THE ELECTRON-POSITRON PAIR CREATION IN QUANTUM ELECTRODYNAMICS

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Abstract

The probability of the emission of the electron-positron pairs is calculated from the vacuum-to-vacuum amplitude in such a way that the modified propagation function of photon is applied. The quantum entanglement of the electron-positron pairs is not analysed. In cosmology, pair production is the heuristic explanation of the Hawking radiation. The pair production process is explained here with the pedagogical simplicity.

1 Introduction

Pair production is the creation of a subatomic particle and its antiparticle from a neutral boson, including creation of the electron and a positron, a muon and an antimuon, or, a proton and an antiproton. Pair production often refers to a photon creating an electron-positron pair near a nucleus. From energy conservation, during the process of pair production follows that incoming energy of the photon must be above a threshold of at least the total rest mass energy of the two particles created. Conservation of energy and momentum are the principal constraints on the process. All other conserved quantum numbers must sum to zero, or, they have opposite values of each other.

For high energy photon (MeV scale and higher), pair production is the dominant mode of photon interaction with matter. These interactions were first observed by Patrick Blackett leading to the 1948 Nobel Prize in Physics.

If the photon is near an atomic nucleus N, the energy of a photon can be converted into an electron-positron pair:

$$\gamma + N(Ze) \to N(Ze) + e^- + e^+.$$

The photon energy is converted to particle mass in accordance with the Einstein equation, $E = mc^2$, where E is energy, m is mass and c is the speed of light. The photon must have higher energy than the sum of the rest mass energies of an electron and positron (2 x 511 keV = 1.22 MeV), resulting in a photon-wavelength of 1.2132 picometer) for the production to occur. Thus, pair production does not occur in medical

X-ray imaging because these X-rays only contain energy $E \sim 150$ keV. The photon must be near a nucleus in order to satisfy conservation of momentum, as an electron-positron pair produced in free space cannot satisfy conservation of both energy and momentum. Because of this, when pair production occurs, the atomic nucleus receives some recoil. The reverse of this process is electron-positron annihilation.

Photon metamorphoses to electron and positron by a spatially nonuniform electric field is the permanent interest of QED (Chervyakov et all., 2011). The history starts by the Klein formulation (Klein, 1929) of his gedanken experiment and its interpretation by Dirac, (Sauter, 1931), Heisenberg and Euler (1936) and Hund (1941). It became clear that the Dirac vacuum of quantum electrodynamics (QED) is unstable against the $e^- + e^+$ production in the present of external electromagnetic field. The first discussions were performed in the first-quantized field theory. The results were rederived within second-quantized field theory in the one-loop approximation by Schwinger (1951) for a constant electromagnetic field, and Nikishov (1969; 1970; 2003) and Narozhny and Nikishov (1970; 1974) for more general field configurations. The two-loop radiative corrections were calculated by Ritus (1977; 1984) and Lebedev and Ritus (1984). The spin-statistics connection in QED with unstable vacuum was discussed by Feynman (1949; 1961). A general discussion, detailed calculations and further references on this subject can be found in texts of Damour (1977), Hansen et al. (1981), Grib et al. (1980), Itsykson et al. (1980), Greiner et al. (1985), Greiner (2000), Fradkin et al. (1991), Kleinert (2008), Dunne (2005) . For recent developments and the modern state of the problem, see Ruffini et al. (2010) and Dunne (2009).

In a constant electric field, the probability for vacuum decay via pair creation results from the imaginary part of the one-loop Euler-Heisenberg Lagrangian. The formula was anticipated by Sauter (1931) and derived by Euler and Heisenberg (1936) and, more elegantly, by Schwinger (1951). It is presently often referred to as Schwinger formula, and the pair creation process as Schwinger mechanism of pair production. The process can be understood within the Dirac picture of vacuum as a quantum tunneling of electron through an electrostatic potential barrier created by the electric field. Its rate is, however, exponentially suppressed for presently accessible field strengths E achieved roughly $4\times$ 1014 V/m by optical lasers (Ringwald, 2001). This leads to the present impossibility of observing the pair creation for which the extraordinary strong electric field strengths E of the order or above the critical value $E \sim 1.3 \times 1018$ V/m are required. Such strong macroscopic fields in the laboratory will hopefully be achieved in the near future after further advances in the laser technology.

