Generalizing Feynman Diagrams with Nodal Incidence Matrices

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Abstract

A simple procedure for capturing all the possible configuations of a Feynman diagram using nodal incidence matrices is given. A nodal incidence matrix represents all the conformations of an interaction because as far as the matrix is concerned, all that matters is the connectivity of the diagram, the directionality of the branches and the composition of the branches. There is no specification of node coordinates or branch length. An example is worked out for the Feynman diagram for pair production and annihilation. The construction of nodal incidence matrices from Feynamn diagrams is noncanonical since the numbering of nodes and branches is arbitrary. However, equivalent Feynman diagrams are inferred from equivalent nodal incidence matrices.

Introduction

Despite the superior intuitive appeal of Feynman diagrams, a mathematization of their representation is useful since by doing so, we can capture all the possible configuations of the events in spacetime in one mathematical statement, thereby generalizing the Feynman diagram.

Why Nodal Incidence Matrices? Feynman Diagrams Transcending Spacetime

It was discovered by the author in (2) that spacetime is not a primitive but is rather created by a current of virtual particles interacting with the cross section of an observable particle. The mathematics behind the theory of relativistic quantum spacetime is very simple and easy to understand but the physical reasoning is very deep. Given that relativistic quantum spacetime is created by a current of virtual particles, we need to have a way to designate a Feynman diagram that lives *outside* of spacetime, i.e., it is only the connectivity of the diagram that matters. It is assumed that the Feynman diagram representing this current of virtual particles is infinitely compact and the number of nodes and branches are uncountably infinite. Describing Feynman diagrams with nodal incidence matrices fills this gap. Since the curvature of spacetime is completely accounted for by Einstein's general theory of relativity, relativistic quantum spacetime, which has the property of nonreflexive distance (thereby accounting for relative masses), points the way towards an actual theory of quantum gravity.

In the article (2), it was discovered by the author that the mass ratio of two subatomic particles could be accounted for solely by the consideration of the particle's cross sections and the introduction of a version of relativistic quantum spacetime that has the property of nonreflexive distance. Refer to the article for a derivation, but the theory of the proton-electron mass ratio is encapsulated by the relations

$$\frac{R_{p,e}}{R_{e,p}} = \sqrt[4]{\frac{P_{t,p}}{P_{r,p}} \cdot \frac{P_{r,e}}{P_{t,e}}} = \sqrt[4]{B} = \frac{m_p}{m_e} = \beta \tag{1}$$

where $R_{p,e}$ is the distance from the proton to the electron, $R_{e,p}$ is the distance from the electron to the proton, $P_{t,p}$ is the power (of the current of virtual particles) transmitted by the proton, $P_{r,p}$ is the power transmitted to the electron and then received back (echo) by the proton, and so forth, B is Bonnar's constant which is an integer having value 11366719876399, and β is the proton-electron mass ratio.

Eqn. 1 essentially amounts to relativistic quantization of spacetime and it is proposed that the virtual particles are actually *creating* spacetime and, in conjunction with particle's cross section, relative mass.

Now since it can be assumed that $P_{t,p}/P_{t,e} = 1$, we'll say that

$$B = \frac{P_{t,p}}{P_{r,p}} \cdot \frac{P_{r,e}}{P_{t,e}} = \frac{\sigma_p}{\sigma_e}. \tag{2}$$

so the radius of the electron can be solved for quite simply and it is found that the result agrees with the experimental value.

Much can be inferred from Eqns. 1 and 2 taken together. Most importantly, mass is not an intrinsic property of subatomic particles. This fact was proven a second way by the author in (3). Rather, particle mass is a function of the particle's cross section and the nonreflexive nature of relativistic quantum spacetime. The only intrinsic property of the particle that contributes to mass is the cross section. Particles such as electrons and protons that are considered to have mass have a noninfinitesimal cross section. But all particles that exist have a cross section and therefore all particles have mass. Particles such as photons, gluons and gravitons, that are conventionally considered to have zero mass, are point-like and therefore have infinitestimal cross sections, thus have infinitesimal mass (otherwise they wouldn't exist). The only type of "object" that actually has zero mass is any portion of the classical vacuum.

Since it is postulated that a current of virtual particles *produces* relativistic quantum spacetime, we need a way to describe a Feynman diagram that exists outside of spacetime. We cannot possibly do this with a graphical depiction of the diagram. For such a Feynman diagram the position of the nodes and the lengths of the branches is irrelevent; all we need to do is state the connectivity and directionality of the events and the composition of the branches. Doing so captures the *essence* of the interaction and it represents all of the possible conformations of the Feynman diagram.

