restore the healthy wave, an appropriately located external current could be given via TMS to counteract the effects of the displaced energy source (**Figure 2G**). Note that the profile wave in **Figure 3E-G** is drawn from the classical Schrodinger equation that holds just for microscopic systems subject to the laws of quantum mechanics. Nevertheless, the example can be generalized to macroscopic systems like EEG traces when considering NLSE instead of the classical Schrodinger equation.
Therefore, artificial manipulation of harmful solitons in the nervous tissue can be achieved by superimposing TMS external currents to counteract the pathological electric waves. This might have consequences for medical treatment. For instance, the proper external current might contribute to modify the harming solitons produced during epileptic discharge, bringing healing to the convulsive symptoms.

The possible relationships between solitary waves and brain activities paves the way to novel lines of research. We focused on the spikes extracted from EEG traces, but the universe of the discourse could be extended also to the oscillations detected by other experimental procedures like fMRI, local field potential electrophysiology, etc. When nonlinear effects occur in silica optical fibers, the momentum and the medium dispersion might coincide. This generates a "phase matching" that allows solitons to emit a special kind of low-intensity radiation, called resonant radiation (Rubino et al., 2012). It could be speculated that this radiation could be found also in EEG traces. The property of solitons to interact with each other in a nonlinear medium suggests that they could be also analyzed in terms of elastically colliding particles that do not interact each other and do not perturb the wave (Didenkulova 2021). When the soliton propagates through the medium, the particles are accelerated until the wave has left the region. After the interaction, the particles become motionless again, but their positions have shifted. The particles' single trajectories are influenced only by wave velocity, while their movement is governed by the current flow. This suggest that solitons at the interface of two biological fluid mediums might be useful to move or mix particles, contributing to the intra- and extra-cellular transport mechanisms.

In sum, research concerning relationships and interactions between solitons and the electric waves produced by the central nervous system needs still to be focused, but the premises are so promising that the field would be worth exploring.

DECLARATIONS

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Consent for publication: The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal, and has not been previously published.
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REFERENCES

1) N. Akhmediev, A. Ankiewicz, and M. Taki, "Waves That Appear from Nowhere and Disappear without a Trace," Phys. Lett. A, 373(6), 2009 pp. 675\[Dash]678.
2) Kaltham K. Al-Kalbani, Khalil S. Al-Ghafri ,Edamana V. Krishnan, Anjan Biswas. 2023. Optical Solitons and Modulation Instability Analysis with Lakshmanan–Porsezian–Daniel Model Having Parabolic Law of Self-Phase Modulation. Mathematics 2023, 11(11), 2471; https://doi.org/10.3390/math11112471.
3) Alquran, M., Jaradat, I., Yusuf, A. et al. Heart-cusp and bell-shaped-cusp optical solitons for an extended two-mode version of the complex Hirota model: application in optics. Opt Quant Electron 53, 26 (2021). https://doi.org/10.1007/s11082-020-02674-1
4) Andersen Søren S.L.; Andrew D. Jackson, Thomas Heimburg. Towards a thermodynamic theory of nerve pulse propagation. Progress in Neurobiology. Volume 88, Issue 2, June 2009, Pages 104-113. https://doi.org/10.1016/j.pneurobio.2009.03.002
5) Bilman, D., Buckingham, R. 2019. Large-Order Asymptotics for Multiple-Pole Solitons of the Focusing Nonlinear Schrödinger Equation. J Nonlinear Sci 29, 2185–2229 (2019). https://doi.org/10.1007/s00332-019-09542-7
6) Thomas Bouchet, Nicolas Marsal, Marc Sciamanna & Delphine Wolfersberger. 2022. Two dimensional Airy beam soliton. Scientific Reports volume 12, Article number: 9064.
7) Piero J. Bradley, Costantino De Angelis. 1996. Soliton dynamics and surface waves at the interface between saturable nonlinear dielectrics. Optics Communications. Volume 130, Issues 1–3, 15, Pages 205-218. https://doi.org/10.1016/0030-4018(96)00320-3
8) A. Chabchoub, O. Kimmoun, H. Branger, N. Hoffmann, D. Proment, M. Onorato, and N. Akhmediev. 2013. Experimental Observation of Dark Solitons on the Surface of Water. Phys. Rev. Lett. 110, 124101.