Indeed, the energy extraction and the optical focusing can be improved considerably in the X-ray free electron lasers to produce the electric field of the order of E $\sim 10^2$ E(critical) capable of yielding a sizeable effect of pair production (Heinzl, 2009), (Bulanov et al., 2006; 2010). Under these circumstances, we could get a significant progress in understanding the properties of QED beyond the scope of perturbation theory in the strong-field regime. Of course, such strong macroscopic fields in the laboratory would never be as uniform as those originally considered in the Schwinger mechanism of pair creation.

Apart from these, enormous electromagnetic and gravitational fields are expected in the powerful Gamma Ray Bursts, strong Coulomb fields in the relativistic heavy-ion collisions for which the Schwinger mechanism of pair creation is also relevant (Ruffini, et al., 2010).

2 The e^-e^+ pairs from the vacuum-to-vacuum amplitude

The probability of the emission of pairs is here calculated from the vacuum-to-vacuum amplitude in such a way that the modified propagation function of photon is used.

The total vacuum amplitude involving the two-particle exchange betwen sources can be expressed using the total propagation function $\tilde{D}_+(x-x')$ in the following manner (Schwinger, 1973):

$$\langle 0_+|0_-\rangle^J = \exp\left\{ i\frac{1}{2}\int (dx)(dx')J^\mu(x)\tilde{D}_+(x-x')J_\mu(x')\right\}. \tag{1}$$

It has the physical meaning of the probability amplitude that the vacuum is disturbed. Since the vacuum amplitude involves the two-particle exchange, the disturbation corresponds to the pair production.

The weak limit of the last formula gives the persistence probabilitty of vacuum in the form (Schwinger, 1973):

$$|\langle 0_+|0_-\rangle|^2 = \exp[-2\ \mathrm{Im}\ W_{int}] \approx 1 - \int \frac{(dk)}{(2\pi)^4} J^{*\mu}(k)\mathrm{Im}\ \tilde{D}_+(k)J_\mu(k), \tag{2}$$

where, because of the presence of Im $\tilde{D}_+(k)$, it corresponds to the probability of a single pair emission by the source.

Using the spin 1/2 structure (Appendix) of $\tilde{D}_+(x-x')$ we have from

$$\tilde{D}_+(k) = \frac{1}{k^2 - i\varepsilon} + \frac{\alpha}{3\pi}\int_{(2m)^2}^{\infty} \frac{dM^2}{M^2}\left(1+\frac{2m^2}{M^2}\right)\ \left(1-\frac{4m^2}{M^2}\right)^{1/2} \frac{1}{k^2+M^2-i\varepsilon}, \tag{3}$$

and from

$$\frac{1}{k^2+M^2-i\varepsilon} = P\ \frac{1}{k^2+M^2} + i\delta(k^2+M^2), \tag{4}$$

the relation

$$\mathrm{Im}\ \tilde{D}_+(k) = \frac{\alpha}{3}\frac{1}{M^2}\left(1+\frac{2m^2}{M^2}\right)\left(1-\frac{4m^2}{M^2}\right)^{1/2} \tag{5}$$

for $k^2 = -M^2$.

In case of the spin 0 two-particle exchange between sources, we have from

$$\tilde{D}_+(k) = \frac{1}{k^2 - i\varepsilon} + \frac{\alpha}{12\pi}\int_{(2m)^2}^{\infty} \frac{dM^2}{M^2}\left(1+\frac{4m^2}{M^2}\right)^{3/2} \frac{1}{k^2+M^2-i\varepsilon}, \tag{6}$$

and from (4) the following formula:

$$\mathrm{Im}\ \tilde{D}_+(k) = \frac{\alpha}{12}\frac{1}{M^2}\left(1-\frac{4m^2}{M^2}\right)^{3/2}; \quad M^2 = -k^2. \tag{7}$$

