Spectral and Symplectic Riemann Mappings

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Abstract

Given a collection of sections of a principle fiber running from the base of a topological space to its top, can we recreate the entire topological space? We answer this question in the affirmative for symplectic manifolds, assuming we are given a filtration of weights. Using the weights which are representative generators at each local neighborhood about each section of a smooth fiber, we reduce our original problem to the Ricci-iterated mapping of Riemann surfaces along a geodesic.

Preamble

The motivation for this is to reduce the following equation:

$$\int_{\perp\mathscr{T}}^{\top\mathscr{T}} \frac{d(P \cap \gamma_i, \mathfrak{w}_i)}{di}$$

to a transitive binary relationship $R \in End(\mathscr{T})$, when $\perp\mathscr{T} = Rep(inf(\mathbb{E}))$, and when $\top\mathscr{T}$ is defined likewise. Our strategy is to supply enough notions from symplectic and spectral algebraic geometry to sufficiently weaken this problem before attacking. In particular, we are able to reduce this task to one of finding a weak equivalence between the homotopy types of symplectic spectra, the definition of which will be revealed later.

So, let $\mathscr{T}$ be a topological space with base space B. Let $\pi : E \to B$ be a fiber bundle. Attach, to each section $\gamma \in \pi$ a norm $\nu(\gamma)$, such that the map

$$\gamma^+ : d(P \cap \gamma, \mathfrak{w}_0) \to d(P \cap \gamma', \mathfrak{w}_1) \tag{0.1}$$

is transitive, but not necessarily distance-preserving for all $\mathfrak{w}_{\{0,1\}}$. One notices immediately that, whence this map is a weak equivalence, γ becomes the fibrant object, and $\mathfrak{w}_i \to \mathfrak{w}_{i+1}$ is a cofibration for all $i \in \mathbb{N}$.

Of course, the underlying assumption here is that $\mathscr{T}$ is flatly presentable, or in other words generated by a tame topological stack, $\mathscr{T} \times \mathfrak{X}$. This, in turn, implies that the map from B onto π is projective, and essentially surjective. This is effectively trivial when one is working with $Cat(0)$ spaces. However, for a map of norms $a \to b$ for unequal a and b, we obtain a rough idea of the curvature for the principle fiber of π. That is to say,

$$Curv(P \in \pi) \cong k$$

is the chief topological invariant for a $Cat(k)$ space. We assume that each $\mathfrak{w}_i$ is a representative weight for a given section γ_i. Thus, for our base, we obtain an object of lowest weight, which we can use to construct a net of maps

$$N(i,\gamma) \coloneqq \mathfrak{w}_i - mod \longrightarrow B\gamma$$

from weighted modules to the classifying space of each section.

Notation 0.1. *For a collection of sections,* $\sum_{i<j}^{j} \gamma_i$*, we will write* $\Gamma_{i\to j}$.

It is evident that each collection of sections is algebraically equivalent to a filtration:

$$Fil_{[i,j]}\ \pi_n(\mathscr{T})$$

and, specifically, $max(n) = j - i$.

Definition 0.1. *We will call* $\mathscr{T}$ geodesible *if, for any* $i < j$*, the composition of all maps*

$$\Gamma_{i\to j} \coloneqq \gamma_{j-1}^{+} \circ ... \circ \gamma_{i+1}^{+} \circ \gamma_i^{+}$$

we have

$$Avg(Curv(\Gamma_{i\to j})) = Curv(P)$$

such that the curvature of our principle fiber is exactly equal to the mean curvature of the space.

This means, classically, that we will call a path-connected curve $\gamma_i \to \gamma_j$ a *geodesic* if it is isomorphic to a path-connected curve $\mathfrak{w}_i \longrightarrow \mathfrak{w}_j$. Note, however, that this is akin to (but not quite as strong as) saying that the map defined in our first equation vanishes at every step of its Ricci iteration. The condition of being geodesible means for us that cofibration $a \rightharpoonup b$ is isomorphic to the canonical trivial fibration given by passing from fibrant to cofibrant objects along the principle bundle. Equivalently, the tautological line bundle indexing every given path is representative of the path via an isomorphism between filtered objects and weighted objects, which induces a strong equivalence

$$N(i,\gamma) \equiv Fil_{[i,j]}\ \pi_n(\mathscr{T})$$

which is realized by every $\Gamma_{i\to j}$.

