Derivation of Weinberg angle, quark mixing matrix angles and neutrino oscillation matrix angles

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Abstract

Following the path of our previous articles on the masses of charged leptons and quarks, in this paper we derive in a theoretical way the Weinberg angle and in a heuristic-empirical way the angles of the mixing matrices of quarks and neutrinos.Except for the CP violation angle of the neutrino mixing matrix which has a large indeterminacy.

1 Introduction

In the previous articles on the masses of electrically charged leptons and the masses of quarks, we have used the four imaginary parts of the first four non-trivial zeros of the Riemann zeta function as well as the Weinberg angle of the electroweak theory and the Cabibbo angle (13.02 degree). These angles are not derived by the standard model since they are purely experimental parameters. The same is true for the angles of the mixing matrices of the quarks and neutrinos.

This type of purely empirical parameters is closely related to the incompleteness of the standard model. Its incompleteness can be summarized as "the deficiencies of the Standard Model, such as the inability to explain the fundamental parameters of the standard model, the strong CP problem, neutrino oscillations, matter–antimatter asymmetry, and the nature of dark matter and dark energy.[1] Another problem lies within the mathematical framework of the Standard Model itself: the Standard Model is inconsistent with that of general relativity, and one or both theories break down under certain conditions, such as spacetime singularities like the Big Bang and black hole event horizons." (Wikipedia, https://en.wikipedia.org/wiki/Physics_beyond_the_Standard_Model)

In this investigation we will use two main hypothesis. The first hypothesis depends on the following theoretical and empirical observation: the amount of fermions (12) is the integer part of the entropy of the ratio In(2 x mZ/me). That is to say In(2 x Z boson mass/electron mass).

The second hypothesis is of the empirical heuristic type and is based on the hypothesis that the mixing angles of the quark and neutrino matrices are functions of the Weinberg angle and or some quark masses (for the mixing angles of the quark matrices). We make this hypothesis based on the fact that precisely this angle is responsible for the mass quotient of the W and Z boson and that it is at this mass scale that the electroweak unification and decay of the W and Z bosons into fermions (leptons and quarks) takes place. Being the mass of reference the mass of the less massive lepton with electric charge and totally stable, that is: the electron.

Despite the stability of neutrinos, as they are in constant oscillation in their masses and flavors, they are not suitable as a minimum reference mass.And even less so when their exact masses are unknown.

1.1 The imaginary parts of the non-trivial zeros of the Riemann zeta function used in this work

$z_1 = 14.134725142$

1.2 Standard model parameters

Inverse Fine structure constant =137.035999084= $\alpha^{-1}(0)$

Higgs Vacuum $= 246.219650794138\, Gev = V_H$

Fermi Constant$= \dfrac{1}{\left(V_H \cdot \sqrt{\sqrt{2}}\right)^2} = G_F$

Quark masses

$m_{qu} = 2.16\, Mev\,, m_{qd} = 4.67\, Mev\,, m_{qs} = 93.4\, Mev\,, m_{qc} = 1.27\, Gev\,, m_{qb} = 4.18\, Gev\,, m_{qt} = 172.69\, Gev$

weak mixing angle at the Z boson mass scale$=\hat{\theta}\left(M_Z\right)\left(\bar{M}S\right), \sin^2\hat{\theta}\left(M_Z\right)\left(\bar{M}S\right) = 0.23121$

Weinberg angle$=$ $\arccos\left(\dfrac{m_W}{m_Z}\right) = \theta_{W,}\, m_W = 80.377\, Gev\,, m_Z = 9.1876\, Gev$

$\theta_{W,} = 28.182638686° = \arccos\left(\dfrac{m_W}{m_Z}\right)$

1.2.1 Angles of the quark mixing matrix

$\theta_{c12} = 13.02°$ $\theta_{c13} = 0.21142°$ $\theta_{c23} = 2.3968°$ $\delta_{CP} = 1.144\, rad \pm 0.027$Particle data group, CKM quark-mixing matrix

ϑ12 = 13.04°±0.05°, ϑ13 = 0.201°±0.011°, ϑ23 = 2.38°±0.06°, δ13 = 1.20±0.08 radians Wikipedia, Cabibbo–Kobayashi–Maskawa matrix

1.2.2 angles of the neutrino mixing matrix

atmosphere angle $\theta_{at\nu} = 33.41° + 0.75\,, -0.72$ solar angle $\theta_{sol\nu} = 49.1° + 1 - 1.3$ nuclear reactor angle $\theta_{r\nu} = 8.54° + 0.11 - 0.12$ $\delta_{CP} = 197° + 42 - 25$ Particle data group, Neutrino masses, mixing, and oscillations (rev.)