9) Amin Chabchoub, Alexey Slunyaev, Norbert Hoffmann, Frederic Dias, Bertrand Kibler, et al. 2021. The Peregrine Breather on the Zero-Background Limit as the Two-Soliton Degenerate Solution: An Experimental Study. Front. Phys., 25 August 2021. Sec. Interdisciplinary Physics. Volume 9 - 2021 | https://doi.org/10.3389/fphy.2021.633549
10) Xiao Chai, Di Lao, Kazuya Fujimoto, and Chandra Raman. 2020. Magnetic soliton: From two to three components with SO(3) symmetry. Phys. Rev. Research 3, L012003 – Published 6 January 2021
11) Chari, G., Yadav, K., Nishijima, D. et al. Improving the ability of ED physicians to identify subclinical/electrographic seizures on EEG after a brief training module. Int J Emerg Med 12, 11 (2019). https://doi.org/10.1186/s12245-019-0228-9
12) Chung SW, Sullivan C, Rogasch NC, Hoy KE, Bailey N Wetal. (2019) The effects of individualized intermittent theta burst stimulation in the prefrontal cortex: a TMS-EEG study. Hum Brain Mapp 40(2):608–627. https://doi.org/10.1002/hbm.24398
13) Peter Ciblis, Irena Cosic. 1997. The Possibility of Soliton/Exciton Transfer in Proteins. Journal of Theoretical Biology,Volume 184, Issue 3, 7 February 1997, Pages 331-338

14) Scott R. Cole, Bradley Voytek. 2017. Brain Oscillations and the Importance of Waveform Shape. Trends in Cognitive Sciences. Volume 21, Issue 2, February 2017, Pages 137-149. https://doi.org/10.1016/j.tics.2016.12.008
15) Andrea Costa, Alfred R. Osborne, Donald T. Resio, Silvia Alessio, Elisabetta Chrivì, Enrica Saggese, Katinka Bellomo, and Chuck E. Long. 2014. Soliton Turbulence in Shallow Water Ocean Surface Waves. Phys. Rev. Lett. 113, 108501 – Published 2 September 2014
16) Didenkulova, E.: 2021. Soliton turbulence in weakly nonlinear and weakly dispersive media, EGU General Assembly 2021, online, 19–30 Apr 2021, EGU21-552, https://doi.org/10.5194/egusphere-egu21-552, 2021.
17) F. Diebel, D. V. Timotijević, D. M. Jović Savić, C. Denz. 2016. Soliton formation by interacting Airy beams. Proceedings Volume 9764, Complex Light and Optical Forces X; 97640S (2016) https://doi.org/10.1117/12.2213527
18) E. Fermi, J. R. Pasta, and S. Ulam, "Studies of nonlinear problems. I.," Report LA-1940, Los Alamos Scientific Laboratory (1955)
19) José L Fernández-Torre. 2010. First Chapter: Interictal EEG. In book: Atlas of Epilepsies. Editors: Panayioutopoulos C Publisher: Springer-Verlag
20) Fritsch, Amilson R; et al. 2022. Dark solitons in Bose–Einstein condensates: a dataset for many-body physics research. Mach. Learn.: Sci. Technol. 3 047001
21) Deependra Singh Gaur, Ankit Purohit a, Akhilesh Kumar Mishra. 2024. Trapping and guiding soliton through Airy pulse. Optics Communications, Volume 556, 1 April 2024, 130300. https://doi.org/10.1016/j.optcom.2024.130300
22) G. Fotopoulos, D.J. Frantzeskakis, N.I. Karachalios, P.G. Kevrekidis, V. Koukouloyannis, K. Vetas. 2020. Extreme wave events for a nonlinear Schrödinger equation with linear damping and Gaussian driving. Communications in Nonlinear Science and Numerical Simulation. Volume 82, March 2020, 105058
23) Natan Gadoth. 2011. Subacute Sclerosing Pan-Encephalitis (SSPE) – Past and Present.. Pathogenesis of Encephalitis. Edited by Daisuke Hayasaka. DOI: 10.5772/20565
24) Gélisse, P., Tatum, W.O., Crespel, A. et al. 2023. Determining ICU EEG periodic patterns and why it matters. J Neurol 270, 4744–4752 (2023). https://doi.org/10.1007/s00415-023-11835-7
25) P. Genevet, S. Barland, M. Giudici, and J. R. Tredicce. 2008. Cavity Soliton Laser Based on Mutually Coupled Semiconductor Microresonators. Phys. Rev. Lett. 101, 123905.
26) Gonzalo Cogno S., Obenhaus, H.A., Lautrup, A. et al. 2024. Minute-scale oscillatory sequences in medial entorhinal cortex. Nature 625, 338–344 (2024). https://doi.org/10.1038/s41586-023-06864-1
27) E. J. Halperin and J. L. Bohn. 2020. Quench-produced solitons in a box-trapped Bose-Einstein condensate. Phys. Rev. Research 2, 043256.