We will be interested in studying the case when a collection $\Gamma_{i\to j}$ of sections is treated differentially. That is to say, when the Ricci iteration

$$Ric_i(\pi) \longrightarrow Ric_{i+g}(\pi)$$

admits a Riemann mapping for each $g < (j - i) \in \mathbb{N}$. For a class of Riemann-mappable sections, not only is there a natural way of deriving the norm on each section, but there is also a canonical Bousfield localization to each section, given by the slice category $\mathscr{T}_{/\nu(\gamma)}$.

Recollection 0.1. *Recall that for a class of Riemann maps* $[\mathscr{R}]$*, there is an isomorphic category* $WE[\mathscr{K}]$ *of weakly equivalent Kan complexes given by passing between homotopy types of sections.*

This is a powerful statement, as it means that for our topological space, there is a unique Yoneda embedding:

$$Yo(\mathscr{T}) : \int_{\perp\pi}^{\top\pi} \frac{d(\mathfrak{w}_i)}{di} \longrightarrow \Gamma_{\perp\pi\to\top\pi} \cong \mathscr{T}$$

which allows us to reconstruct the original topological space from its weighted representatives. When taken seriously, as a formal gluing condition, this means that the class of representatives of a given topological space

$$[Rep(\mathscr{T})] = [\mathfrak{w}]$$

is actually a presheaf of sections

$$= Pshv(\gamma)$$

which can be "*sheafified*" via the group completion of the fiber bundle. Thus, we have:

$$(Pshv(\gamma) \hookrightarrow (\Gamma_\infty \cong \Pi_\infty(\mathfrak{w}_{(i-j)\leq\infty}, \gamma)) = Shv(\Gamma_{i\to j}) = \pi^\circ(\mathscr{T})$$

for a C^∞ (genuinely smooth) topological space. This eases rigidity, and allows us to pass from strict norms to equivalence classes of norms by mapping out of the principle fiber and into the moduli space of all admissible fibers.

Definition 0.2. *We call a fiber $F : A \longrightarrow B$* admissible *if, for all ε and for all $\delta \in [j-i] \leq (\aleph_0 \sim \infty)$, the fibration*

$$a \ltimes b = ((\delta \times \varepsilon)(a \in A) \longrightarrow b \in B) = \delta_i(a \to b)$$

has an inverse

$$a \rtimes b = ((\delta \times \varepsilon)(b \in B) \longrightarrow a \in A) = \delta_i(a \leftarrow b)$$

such that $(a \rtimes b) \circ (a \ltimes b) = Id_a$ is the trivial action.

Proposition 0.1. *Fibrations of admissible fibers preserve dependent sums.*

We prove this proposition using standard-fare abstract nonsense:

Proof. The following diagram

$$\begin{array}{ccc} a & \xrightarrow{\ltimes} & b \\ {\scriptstyle\ltimes}\downarrow & & \downarrow{\scriptstyle\rtimes} \\ b & \xrightarrow[\rtimes]{} & a \end{array} \qquad \cong \qquad \begin{array}{ccc} \Sigma_i a_i & \xrightarrow{\delta\times\varepsilon} & \Sigma_j b_j \\ {\scriptstyle\delta\times\varepsilon}\downarrow & & \downarrow{\scriptstyle\varepsilon\times\delta} \\ \Sigma_i b_j & \xrightarrow[\varepsilon\times\delta]{} & \Sigma_i a_i \end{array}$$

is an equivalence for all fiber spectra a_i and b_j. Thus, the image of the map $\delta_i(a \to b) : a \oplus \epsilon$ is given by the quasi-isomorphism