Then, the corresponding pair emission probability in case of the spin 1/2 and spin 0 situation is as follows (Schwinger, 1973):

$$P_{(pair\ emiss,\ s=1/2)} \quad =$$

$$\frac{\alpha}{12}\int \frac{(dk)}{(2\pi)^4}\frac{1}{M^2}\left(1+\frac{2m^2}{M^2}\right)\left(1-\frac{4m^2}{M^2}\right)^{1/2} J^{*\mu}(k)J_\mu(k). \tag{8}$$

The calculations was here performmed for spin 1/2 particles because the important physical applications refers to the charged particles with small mass - the electron. The same procedure can be realized for spin 0 particles. The result is as follows (Schwinger, 1973):

$$P_{(pair\ emiss,\ s=0)} = \frac{\alpha}{3}\int \frac{(dk)}{(2\pi)^4}\frac{1}{M^2}\left(1-\frac{4m^2}{M^2}\right)^{3/2} J^{*\mu}(k)J_\mu(k). \tag{9}$$

3 Discussion

According to quantum mechanics, particle pairs are constantly appearing and disappearing as a quantum foam. In cosmology, pair production is the method of explanation of hypothetical Hawking radiation. In a region of strong gravity tidal forces, the two particles in a pair may sometimes be wrenched apart before they have a chance to mutually annihilate. When this happens in the region around a black hole, one particle may escape while its antiparticle partner is captured by the black hole. The mechanism behind the hypothesized pair-instability supernova type of stellar explosion, is pair production. It suddenly lowers the pressure inside a supergiant star, leading to a partial implosion, and then explosive thermonuclear burning. According to our knowledge, supernova SN 2006gy has been a pair production type supernova.

The Schwinger formula around Kerr-Newman black holes was pioneered by Damour and Ruffini, only for black hole masses larger than the critical mass of neutron stars. In this case the electron Compton wavelength is much smaller than the spacetime curvature and all previous results can be applied following well established rules of the equivalence principle. It corresponds to rate of $e^- + e^+$ pair production. The progress was done in describing the evolution of optically thick $e^- + e^+$ plasma in the presence of supercritical electric field, which is relevant both in astrophysics as well as in ongoing laser beam experiments. In particular the recent progress was based on the Vlasov-Boltzmann-Maxwell equations to study the feedback of the created $e^- + e^+$ pairs on the original constant electric field.

The existence of plasma oscillations is known with its interaction with photons leading to energy and number equipartition of photons, electrons and positrons. The recent progress was obtained by using the Boltzmann equations to study the evolution of an $e^- + e^+ + \gamma$ plasma towards thermal equilibrium and determination of its characteristic time scales.

4 Appendix: The propagation function of photon with radiative corrections

It is known from the traditional theory of the Feynman propagator of photon that the one-loop radiative correction to the photon propagator can be graphically represented by the Feynman diagram of the second order. The physical meaning of this diagram is the process $\gamma \to (e^- + e^+) \to \gamma$, where γ is denotation for photon, and e^-, e^+ is the electron-positron pair. It means that photon can exist in the intermediate state with e^+, e^- being virtual particles.

The modified photon propagation function involving only the two-particle exchange process between sources is then diagrammatic expressed by the analogical way in the Schwinger source theory of QED.

Now, the goal of this contribution is to determine, in the framework of the source methods, the photon propagator corresponding to the intermediate electron-positron pair. The emission photon source emits by manner of the inter medial virtual photon the electron-positron pair which is absorbed by the detection source. Such process is possible when a source emits too much energy to produce only a photon. For the virtual photon the relation $k^2 \neq 0$ and the excitation cannot propagate very far because the balance of energy and momentum is broken.

We can split the process into two parts. The lower part and the upper part. The lower part is the emission part and the upper one is the absorption part. The emission part corresponds to the emission effective source. The effective two-particle source is here the electromagnetic vector potential.

