Towards an Intuitive Theory of Quantum Gravity

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Abstract

The successful development of a mathematical theory of quantum gravity remains an elusive goal despite decades of intense research. The mathematics involved with string theory is far out of reach for most scientists but perhaps an intuitive theory of quantum gravity could grant them an understanding of the phenomenon. This article moves in that direction and approaches quantum gravity from the angle of an easy-to-understand theory of relativistic quantum spacetime. In the process, a new theory of mass is discovered and several suggestions are made for further development of an intuitive model.

2

The Second Relativization

Since Einstein's general theory of relativity thoroughly explains gravity as being a result of the curvature of spacetime and that its curvature is an effect of mass, our first step towards obtaining an intuitive understanding of quantum gravity is to devise an easily understandable way to conceptualize quantum spacetime. Here we will consider a new type of relativity, one that is produced by the interactions of subatomic particles with a quantum field, specifically a field of virtual gravitons, and in so doing we shall obtain an intuitive idea of relativistic quantum spacetime. Just as the quantization of the other types of fields is referred to as the *second quantization*, the process of obtaining such a model of spacetime can be called the *second relativization*. The mathematics involved in the second relativization is simple, but the concepts described are deep. Our reasoning shall proceed as in (1). Let's get started.

Our toy model shall be a system composed of a single proton p^+ and electron e^-. Now suppose the proton and electron are isotropic radiators of a current of unobservable virtual gravitons, as allowed by the Heisenberg uncertainty principle, and further suppose these virtual particles possess a certain amount of power. If the power of a transmitter, call it T_x, is denoted by P_T and if isotropic radiators (transmitters which radiate energy uniformly in all directions) are assumed, then the power density at a distance R from the transmitter T_x is equal to the radiated power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius R, i.e., the power density at range R from an isotropic radiator is

$$= P_T/4\pi R^2 \quad \text{Watt/m}^2. \tag{1}$$

In what follows we shall be considering the exchange of virtual graviton particles, back and forth, between a proton and an electron. For purposes of clarity, let T_x stand for the "original transmitter", and since we are exchanging particles back and forth the original transmitter eventually becomes the receiver ($T \mapsto R$) let R_x stand for "receiver", and let t_x stand for "target".

When T_x emits a current of virtual particles the target t_x intercepts a portion of the incident energy and re-radiates it in all directions. It is only the power density re-radiated in the direction of the original transmitter T_x (echo) that is of interest. The signal cross-section of the target determines the power density returned to the original transmitter for a particular power density incident on the target. It is denoted by σ. The reradiated power density returning back at the original transmitter (now the receiver) is (2):

$$\frac{P_{R_x}}{\sigma_{R_x}} = \frac{P_{T_x}}{4\pi R^2} \cdot \frac{\sigma_{t_y}}{4\pi R^2}. \tag{2}$$

Simple algebraic rearrangement of Eqn. 2 leads to:

$$R = \sqrt[4]{\frac{P_{T_x}}{P_{R_x}} \cdot \frac{\sigma_{t_y}\sigma_{R_x}}{(4\pi)^2}}. \tag{3}$$

In particular we have

$$R_{p^+\to e^-} = \sqrt[4]{\frac{P_{T_{p^+}}}{P_{R_{p^+}}} \cdot \frac{\sigma_{t_{e^-}}\sigma_{R_{p^+}}}{(4\pi)^2}} \qquad (4)$$

and

$$R_{e^-\to p^+} = \sqrt[4]{\frac{P_{T_{e^-}}}{P_{R_{e^-}}} \cdot \frac{\sigma_{t_{p^+}}\sigma_{R_{e^-}}}{(4\pi)^2}} \qquad (5)$$

Now it is well known fact that the cross-section of a proton is far greater than that of an electron which, according to Eqns. 4 and 5 (and making the assumption $P_{T_{p^+}}/P_{T_{e^-}} = 1$), leads us to the chain of implications

$$\sigma_{p^+} > \sigma_{e^-} \Rightarrow P_{R_{p^+}} > P_{R_{e^-}} \Rightarrow R_{e^-\to p^+} > R_{p^+\to e^-}. \qquad (6)$$

So we have arrived at our first mind-boggling result: that if we could shrink down to the size of subatomic particles and measure the distance between two particles of different types, we would get different results depending on whether we measured from particle A to particle B or from particle B to particle A! We call this property of relativistic quantum spacetime *non-reflexive distance* and it is instrumental in coming to an understanding of relative masses. We shall only deal with mass ratios as they are dimensionless and are the only designation of a quantity of mass that has any real meaning. For example, if the mass of everything in the universe suddenly doubled, all of the laws of physics would remain the same and nothing would change.