1.3 Hypothesis of the entropy and the number of fermions of the standard model. Weinberg angle

For a dimensionless variable x the entropy of equiprobable states is, as is well known, In(x). We start from the mathematical-physical equivalence that this entropy is equivalent to the number of simultaneous states or number of particles that exist simultaneously, in this case, and whose origin is the sum of probabilities. Mathematically this is expressed as: $\int_1^x \dfrac{dx}{x} = \ln(x)$

In the quantum vacuum filled with virtual particles, all kinds of particles coexist at the same time. Our first hypothesis is based (apart from entropy) on the following empirical fact: Whole part of $[\ln(2 \cdot m_Z/m_e)] = 12$,6 quarks + 6 leptons.

And it turns out that the Weinberg angle (given in degrees) is very approximately:

1.

$$\frac{360^\circ}{\ln\left(2\cdot m_Z/m_e\right)}=28.157537^\circ$$

We check its accuracy by the ratio of W boson mass to Z boson mass.

$$\left(m_Z=9.1876\,Gev\right)\cdot\cos\left(28.157537^\circ\right)=80.39585\,Gev$$

As can be seen, the value obtained is very close to that of 80.377 Gev, taking into account the uncertainties of the latter value. According to the average given by the particle data group, we have for the boson W mass : $m_W=80.377\pm0.012\,Gev$

Anyway, obtaining the Weinberg angle depends on the mass of the Z boson, which again is an empirical parameter. This makes us think that there must exist an equation that does not depend on the mass of the Z boson and that allows us to obtain the Weinberg angle with the same or more accuracy.

Equation 1 depends on twice the mass of the Z boson, which makes us theorize that there may be a connection with gravitons, since one graviton can decay into two Z bosons. $G\rightarrow Z+Z$

Surprisingly, we find a very simple equation that depends on the imaginary part of the first nontrivial zero of the Riemann zeta function and the cosine of spin 2 of the graviton. This equation allows us to calculate with, surely, total accuracy the Weinberg angle. We will also show that from this imaginary part we obtain equations related to the sum of the masses of all the elementary particles which are fermions, as well as the value of 2Mz/me. There even seems to be an equation for the value of the Higgs vacuum that also depends on the value of this imaginary part (z1=14.134725142)

1.4 Weinberg angle as a function of z1 and cosine of spin 2

$$\theta_W=\frac{360^\circ}{\sqrt{z_1^2\cdot\cos\left(spin\,G=2\right)}}=\frac{360^\circ}{\sqrt{14.134725142^2\cdot\left(2/\sqrt{6}\right)}}=28.186292^\circ$$

$$\cos\left(spin\,G=2\right)=\frac{2}{\sqrt{2\cdot(2+1)}}=\frac{2}{\sqrt{6}}$$

With this value obtained for the Weinberg angle we recalculate the mass of the boson W , $m_Z\cdot\cos\left(28.186292^\circ\right)=80.37425\,Gev$

1.5 Sum of masses of all the fermions as a function of $\sqrt{z_1^2 \cdot \cos\left(spin\ G = 2\right)}$

$\sum_{l=leptons} m_l + \sum_{q=quarks} m_q = \sum_{f=fermions} m_f = 1.88302923135\ Gev + 178.24023\ Gev =$ $180.1232592313\ Gev$

$$\exp\left(\sqrt{14.134725142^2 \cdot \left(2/\sqrt{6}\right)}\right) = \frac{\sum_f m_f}{m_e} = 352274.492450417$$

$$\frac{m_e \cdot 352274.49240417 \cdot c^2}{e = electric\,charge \cdot 1E9} = 180.011895752595\ Gev$$

1.6 Ratio 2 x Z boson mass, electron mass

15= 6 quarks+6 leptons+ 1 W boson+1 Z boson+ photon. SU(2) X U(1)

$$\exp\left(\sqrt{14.134725142^2 \cdot \left(2/\sqrt{6}\right)}\right) + 2 \cdot \exp\left(\frac{\sqrt{dim\ E8\ group = 240}}{2}\right) + \sin\left(2\pi/15\right) = 356899.354500642 = \frac{2m_Z}{m_e}$$

$$\frac{\exp\left(12 + \frac{\pi}{4}\right)}{\left(1 + \frac{1}{\exp\left(\pi e\right) + \ln^2\left(\frac{V_H \cdot \sqrt{\sqrt{2}}}{m_e}\right)}\right)} = 356899.367536732 = \frac{2m_Z}{m_e}$$

$$\theta_W \simeq \frac{360^\circ}{12 + \frac{\pi}{4}} = 28.157120755^\circ$$

1.7 Higgs vacuum, function of z1

$$z_1 - \frac{1}{\ln\left(\pi\right)} - \frac{1}{2 \cdot z_1^2} - \frac{1}{\exp\left(z_1 + \exp\left(2 \cdot \sin^2\hat{\theta}\left(M_Z\right)\left(\bar{M}S\right)\right)\right)} = \ln\left(\frac{V_H \cdot \sqrt{\sqrt{2}}}{m_e}\right) = 13.258653861462$$

Note that with 12 fermions, we have an angle of 30 degrees (360/12=30)