28) Palak Handa, Monika Mathur, Nidhi Goel. 2021. Open and free EEG datasets for epilepsy diagnosis. arXiv:2108.01030
29) Heimburg Thomas; Andrew D. Jackson. 2005. On soliton propagation in biomembranes and nerves. Proc Natl Acad Sci U S A. 2005 Jul 12; 102(28): 9790–9795. Published online 2005 Jul 1. doi: 10.1073/pnas.0503823102
30) Heimburg, Thomas; Andrew D. Jackson. 2006. On the action potential as a propagating density pulse and the role of anesthetics. arXiv:physics/0610117
31) Heimburg T. 2022. The thermodynamic soliton theory of the nervous impulse and possible medical implications. Progress in Biophysics and Molecular Biology. Volume 173, September 2022, Pages 24-35. https://doi.org/10.1016/j.pbiomolbio.2022.05.007
32) Christopher Ivan Hernandez, Shaida Kargarnovin, Sara Hejazi Waldemar Karwowski. 2023. Examining electroencephalogram signatures of people with multiple sclerosis using a nonlinear dynamics approach: a systematic review and bibliographic analysis. Front. Comput. Neurosci., 29 June 2023. Volume 17 - 2023 | https://doi.org/10.3389/fncom.2023.1207067
33) J. Hietarinta, "Introduction to the Hirota Bilinear Method," arXiv, 1997 pp. 1\[Dash]10.
34) R. Hirota, "Exact Solution of the Korteweg\[Dash]de Vries Equation for Multiple Collisions of Solitons," Physical Review Letters, 27(18), 1971 pp. 1192\[Dash]1194. doi:10.1103/PhysRevLett.27.1192.
35) Dmitry Kachulin, Alexander Dyachenko, Vladimir Zakharov. 2020. Soliton Turbulence in Approximate and Exact Models for Deep Water Waves. Fluids 2020, 5(2), 67; https://doi.org/10.3390/fluids5020067
36) N. Karjanto. 2019. The nonlinear Schrödinger equation: A mathematical model with its wide-ranging applications. arXiv:1912.10683
37) Yaroslav V. Kartashov, Victor A. Vysloukh, Lluis Torner. 2004. Soliton trains in photonic lattices. Optics Express, Vol. 12, Issue 13, pp. 2831-2837 (2004) •https://doi.org/10.1364/OPEX.12.002831
38) Abdul Kayum, Shamim Ara, Hemonta Kumar Barman, M. Ali Akbar. 2020. Soliton solutions to voltage analysis in nonlinear electrical transmission lines and electric signals in telegraph lines. Results in Physics. Volume 18, September 2020, 103269. https://doi.org/10.1016/j.rinp.2020.103269
39) Kohli S, Casson A (2019) Machine learning validation of EEG tACS artefact removal. J Neural Eng. https://doi.org/10.1088/1741-2552/ab58a3
40) T.F. Kieffer, M.Loss. 2020. Non-linear Schrödinger Equation in a uniform magnetic field. arXiv:2010.12961

41) Y.S. Kivshar and B. Luther-Davies, "Dark Optical Solitons: Physics and Applications," Physics Reports, 298, 1998 pp. 81\[Dash]197.