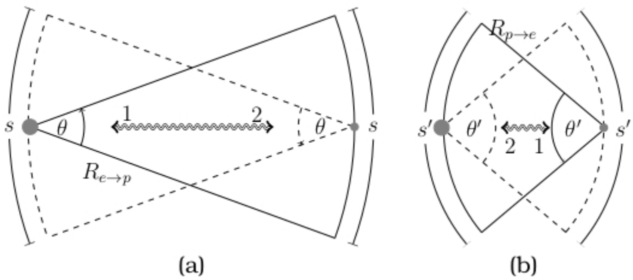

Figure 1: Nonreflexive distance generated as a result of the relativistic quantization of spacetime. The smaller filled circle repesents an electron while the larger filled circle represents a proton. The double squiggly lines with arrowheads represent graviton exchange.

The discrepancy in distance generated by relativistic quantum spacetime means that the motion of a proton along some arc length s and equivalent amount of motion of an electron along some similar arc length s', where $|s| = |s'|$, would result in the traversal of different central angles thus an apparent difference in the relative inertia observed by the other particle even though the two particles may be moving the same distance and speed along s (proton) and s'. The situation is illustrated in Figure 1 above.

From Figure 1 and the relation $\theta = s/R$ we can make one more implication, that

$$R_{e^-\to p^+} > R_{p^+\to e^-} \Rightarrow \theta' > \theta. \quad (7)$$

This property of relativistic quantum spacetime is responsible for relative mass.

At this point it is helpful to introduce a new constant B, which we'll call Bonnar's constant, with the meaning

$$\sqrt[4]{B} = \frac{R_{p^+ \to e^-}}{R_{e^- \to p^+}} = \sqrt[4]{\frac{P_{T_{p^+}} P_{R_{e^-}}}{P_{R_{p^+}} P_{T_{e^-}}}} = \sqrt[4]{\frac{P_{R_{e^-}}}{P_{R_{p^+}}}} = \sqrt[4]{\frac{\sigma_{p^+}}{\sigma_{e^-}}} \qquad (8)$$

Now for an ansatz. Let's make the assertion that

$$\sqrt[4]{B} = \frac{R_{p^+ \to e^-}}{R_{e^- \to p^+}} = \sqrt[4]{\frac{\sigma_{p^+}}{\sigma_{e^-}}} = \beta \qquad (9)$$

Now we can entertain getting a better understanding of the proton-electron mass ratio β. Experimentally, the approximate value of β is $1836.15267343(11)$ (3). So for that reason it is proposed that B is a more fundamental constant of nature than β and has the value $B = 11,366,719,876,399$ which has the same number of digits that the experimental value of β has (though the last two are uncertain). Let's see if this value of B gives us a reasonable value for the radius of an electron. The experimental radius of an electron is found to have an upper limit of about 10^{-22}m (4).

The accepted value for the radius of a proton is $r_p = 8.4 \times 10^{-16}$ m, giving us a cross section $\sigma_p = \pi r_p^2 = 2.2167 \times 10^{-30}\,\mathrm{m}^2$ for the proton.

So we have

$$\frac{2.2167 \times 10^{-30}\,\mathrm{m}^2}{\sigma_e} = 11,366,719,876,399.$$

This gives

$$\sigma_e = \pi r_e^2 = 1.9501668 \times 10^{-43}\,\mathrm{m}^2$$

and we have

$$r_e = 2.49 \times 10^{-22}\,\mathrm{m}$$

in good agreement with experiment. So the main theoretical finding is that if we adopt a relativistic quantum version of spacetime we can deduce from it that

$$\beta = \sqrt[4]{B} = \frac{R_{p^+ \to e^-}}{R_{e^- \to p^+}} = \sqrt[4]{\frac{\sigma_{p^+}}{\sigma_{e^-}}}. \tag{10}$$