There is no renormalization procedure necessary, neither for the mass nor for the charge. Here is the way from free traveling photon ($k^2 = 0$) to the modified effective photon propagator which experiences from the source an excess of energy ($k^2 = -M^2$), so that after an extremely short time, it can produce an electron-positron pair. Everything happens between the "vacua" $< 0_-|$ and $|0_+ >$. These are not the Dirac vacua with particle-antiparticle pairs, etc. They are absolutely empty until an external source delivers or takes the necessary attributes of energy, momentum, spin, etc. to, or, from the particles to be produced or annihilated.

The vacuum amplitude corresponding to the primitive interaction that occurs is involved in the vacuum to vacuum amplitude with $i/\hbar = i$, for $\hbar = 1$ (Dittrich, 1978)

$$\langle 0_+|0_-\rangle = e^{iW_{int}}, \tag{1}$$

where

$$W_{int} = \int (dx) j^\mu(x) A_\mu(x) \tag{2}$$

with

$$j^\mu(x) = \frac{1}{2}\psi(x)\gamma^0 eq\gamma^\mu\psi(x) \tag{3}$$

and the vacuum amplitude corresponding to the considered process is

$$\langle 0_+|0_-\rangle = i\int (dx)\psi(x)\gamma^0 eq\gamma^\mu\psi(x)A_\mu(x), \tag{4}$$

where q is the charge matrix: $q = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

The vacuum amplitude for the non-interacting spin 1/2 particle is involved in the general formula (Dittrich, 1978)

$$\langle 0_+|0_-\rangle = \langle 0_+|0_-\rangle^\eta \langle 0_+|0_-\rangle^\eta = e^{iW(\eta)}e^{iW(\eta)} \tag{5}$$

with

$$W(\eta) = \int \psi\gamma^0\eta, \tag{6}$$

from which we extract the vacuum amplitude for the two non-interacting spin 1/2 particles in the form (Schwinger, 1970; 2018)

$$\langle 0_+|0_-\rangle = \frac{1}{2}\left[i\int (dx)\psi(x)\gamma^0\eta(x)\right]^2 \quad = \quad -\frac{1}{2}\int (dx)(dx')\psi(x)\gamma^0\eta(x)\eta(x')\gamma^0\psi(x') \quad \to$$

$$-\frac{1}{2}\int \psi_1(x)\gamma^0\eta_2(x)\eta_2(x')\gamma^0\psi_1(x'). \tag{7}$$

The comparison of eq. (7) with eq. (4) supplies the matrix

$$i\eta_2(x)\eta_2(x')|_{eff\ emiss} = eq\gamma^\mu\gamma^0 A_{2\mu}\delta(x-x'), \tag{8}$$

or, in the momentum representation

$$i\eta_2(p)\eta_2(p')|_{eff\ emiss} = eq\gamma^\mu\gamma^0 A_{2\mu}(k) \tag{9}$$

with $k = p + p'$.

By the same procedure just performed we get for the absorption effective source the following formula:

$$i\eta_1(p)\eta_1(p')|_{eff\ abs} = eq\gamma^\mu\gamma^0 A_{1\mu}(-k) \tag{10}$$

with $k = p + p'$.

Let us remark that the antisymmetry of the left side of eq. (8) for all indices expressing the Fermi-Dirac statistics is involved in the charge matrix q.

The amplitude which describes emission and absorption of the two noninteracting particles which propagate freely between the effective source can be separated from the vacuum amplitude

$$\langle 0_+|0_-\rangle = \exp\left\{\int (dx)(dx')\eta_1(x)\gamma^0 G_+(x-x')\eta_2(x')\right\} \tag{11}$$

as its quadratic term of its expansion. Here

$$G_+(x-x') = i\int d\omega_p e^{ip(x-x')}(m-\gamma p), \tag{12}$$

where $d\omega_p = \frac{d(\mathbf{p})}{2\pi^3}\frac{1}{2p^0}$ and $p^0 = +\sqrt{\mathbf{p}^2 + m^2}$ (Dittrich, 1978).