42) T. Körpınara, R. Cem Demirkola, Z. Körpınara. 2020. Soliton propagation of electromagnetic field vectors of polarized light ray traveling along with coiled optical fiber on the unit 2-sphereS^2. Rev. mex. fis. vol.65 no.6. https://doi.org/10.31349/revmexfis.65.626
43) Katarzyna Krupa, K. Nithyanandan, Ugo Andral, Patrice Tchofo-Dinda, and Philippe Grelu. 2017. Real-Time Observation of Internal Motion within Ultrafast Dissipative Optical Soliton Molecules. Phys. Rev. Lett. 118, 243901 – Published 13 June 2017
44) S. F. Krylov and V. V. Yan'kov. 1980. The role of solitons in strong turbulence. Zh. Eksp. Teor. Fu. 79,8246
45) Gurbax Singh Lakhina, Satyavir Singh, Rajith Rubia. Selvaraj Devanandhan. 2021. Electrostatic Solitary Structures in Space Plasmas: Soliton Perspective. Plasma 2021, 4(4), 681-731; https://doi.org/10.3390/plasma4040035
46) Mahgoub, A., Qaraqe, M. Automatic detection of ictal activity in EEG using synchronization and chaos-based attributes. Med Biol Eng Comput 61, 3387–3396 (2023). https://doi.org/10.1007/s11517-023-02916-w
47) Christian Meisel, Eckehard Olbrich, Oren Shriki and Peter Achermann. 2013. Fading Signatures of Critical Brain Dynamics during Sustained Wakefulness in Humans. Journal of Neuroscience 30 October 2013, 33 (44) 17363-17372; DOI: https://doi.org/10.1523/JNEUROSCI.1516-13.2013
48) Bing-Xiang Li, Volodymyr Borshch, Rui-Lin Xiao, Sathyanarayana Paladugu, Taras Turiv, et al. 2018. Electrically driven three-dimensional solitary waves as director bullets in nematic liquid crystals. Nat Commun. 2018; 9: 2912. doi: 10.1038/s41467-018-05101-y
49) Lima, F.M.S. 2018. What Exactly is the Electric Field at the Surface of a Charged Conducting Sphere?. Reson 23, 1215–1223. https://doi.org/10.1007/s12045-018-0731-y
50) Fei-Fei Liu, Xing Lü, Jian-Ping Wang. 2024. Dynamical behavior and modulation instability of optical solitons with spatio-temporal dispersion. Physics Letters A, Volume 496, 5 February 2024, 129317. https://doi.org/10.1016/j.physleta.2024.129317
51) Luo, C., Li, F., Li, P. et al. 2022. A survey of brain network analysis by electroencephalographic signals. Cogn Neurodyn 16, 17–41 (2022). https://doi.org/10.1007/s11571-021-09689-8
52) Weida Xia, Yuxiang Ma, Guohai Dong, Jie Zhang, Xiaozhou Ma. 2021. Emergence of Solitons from Irregular Waves in Deep Water. J. Mar. Sci. Eng. 2021, 9(12), 1369; https://doi.org/10.3390/jmse9121369
53) Yan Xiao, Pengxiang Wang, Jing Zhang, Teng Guo, Ru Gao, Shumin Ren. 2021. Controllable soliton propagation of Airy-Gaussian beams under the fractional effect. Optik, Volume 243, October 2021, 167431. https://doi.org/10.1016/j.ijleo.2021.167431
54) M. E. Manley, A. J. Sievers, J. W. Lynn, S. A. Kiselev, N. I. Agladze, et al. 2009. Intrinsic localized modes observed in the high-temperature vibrational spectrum of NaI. Phys. Rev. B 79, 134304
55) Weida Xia, Yuxiang Ma,Guohai Dong,Jie Zhang, Xiaozhou Ma. 2021. Emergence of Solitons from Irregular Waves in Deep Water. J. Mar. Sci. Eng. 2021, 9(12), 1369; https://doi.org/10.3390/jmse9121369
56) Andrew S. Johnson, William Winlow. 2018. The Soliton and the Action Potential – Primary Elements Underlying Sentience. Front. Physiol., 25 June 2018, Volume 9 - 2018 | https://doi.org/10.3389/fphys.2018.00779
57) S Minardi, F Eilenberger, Y V Kartashov, A Szameit, U Röpke, et al. 2010. Three-dimensional light bullets in arrays of waveguides. Phys Rev Lett. 2010 Dec 31;105(26):263901. doi: 10.1103/PhysRevLett.105.263901.
58) Peregrine, D. H. 1983. Water waves, nonlinear Schrödinger equations and their solutions. J. Austral. Math. Soc. B. 25: 16–43. doi:10.1017/S0334270000003891.
59) Poikonen, H., Zaluska, T., Wang, X. et al. . 2023. Nonlinear and machine learning analyses on high-density EEG data of math experts and novices. Sci Rep 13, 8012 (2023). https://doi.org/10.1038/s41598-023-35032-8
60) A. D. Polyanin and V. F. Zaitsev, Handbook of Nonlinear Partial Differential Equations, London: Chapman & Hall/CRC Press, 2003.
61) R. R. Poznanski, L. A. Cacha,2 Y. M. S. Al-Wesabi,1 J. Ali,2,3 M. Bahadoran, et al. 2017. Solitonic conduction of electrotonic signals in neuronal branchlets with polarized microstructure. Sci Rep. 2017; 7: 2746. doi: 10.1038/s41598-017-01849-3
62) Ivan Redor, Eric Barthélemy, Hervé Michallet, Miguel Onorato, and Nicolas Mordant. 2019. Experimental Evidence of a Hydrodynamic Soliton Gas. Phys. Rev. Lett. 122, 214502 – Published 29 May 2019
63) Tiago L. Ribeiro, Dante R. Chialvo, Dietmar Plenz. 2020. Scale-Free Dynamics in Animal Groups and Brain Networks. Front. Syst. Neurosci., 20 January 2021 Volume 14 - 2020 | https://doi.org/10.3389/fnsys.2020.591210
64) David S. Ricketts (eds), Donhee Ham (eds.). 2010. Electrical Solitons: Theory, Design, and Applications.