It is not difficult to imagine that this principle is universal and can be generalized to *any* two subatomic particles. For this reason we shall state the principle like this:

$$\frac{m_x}{m_y} = \sqrt[4]{\frac{\sigma_x}{\sigma_y}}. \tag{11}$$

Eqn. 11 results from the relativistic quantization of spacetime and it is proposed that a current of virtual gravitons are actually *creating* spacetime (thus spacetime is quantized) and, in conjunction with any two particles' cross-sections, relative mass. Much can be inferred from Eqn. 11. Most importantly, that mass is not an intrinsic property of subatomic particles. This fact was proven a second way by the author in (5). Rather, particle mass is a function of the particle's cross-section and the nonreflexive nature of relativistic spacetime, i.e., mass results from an interaction of the particle's cross-section with a quantum field. The only intrinsic property of a particle that contributes to mass is the cross-section. Particles such as electrons and protons that are conventionally considered to have mass have a noninfinitesimal cross section. But all particles that exist have a nonzero cross-section and therefore all particles have mass. Particles such as photons, gluons and gravitons, that are conventionally considered to have zero mass, are point-like and therefore have infinitesimal cross-sections (not zero), otherwise they would not exist, and interact with the quantum field to an infinitesimal extent thus have infinitesimal mass. The only type of "object" that actually has zero mass is any portion of the classical vacuum.

It may be helpful to consider a Feynman diagram illustrating the fundamental interaction responsible for nonreflexive distance. Refer to Figure 2 on the next page. In the figure, part (a), an electron emits a virtual graviton which is absorbed by a proton which in turn re-emits the graviton and sends it back to the electron. A certain spacetime distance is created. In part (b), a proton emits a virtual graviton which is subsequently absorbed by an electron, which in turn re-radiates it in the direction of the proton. A much shorter relative distance is created ($1/\beta$ times as much) in this case because $\sigma_{p^+} \gg \sigma_{e^-}$.

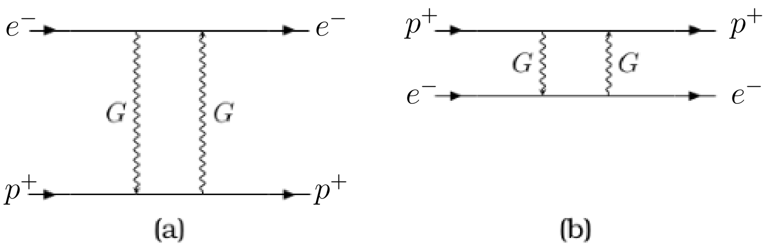

Figure 2: Feynman diagrams illustrating the fundamental interaction giving rise to nonreflexive distance and thus the origin of relative mass.

Infinities and the Infinitesimals

Until the end of the 1800s no mathematician had managed to describe the infinite, except for the intuitive idea that it is an absolutely unattainable value. Georg Cantor was the first to address it, and he did it by developing set theory, which led him to the mind-blowing conclusion that there are infinities of different sizes. Faced with the rejection of his ideas, Cantor went mad and ended up dying in an insane asylum. But today, mathematics cannot be understood without his revolutionary insights.

For Cantor, sets are collections of objects that can have finite or infinite elements (6). He established the concept of cardinal as the number of elements that a set has. The cardinal of the set of fingers of one hand is 5, while the cardinal of the set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$ has infinite elements and we denote the cardinality of that infinity $\aleph_0$, which happens to be the smallest infinity. Notice that we are able to count (i.e., write down) the consecutive elements of that set if it is ordered. The next biggest set is the set of real numbers $\mathbb{R}$. In that case, we cannot write down the consecutive elements of the ordered set. $\mathbb{R}$ is an uncountable set, we denote its cardinality by $\aleph_1$, which is a larger infinity than $\aleph_0$. Cantor proved that it was impossible to establish a bijective function between the set of natural numbers and the reals. He thus came to the conclusion that the cardinal of the set of real numbers was greater than that of natural numbers: they were infinities of different sizes. Cantor proposed that there are an infinite number of infinities of increasing cardinality.

Cantor proposed the continuum hypothesis which is a hypothesis about the possible sizes of infinite sets. It states that "there is no set whose cardinality is strictly between that of the integers and the real numbers" or equivalently, that "any subset of the real numbers is finite, is countably infinite, or has the same cardinality as the real numbers."

Now by an *infinitesimal* it is meant a number that is infinitely small in magnitude. Just as there are infinities of different sizes, there are infinitesimals of different degrees of smallness. We shall define our infinitesimal quantities ϵ as the reciprocals of infinities.