Then,

$$\langle 0_+|0_-\rangle = \frac{1}{2}\int d\omega_p \int d\omega_{p'} \left[\eta_1(-p)\gamma^0(m-\gamma p)\eta_2(p)\eta_2(p')\gamma^0(m+\gamma p')\eta_1(p')\right] \tag{13}$$

Using relation

$$\eta_{1a}(-p)M_{ab}\eta_{1b}(-p') = -M_{ab}\eta_{1b}(-p')\eta_{1a}(-p) \quad = -\mathrm{tr}[M\eta_1(-p')\eta_1(-p)], \tag{14}$$

where a and b are indexes for the eight-dimensional mathematical object η and symbol tr denotes the eight-dimensional trace.

Using eq. (14) we can write

$$\langle 0_+|0_-\rangle = \frac{1}{2}\int d\omega_p \int d\omega_{p'} \mathrm{tr}\left[(m-\gamma p)\eta_2(p)\eta_2(p')\gamma^0(-m-\gamma p')\eta_1(-p)\eta_1(-p)\gamma^0\right]. \quad (15)$$

After inserting of the effective emission and absorption sources with $k = p + p'$

$$i\eta_2(p)\eta_2(p')|_{eff\ emiss} = eq\gamma^\mu\gamma^0 A_{2\mu}(k) \quad (16)$$

$$i\eta_1(-p')\eta_1(-p)|_{eff\ abs} = eq\gamma^\mu\gamma^0 A_{1\mu}(-k) \quad (17)$$

into eq. (13) we get with $(\gamma^0)^2 = 1$,

$$\langle 0_+|0_-\rangle \quad = -\frac{1}{2}\int d\omega_p \int d\omega_{p'} \mathrm{tr}\left[(m-\gamma p)eq\gamma A_2(k)(-m-\gamma p')eq\gamma A_1(-k)\right]. \quad (18)$$

Substituting the unit factor

$$1 = (2\pi)^3 \int dM^2 d\omega_k \delta(k-p-p'), \quad (19)$$

we find

$$\langle 0_+|0_-\rangle = -e^2 \int dM^2 d\omega_k A_1^\mu(-k) I_{\mu\nu}(k) A_2^\nu(k), \quad (20)$$

where

$$I_{\mu\nu}(k) = I_{\nu\mu}(k) = (2\pi)^3 \int d\omega_p d\omega_{p'} \delta(k-p-p').\mathrm{tr}[\gamma_\mu(m-\gamma p)\gamma_\nu(-m-\gamma p')]. \quad (21)$$

Using relations $p^2 + m^2 = 0$, $p'^2 + m^2 = 0$ we find

$$\mathrm{tr}[\gamma_\mu(m-\gamma p)\gamma_\nu(-m-\gamma p')] = \mathrm{tr}[(\gamma p + m)\gamma_\mu(\gamma p' - m)(m-\gamma p)\gamma_\nu(-m-\gamma p')]. \quad (22)$$

It may be easily seen that

$$k^\mu I_{\mu\nu}(k) = 0, \quad (23)$$

which implies the gauge invariance of $\langle 0_+|0_-\rangle$ in the form

$$A_\mu(k) \to A_\mu(k) + ik_\mu\lambda(k). \quad (24)$$

The symmetrical tensor constructed from the vector k_μ is

$$I_{\mu\nu} = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) I(M^2), \quad (25)$$

where $I(M^2)$ can be calculated from relation

$$3I(M^2) = (2\pi)^3 \int d\omega_p d\omega_{p'} \delta(p+p'-k)\mathrm{tr}[\gamma^\mu(m-\gamma p)\gamma_\nu(-m-\gamma p')]. \quad (26)$$

Then, using (Dittrich, 1978)

$$\gamma^\mu\gamma_\mu = -4; \quad \gamma^\mu\gamma p\gamma_\mu = 2\gamma p \quad (27)$$

$$\mathrm{tr}[\gamma_\mu\gamma_\nu] = -4g_{\mu\nu}; \quad \mathrm{tr}\gamma_\mu = 0 \tag{28}$$

$$-M^2 = k^2 = (p+p')^2 = -2m^2 + 2pp' \tag{29}$$

and

$$(2\pi)^3 \int d\omega_p d\omega_{p'} \delta(p+p'-k) = \frac{1}{(4\pi)^2}\left(1 - \frac{4m^2}{M^2}\right)^{1/2}, \tag{30}$$

we obtain

$$I(M^2) = \frac{4}{3}\left(M^2 + 2m^2\right)\frac{1}{(4\pi)^2}\left(1 - \frac{4m^2}{M^2}\right)^{1/2}. \tag{31}$$