1.8 Mixing angles of the quark matrix

Now we are entering completely different terrain. We use heuristic intuition to try to find equations that give the observed angles as a function of the Weinberg angle. Trying to make the values as exact as possible

cabibbo angle θ_{c12}

$$\theta_W = \frac{360^\circ}{\sqrt{z_1^2 \cdot \cos(spin\, G = 2)}}$$

$$\theta_{c12} = 2 \cdot \theta_W \cdot \sin^2\hat{\theta}(M_Z)\left(\bar{M}S\right) = 13.03390151^\circ$$

$$\frac{\theta_W}{\ln(z_1) \cdot \cos(spin\ 2)} = \theta_{c12} = 13.0335144^\circ$$

cabibbo angle θ_{c23}

$$\theta_{c23} = \frac{\theta_W \cdot \sin^2\hat{\theta}(M_Z)\left(\bar{M}S\right)}{e} = 2.3974528^\circ$$

cabibbo angle θ_{c13} $\quad, m_h = 125.25\ Gev$ (Higgs boson mass)

$$\theta_{c13} = \frac{\theta_W \cdot \sin^4\hat{\theta}(M_Z)\left(\bar{M}S\right)}{\exp(V_H/m_h)} = 0.21101045^\circ$$

Two outstanding empirical facts

1) $\theta_{c12} + \theta_{c23} + \theta_{c13} = 15.62822^\circ \simeq \frac{360^\circ}{23}$,23=12 fermions+1 W boson+ 1 Z boson+ 1 photon+8 gluons

2) $\tan(\theta_{c12} = 13.02^\circ) = \sin^2\hat{\theta}(M_Z)\left(\bar{M}S\right) = 0.23123589$

$$\theta_{c23} \simeq \frac{\theta_{c12}}{1 + \sin^{-1}(\theta_{c12})} = 2.393954^\circ$$

$$\delta_{CP} = 1.144 \simeq \ln(\pi)\ rad = 1.1447298858$$

1.9 mixing angles of the neutrino matrix

atmospheric angle $\theta_{AT\nu}$ solar angle $\theta_{SOL\nu}$ reactor angle $\theta_{R\nu}$

$\theta_{AT\nu} + \theta_{SOL\nu} + \theta_{R\nu} = 91.05^\circ \simeq 90^\circ$

$$\frac{\theta_W \cdot \sin^2 \hat{\theta} \left(M_Z\right) \left(\bar{M S}\right)}{2 \cdot \cos\left(\theta_W\right) - 1} = \theta_{R\nu} = 8.5430924^\circ$$

$$\theta_W \cdot \ln^2\left(\pi\right) \cdot \sin^2 \hat{\theta} \left(M_Z\right) \left(\bar{M S}\right) = \theta_{R\nu} = 8.539857^\circ$$

$$\theta_W \cdot \left[\frac{2\pi}{\ln\left(2 \cdot m_Z/m_e\right)} + 3 \cdot \sin^2 \hat{\theta} \left(M_Z\right) \left(\bar{M S}\right)\right] = \theta_{SOL\nu} = 33.402777^\circ$$

$$\frac{\theta_W}{\left(\ln\left(z_1\right) + 1\right) \cdot \sin^2 \hat{\theta} \left(M_Z\right) \left(\bar{M S}\right)} = \theta_{SOL\nu} = 33.411884^\circ$$

$$\frac{\theta_W}{\left(\exp\left(1/2\right) + 2\right) \cdot \sin^2 \hat{\theta} \left(M_Z\right) \left(\bar{M S}\right)} = \theta_{SOL\nu} = 33.4110901^\circ$$

$$\frac{\theta_W}{\frac{\sin^2\left(\theta_W\right)}{2} + 2 \cdot \sin^2 \hat{\theta} \left(M_Z\right) \left(\bar{M S}\right)} = \theta_{AT\nu} = 49.107391922^\circ$$

$$\theta_W \cdot \left(2 \cdot \cos\left(\theta_W\right) - \exp - \left(\left[\frac{V_H}{m_h}\right]^2\right)\right) = \theta_{AT\nu} = 49.096544^\circ$$

$$\theta_W \cdot \left(2 \cdot \cos\left(\theta_W\right) - \exp - \left(\exp\left(\sin\left(\theta_W\right) + \cos\left(\theta_W\right)\right)\right)\right) = \theta_{AT\nu} = 49.1009458^\circ$$

$$\frac{\Delta m^2_{atm} = (24.4 \pm 0.6) \times 10 - 4 \; eV^2}{\Delta m^2_{sol} = (0.753 \pm 0.018) \times 10 - 4 \; eV^2} = 33.86334656 \simeq \ln\left(z_1\right) \cdot \ln\left(2 \cdot m_Z/m_e\right)$$

2 Conclusions

As far as the Weinberg angle is concerned, we believe that the theoretical hypothesis we have made is well founded. As for the mixing angles of the quark and neutrino matrices, we have applied a heuristic hypothesis. Although the equations give precise results of the actual values of these angles, they are still a purely intuitive mathematical game, so these equations should be taken with caution. Perhaps if we make a real theoretical breakthrough, some of them may make physical sense. For the moment they remain as a pure game experiment.

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