In particular we have,

$$\epsilon_0 = \frac{1}{\aleph_0} \text{ and } \epsilon_1 = \frac{1}{\aleph_1} \tag{12}$$

ϵ_0 and ϵ_1 are both infinitesimal but $\epsilon_1 < \epsilon_0$.

Point Particles Such as Photons, Gluons and Gravitons Have Nonzero Infinitesimal Cross Sections

It was argued elsewhere that point particles have nonzero cross sections otherwise they simply wouldn't exist. It was further argued that, through interaction with a quantum field that creates nonreflexive relativistic quantum spacetime, mass is conferred on any particle having a cross section. But since particles (which are conventionally considered to have no mass) are points, their cross sections are infinitesimal and they therefore behave as if they have infinitesimal mass since they interact with the quantum field to an infinitesimal extent.

It remains to determine the actual magnitude of the infinitesimal cross sections and masses. Quantum spacetime is a tricky concept that befuddles the human mind. We can imagine the spacetime coming in discrete chunks of a certain size, but for one there are no "spaces between the spaces", which is why Stephen Hawking said he saw no reason to abandon the conventional continuum conception of spacetime. Furthermore, the discrete chunks of spacetime are themselves infinitesimal (we might try to imagine that the current of virtual particles that generates relativistic quantum spacetime being described by infinitely compact Feynman diagrams).

Since the current of virtual particles that generates relativistic quantum spacetime is infinitely compact, therefore uncountable, I propose that the correct infinitesimal to assign these particle cross sections and masses is ϵ_1. So it can be stated

$$m_\gamma = m_g = m_G = \epsilon_1 \tag{13}$$

and

$$\sigma_\gamma = \sigma_g = \sigma_G = \pi\epsilon_1^2 \tag{14}$$

The main consequence of having an infinitesimal cross section, thus having an existence, is that the particle will have an infinitesimal mass due to the effects of the nonreflexive character of relativistic quantum spacetime generated by the current of virtual gravitons. All particles with infinitesimal mass travel at the speed of light c. All particles with infinitesimal mass interact with a gravitational field an infinitesimal amount. Particles cannot exist without possessing a cross section that is at least infinitesimal. The existence of γ, g and G is thereby established.

Using Nodal Incidence Matrices to Describe Feynman Diagrams That Precede Spacetime

Given that relativistic quantum spacetime is created by a current of virtual particles, we need to have a way to designate a Feynman diagram that lives *outside* of spacetime, i.e., it is only the connectivity of the diagram that matters. It is assumed that the Feynman diagram representing this current of virtual particles is infinitely compact and the number of nodes and branches are uncountably infinite. Describing Feynman diagrams with nodal incidence matrices fills this gap. Since the curvature of spacetime is completely accounted for by Einstein's general theory of relativity, relativistic quantum spacetime, which has the property of nonreflexive distance (thereby accounting for relative masses), points the way towards an actual theory of quantum gravity.

To reiterate, since it is postulated that a current of virtual particles *produces* relativistic quantum spacetime, we need a way to describe a Feynman diagram that exists outside of spacetime. We cannot possibly do this with a graphical depiction of the diagram. For a Feynman diagram preceding spacetime, the position of the nodes and the lengths of the branches is irrelevent; all we need to do is state the connectivity and directionality of the events and the composition of the branches. Doing so captures the *essence* of the interaction and it represents all of the possible conformations of the Feynman diagram.

Nodal Incidence matrices can be used to describe networks composed of nodes and branches (which connect the nodes). Typically the branches possess directionality. Nodal incidence matrices can be used to describe the configuration of one-way and two-way streets in a city, electrical networks and many other types of networks (9). In our case, we are going to use nodal incidence matrices to describe Feynman diagrams.

Using nodal incidence matrices to describe Feynman diagrams is a useful concept and though not as expedient to understand as a graphical diagram, nodal incidence matrices generalize the diagram. One might imagine stretching and/or shrinking some or all of the branches and/or moving the nodes around in spacetime. A nodal incidence matrix captures all of these configurations because as far as the matrix is concerned, all that matters is the connectivity of the diagram, the directionality of the branches and the composition of the branches (i.e., a branch represents a certain type of particle).

Constructing a nodal incidence matrix for a Feynman diagram is straightforward. Since our nodal incidence matrix represents a Feynman diagram, let's denote it $\mathbf{F}$. We shall define the elements of the nodal incidence matrix $\mathbf{F} = [p_n f_{jk}]$. Each *type* of particle is represented by a unique positive integer p_n that we are allowed to arbitrarily choose. In our example, the Feynman diagram, or interaction, involves an electron, proton and a graviton. We shall arbitrarily choose $p_{e^-} = 1$, $p_{p^+} = 2$ and $p_G = 3$.