Now, we can write the vacuum amplitude in the form

$$\langle 0_+|0_-\rangle = -e^2 \int dM^2 d\omega_k A_1^\mu(-k) \quad \times$$

$$\left(g_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\right)\frac{4}{3}\left(M^2 + 2m^2\right)\frac{1}{(4\pi)^2}\left(1 - \frac{4m^2}{M^2}\right)^{1/2} A_2^\nu(k). \tag{32}$$

Since $k^2 A^\mu(k) = J^\mu$ and $k^2 = -M^2$, we have for the effective sources $J_{1,2}(\mp k)$:

$$A_2^\mu(k) = -\frac{1}{M^2}J_2^\mu(k); \quad A_1^\mu(-k) = -\frac{1}{M^2}J_1^\mu(-k) \tag{33}$$

and after substitution of eq. (33) into vacuum amplitude (32) we get

$$\langle 0_+|0_-\rangle \quad = i\frac{\alpha}{3\pi}\int \frac{dM^2}{M^2}\left(1 + \frac{2m^2}{M^2}\right)\left(1 - \frac{4m^2}{M^2}\right)^{1/2} id\omega_k J_1^\mu(-k) J_{2\mu}(k). \tag{34}$$

Now, we substitute for the momentum representation of $J^\mu(k)$

$$J_1^\mu(-k) = \int (dx) J_1^\mu(x) e^{ikx} \tag{35}$$

$$J_2^\mu(-k) = \int (dx') J_2^\mu(x') e^{ikx'} \tag{36}$$

and put

$$\Delta_+(x-x';M^2) = i\int d\omega_k e^{ik(x-x')}. \tag{37}$$

Then,

$$\langle 0_+|0_-\rangle = i\frac{\alpha}{3\pi}\int \frac{dM^2}{M^2}\left(1 + \frac{2m^2}{M^2}\right)\left(1 - \frac{4m^2}{M^2}\right)^{1/2} \quad \times$$

$$\int (dx)(dx') J_1^\mu(x)\Delta_+(x-x';M^2) J_{2\mu}(x'). \tag{38}$$

The amplitude (38) involves the electron-positron pair production and the complete radiation process is described by the amplitude

$$\langle 0_+|0_-\rangle = \int (dx)(dx') J_1^\mu(x)\tilde{D}_+(x-x';M^2) J_{2\mu}(x'), \tag{39}$$

where the momentum representation of $\tilde{D}_+(x - x')$ can be now written with the regard to eq. (38) in the form:

$$\tilde{D}_+(k) = \frac{1}{k^2 - i\varepsilon} \quad +$$

$$\frac{\alpha}{3\pi} \int_{4m^2}^{\infty} \frac{dM^2}{M^2} \left(1 + \frac{2m^2}{M^2}\right) \left(1 - \frac{4m^2}{M^2}\right)^{1/2} \frac{1}{k^2 + M^2 - i\varepsilon}, \tag{40}$$

or,

$$\tilde{D}_+(k) = \frac{1}{k^2 - i\varepsilon} + \int dM^2 \frac{a(M^2)}{k^2 + M^2 - i\varepsilon} \tag{41}$$

where

$$a(M^2) = \frac{\alpha}{3\pi} \frac{1}{M^2} \left(1 + \frac{2m^2}{M^2}\right) \left(1 - \frac{4m^2}{M^2}\right)^{1/2}. \tag{42}$$

is the weight function of the e^+e^- - particle production. Let us remark that for $M \gg 2m$ the radiative correction to the Green function of the free photon $\tilde{D}_+$ behave like

$$\int_{4m^2}^{\infty} \frac{dM^2}{M^2} \frac{1}{k^2 + M^2} \tag{43}$$

and therefore there is no convergence problem of integral in eq. (41).

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