Generalizing Feynman Diagrams - Nodal Incidence Matrices

Nodal Incidence matrices can be used to describe networks composed of nodes and branches (which connect the nodes). Typically the branches possess directionality. Nodal incidence matrices can be used to describe the configuration of one-way and two-way streets in a city, electrical networks and many other types of networks (1). In our case, we are going to use nodal incidence matrices to describe Feynman diagrams.

Using nodal incidence matrices to describe Feynman diagrams is a useful concept and though not as expedient to understand as a graphical diagram, nodal incidence matrices generalize the diagram. One might imagine stretching and/or shrinking some or all of the branches and/or moving the nodes around in spacetime. A nodal incidence matrix captures all of these configurations because as far as the matrix is concerned, all that matters is the connectivity of the diagram, the directionality of the branches and the composition of the branches (i.e., a branch represents a certain type of particle).

Constructing a nodal incidence matrix for a Feynman diagram is straightforward. Since our nodal incidence matrix represents a Feynman diagram, let's denote it **F**. We shall define the elements of the nodal incidence matrix $\mathbf{F} = [p_n a_{jk}]$. Each *type* of particle is represented by a unique positive integer p_n that we are allowed to arbitrarily choose. In our example, the Feynman diagram, or interaction, involves electrons, positrons and a photon. We shall arbitrarily choose $p_\gamma = 1$, $p_{e^-} = 2$ and $p_{e^+} = 3$. These numbers represent the absolute value of their respective branches.

Next we define a_{jk} which is dependent upon whether the branch leaves, enters, enters or leaves, or neither enters or leaves, a node. It is defined as follows:

$$a_{jk} = \begin{cases} +1 & \text{if branch } k \text{ leaves node } j \\ -1 & \text{if branch } k \text{ enters node } j \\ i & \text{if branch } k \text{ enters } \textit{or} \text{ leaves node } j \\ 0 & \text{if branch } k \text{ does not touch node } j \end{cases} \tag{3}$$

So given these definitions and by referring to figure 1, it is trivial to construct the corresponding nodal incidence matrix. In the diagram, the circled numbers represent nodes, whereas the uncircled numbers represent branches. The numbering is arbitrary.

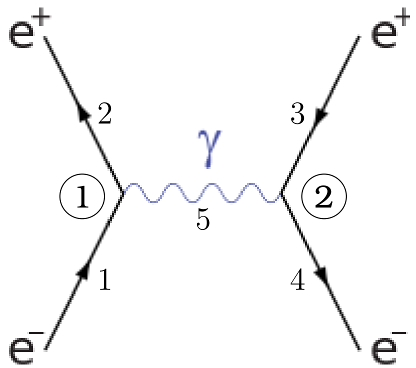

Figure 1: Feynman diagram for pair production and annihilation (In the Stückelberg-Feynman interpretation, pair annihilation is the same process as pair production.)

The resulting nodal incidence matrix is

$$\mathbf{F} = \begin{matrix} \text{branch}\rightarrow \\ \text{node } 1 \rightarrow \\ \text{node } 2 \rightarrow \end{matrix} \begin{matrix} \quad 1 & 2 & 3 & 4 & 5 \\ \left(\begin{matrix} -2 & 3 & 0 & 0 & i \\ 0 & 0 & -3 & 2 & i \end{matrix} \right) \end{matrix} \tag{4}$$

The elements of **F** tells us what is where and what happens where. For example, given our definitions, $f_{1,1} = -2$ tells us that an electron enters node 1, $f_{2,3} = -3$ tells us that a positron enters node 2, $f_{2,4} = 2$ tells us that an electron leaves node 2, $f_{1,5} = i$ tells us that a photon can enter or leave node 1, $f_{1,4} = 0$ tells us node 1 is not touched by branch 4, and so forth.

Afterthoughts

The process of constructing a nodal incidence matrix F is not canonical since the nodes and branches of the diagram are numbered arbitrarily. However, if the numbering scheme is designated along with the nodal incidence matrix, everyone would draw an equivalent Feynman diagram upon deciphering it. The interchange of any rows (columns) merely represents a different arbitrary numbering of the nodes (branches), which lead to equivalent diagrams.

We now have a way, to not only generalize conventional Feynman diagrams that live within spacetime, but also to designate Feynman diagrams that live outside or *precede* spacetime. This development will be very important in the development of a theory of quantum gravity since such a theory will ultimately rest upon the quantization of spacetime.

References

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