Next we define f_{jk} which is dependent upon whether the branch leaves, enters, enters or leaves, or neither enters or leaves, a node. It is defined as follows:

$$f_{jk} = \begin{cases} +1 & \text{if branch } k \text{ leaves node } j \\ -1 & \text{if branch } k \text{ enters node } j \\ i & \text{if branch } k \text{ enters } \textit{or} \text{ leaves node } j \\ 0 & \text{if branch } k \text{ does not touch node } j \end{cases} \tag{15}$$

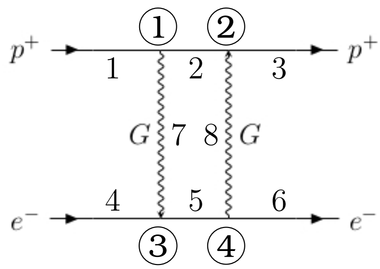

Figure 3: Enumerating a feynman diagram

So given these definitions and by referring to Figure 3, it is trivial to construct the corresponding nodal incidence matrix. In the diagram, the circled numbers represent nodes, whereas the uncircled numbers represent branches. The numbering is arbitrary.

The resulting nodal incidence matrix is

$$\mathbf{F} = \begin{array}{r} \text{branch}\!\rightarrow \\ \text{node } 1 \rightarrow \\ \text{node } 2 \rightarrow \\ \text{node } 3 \rightarrow \\ \text{node } 4 \rightarrow \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} -2 & 2 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & -2 & 2 & 0 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -1 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 3 \end{pmatrix} \end{array}$$

The elements of **F** tells us what is where and what happens where. For example, given our definitions, $f_{1,1} = -2$ tells us that a proton enters node 1, $f_{2,3} = 2$ tells us that a proton leaves node 2, $f_{4,5} - 1$ tells us that any electron enters node 5, $f_{3,7} = -3$ tells us that a graviton enters node 3 and $f_{3,6} = 0$ tells us node 3 is not touched by branch 6, etc.

The process of constructing a nodal incidence matrix **F** is not canonical since the nodes and branches of the diagram are numbered arbitrarily. However, if the numbering scheme is designated along with the nodal incidence matrix, everyone would draw an equivalent Feynman diagram upon deciphering it. The interchange of any rows (columns) merely represents a different arbitrary numbering of the nodes (branches), which lead to equivalent diagrams.

We now have a way, to not only generalize conventional Feynman diagrams that live within spacetime, but also to designate Feynman diagrams that live outside or *precede* spacetime. This development will be very important in the development of a theory of quantum gravity since such a theory will ultimately rest upon the quantization of spacetime.

References

1. Bonnar, J. (2024). "A Theoretical Account of the Proton-Electron Mass Ratio". *vixra.org*:2401.0053
2. Sharma, K.K., S.K. Kataria & Sons, *Introduction to Radar Systems.*
3. "2018 CODATA Value: proton-electron mass ratio". *The NIST Reference on Constants, Units, and Uncertainty.* NIST. 20 May 2019.
4. Dehmelt, H. (1988). "A Single Atomic Particle Forever Floating at Rest in Free Space: New Value for Electron Radius". *Physica Scripta.* T22: 102-110.
5. Bonnar, J. (2024). "A Short Note on the de Broglie Wavelengths of Composite Objects". *vixra.org*:2401.0150
6. Jourdain, P. (1955). "Contributions to the Founding of the Theory of Transfinite Numbers". New York: Dover Publications
7. Bonnar, J. (2024). "Photons, Gluons and Gravitons Have Infinitesimal Mass". *vixra.org*:2402.0076
8. Bonnar, J. (2024). "Generalizing Feynman Diagrams with Nodal Incidence Matrices". *vixra.org*:2402.0105
9. Kreyzig, E. (1993). *Advanced Engineering Mathematics.* John Wiley & Sons
10. Diestal, R. (2005). *Graph Theory.* Graduate Texts in Mathematics, vol. 173. Springer-Verlag

11. Gross, J. (2006). *Graph Theory and its Applications*, 2nd Edition.

12. Skiena, S. (1990). *Implementing Discrete Mathematics*. Reading, MA: Addison-Wesley