ON NAVIER–STOKES EQUATIONS

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Abstract. All classical systems must be Galilean invariant, but Navier–Stokes equations are not. The solution is the correct derivation of Navier–Stokes equations.

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1. Main result

Let me divide the liquid into segments. A segment passes point A during some seconds; with this, its velocity changes. The coordinates of point A are x, y, and z. So, from Newton's second law, in the limit of small segment (so the velocity vector $\vec{v}$ is the same throughout the segment's value),

$$\rho \, \frac{\partial \vec{v}(x, y, z, t)}{\partial t} = \vec{F} \, . \tag{1}$$

Unlike Navier-Stokes equations, this is invariant under a Galilean coordinate transformation (see Appendix). To remind you, classical Physics is Galilean-invariant, as the Galilean-invariance follows from Lorentz invariance. So, as the main result, Navier-Stokes equations are wrong. And they are wrongly derived.

2. Problem with Navier-Stokes derivation

It should start with Newton's second law. So,

$$\rho \, \frac{d\vec{v}(x(t), y(t), z(t), t)}{dt} = \rho \, \frac{\partial \vec{v}}{\partial t} + \rho \, \vec{v} \, \frac{\partial \vec{v}}{\partial \vec{r}} = \vec{F} \, . \tag{2}$$

The $\rho \, \frac{d\vec{v}(x(t),y(t),z(t),t)}{dt}$ is Galilean-invariant. The $\rho \, \frac{\partial \vec{v}}{\partial t} + \rho \, \vec{v} \, \frac{\partial \vec{v}}{\partial \vec{r}}$ is not. Hence, I come to contradiction.

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3. Appendix

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The correction factor, the value of which, according to Smeaton's calculations were 1.005, was used for more than 100 years, and only the experiments of the Wright brothers, during which they discovered that the lifting force acting on the gliders was weaker than the calculated one allowed us to clarify Smeaton coefficient to a value of 1.0033. [2]

Perhaps, this mismatch $1.005 \neq 1.0033$ came from wrong equations. Namely, Navier-Stokes ones.

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Galilean invariance or Galilean relativity states that the laws of motion are the same in all inertial frames of reference. Galileo Galilei first described this principle in 1632 in his Dialogue Concerning the Two Chief World Systems using the example of a ship traveling at constant velocity, without rocking on a smooth sea; any observer below the deck would not be able to tell whether the vessel was moving or stationary.

To cite an Encyclopedia of 2023 AD: "*Since understanding the Navier–Stokes equations* [1] *is considered the first step to understanding the elusive phenomenon of turbulence, the Clay Mathematics Institute in May 2000 made this problem one of its seven Millennium Prize problems in mathematics. It offered a prize to the first person providing a solution for a specific statement of the problem: Prove or give a counter-example of the following statement: In three space dimensions and time, given an initial velocity field, there exists a vector velocity and a scalar pressure field, which are both smooth and globally defined, that solve the Navier–Stokes equations.*"

The above formulation of Navier–Stokes problem has terms from Physics: velocity (in the following, $\vec{u}$), space (in the following, coordinate vector $\vec{r}$), time t, and pressure (in the following, the influence of pressure is hidden within $\vec{f}$). The density field is ρ. Therefore, having contradictions with the Physical picture, I have found countless counter-examples against these equations

The Navier-Stokes equations are [1]

$$\rho(\vec{r}, t) \left(\frac{\partial \vec{u}(\vec{r}, t)}{\partial t} + \vec{u}\,\nabla \vec{u} \right) = \vec{f}(\vec{r}, t)\,, \tag{3}$$

where

$$\nabla \vec{u} \equiv \frac{\partial \vec{u}(\vec{r}, t)}{\partial \,\vec{r}}\,, \tag{4}$$

The second-order term is $\vec{u}\,\nabla\vec{u}$, since the u is written twice. But since we need a first-order equation, this term has to be deleted. Moreover, this term causes the violation of the Galilean Relativity Principle: if you replace velocity $\vec{u}$ with $\vec{U} + \vec{C}$, where $\vec{C}$ is the transformation constant velocity, then you have term $\vec{C}\,\nabla\vec{U} \neq 0$ left. It means that transformed

$$\rho(\vec{r},t)\left(\frac{\partial\vec{U}(\vec{r},t)}{\partial t} + \vec{U}\,\nabla\vec{U} + \vec{C}\,\nabla\vec{U}\right) = \vec{f}(\vec{r},t) \tag{5}$$

does not match Eq. (3) due to $\vec{C}\,\nabla\vec{U} \neq 0$.

This means that all solutions of Navier-Stokes equations with $\vec{u}\,\nabla\vec{u} \neq 0$ are counter-examples against the theory of the Navier-Stokes equations.

References

[1] Navier. Mémoire sur les lois du mouvement des fluides. Mémoires de l'Académie des sciences de l'Institut de France. 1822. Vol. 6; Stokes. On the theories of internal friction of fluids in motion, and of the equilibrium and motion of elastic solids. Transactions of the Cambridge Philosophical Society. 1845. Vol. 8.

[2] Clancy L. J., Aerodynamics, John Wiley & Sons, 1975.