65) Rowley, M., Hanzard, PH., Cutrona, A. et al. Self-emergence of robust solitons in a microcavity. Nature 608, 303–309 (2022). https://doi.org/10.1038/s41586-022-04957-x
66) M. I. Shaukat, E. V. Castro, and H. Terças. 2017. Quantum dark solitons as qubits in Bose-Einstein condensates. Phys. Rev. A 95, 053618.

67) Sima Roy, Amar P. Misra. 2022. Electromagnetic solitons and their stability in relativistic degenerate dense plasmas with two electron species. Front. Astron. Space Sci., 19 September 2022. Volume 9 - 2022 | https://doi.org/10.3389/fspas.2022.1007584
68) E. Rubino, J. McLenaghan, S. C. Kehr, F. Belgiorno, D. Townsend, S. Rohr, C. E. Kuklewicz, U. Leonhardt, F. König, and D. Faccio. 2012. Negative-Frequency Resonant Radiation. Phys. Rev. Lett. 108, 253901.
69) Sato, N. 2022. Cortical traveling waves reflect state-dependent hierarchical sequencing of local regions in the human connectome network. Sci Rep 12, 334 (2022). https://doi.org/10.1038/s41598-021-04169-9
70) Asif Shah, Saeed-Ur Rehman, Qamar-UL Haque, Shahzad Mahmood. 2020. Solitons and Turbulence in Solar Wind at 1 au: Multi-satellite Data and FDTD Simulations. The Astrophysical Journal, Volume 890, Number 1. DOI 10.3847/1538-4357/ab6294
71) Sharma, A., Malik, H.K., Kumar, H. et al. 2019. Effect of magnetic field on electromagnetic soliton evolution by different pulses. J Theor Appl Phys 13, 31–37 (2019). https://doi.org/10.1007/s40094-018-0306-z
72) Shen, Y., Dierking, I. 2020. Dynamics of electrically driven solitons in nematic and cholesteric liquid crystals. Commun Phys 3, 14 (2020). https://doi.org/10.1038/s42005-020-0288-4
73) Siddhartha Sen. 2022. A global method of generating action potentials and EEG oscillations in a topological surface network. BiorXiv. doi: https://doi.org/10.1101/2022.06.25.497598. CONTATTARE QUESTO QUI
74) Sinkala, Zachariah. 2006. Soliton/exciton transport in proteins. Journal of Theoretical Biology, Volume 241, Issue 4, p. 919-927. 10.1016/j.jtbi.2006.01.028
75) Kevin E. Strecker, Guthrie B. Partridge, Andrew G. Truscott & Randall G. Hulet. 2002. Formation and propagation of matter-wave soliton trains. Nature volume 417, pages150–153.
76) Joachim Wabnig, Josef Resch, Dominik Theuerkauf, Fabian Anmasser, Manfried Faber. 2022. Numerical Evaluation of a Soliton Pair with Long Range Interaction. arXiv:2210.13374
77) Yamgoué, Serge Bruno; Guy Roger Deffo, Eric Tala-Tebue and François Beceau Pelap. 2018. Exact solitary wave solutions of a nonlinear Schrödinger equation model with saturable-like nonlinearities governing modulated waves in a discrete electrical lattice. Chinese Phys. B 27 126303. DOI 10.1088/1674-1056/27/12/126303
78) Yanlin Ye, Lili Bu, Wanwan Wang, Shihua Chen, Fabio Baronio, Dumitru Mihalache. 2020. Peregrine Solitons on a Periodic Background in the Vector Cubic-Quintic Nonlinear Schrödinger Equation. Front. Phys., 17 November 2020. Volume 8 - 2020 | https://doi.org/10.3389/fphy.2020.596950
79) N. J. Zabusky, “Fermi–Pasta–Ulam, Solitons, and the Fabric of Nonlinear and Computational Science: History, Synergetics, and Visiometrics,” Chaos 15, 015102 (2005).
80) Honghui Zhang, Andrew J. Watrous, Ansh Patel, Joshua Jacobs. 2018. Theta and Alpha Oscillations Are Traveling Waves in the Human Neocortex. Neuron volume 98, issue 6, P1269-1281.E4, JUNE 27, 2018. DOI:https://doi.org/10.1016/j.neuron.2018.05.019