$$(b \cong (b \oplus \varepsilon)) \sim (\delta \cong \gamma^+)$$

which gives us the following 3-cell:

$$\begin{tikzcd} b \arrow[r, equal] \arrow[d, dash] & \delta \arrow[d, dash] \\ b \oplus \varepsilon \arrow[r, equal] & \gamma^+ \end{tikzcd}$$

denoted by the squiggly lines in the center of the above diagram. $\square$

Contents

1 Background

Riemann, in his PhD thesis [6], proved that for any 2-dimensional surface S, there is a unique embedding $S \longrightarrow S^1$ into the unit sphere, thus establishing a fundamental fact of topology. Later works of birational geometers would rely on establishing *birational invariants*, or in other words symmetries between pullback objects and their projective immersions into minimal surfaces, which rely on as little geometric data as possible.

We will not indulge in the birational paradigm here, but we will rather be focused on *homotopical invariants*. In the past, the author has written about forcing and its relationship to homotopy theory. In a nutshell, for a collection $\mathscr{D}$ of data, there is a distinguished character $\mathfrak{d} \in \mathscr{D}$ such that there is a forcing notion $\mathfrak{d} \Vdash H_{n+g}(Rep(\mathscr{D}))$. Here, the isofibration $\mathfrak{d} \xrightarrow{\sim} g$ is essentially our core homotopical invariant. In this paper, we represent this fibration as a path whose union with the affine line $\mathbb{A}^1$ can be reduced to a projection of the trivial bundle of a manifold containing Lagrangian submanifolds. This bears some resemblence to the work done in [5] and [7], where the authors treated a map

$$\widetilde{Ham}(\mathbb{CP})_n \longrightarrow Ham_n(\mathbb{CP})$$

out of a pre-symmetric manifold whose Hamiltonian is given by a choice of semi-linear representative paths. This gives rise to the notion of a *quasi-state*.

Essentially, our thinking was born about in considering a pre-sheaf of quasi-states with a gluing condition, which allows us to reconstruct a topological space by taking its local Yoneda embeddings at every given object for each of its subobject-identifiers which coincide with the classification of a filtered, representative weight. From this project, we are able to decompose higher dimensional manifolds into towers of low-dimensional homotopical data. This is physically significant, as it suggests that the structure of good ol' Minkowski spacetime itself is generated via the *transport* of a probe in $\mathbb{E}^n$. Loops in this space represent closures of open intervals, which in turn are the group completions of the supercircle group $S^{p|q}$. This coincides with Schreiber's recent thinking, in which he constructed a rather elaborate spacetime probe.[1]

2 Symplectic Manifolds

Recollection 2.1. *A symplectic manifold (M, ω) is an even-dimensional, orientable manifold M endowed with a non-degenerate 2-form ω called a symplectic form.*

Recollection 2.2. *Symplectic manifolds are locally indistinguishable.*

Because of this reason, symplectic geometry is naturally non-local. For a tower

$$\begin{array}{ccccc} \dots & & \dots & & \\ \downarrow & & \downarrow & & \\ M_\zeta & \otimes & \omega_\zeta & = & Sym_\zeta \\ \downarrow & & \downarrow & & \\ M_\eta & \otimes & \omega_\eta & = & Sym_\eta \\ \downarrow & & \downarrow & & \\ M_\theta & \otimes & \omega_\theta & = & Sym_\theta \\ \downarrow & & \downarrow & & \\ \dots & & \dots & & \end{array}$$

We have a sequence of characters:

$$\mathscr{S} = \{..., \zeta, \eta, \theta, ...\} \longmapsto B\mathbb{N}$$

taking each sub-object and sending it to a unique classifier. By applying our gluing condition, we obtain a unique sheaf of symplectic manifolds equipped with a 2-form $\omega \times \mathscr{S}$, where $\mathscr{S}$ acts as the flow variable. Call this sheaf $\mathcal{O}_\omega$.

Definition 2.1. *A symplectic connection, ∇_{Sym}, consists of a geodesible symplectic manifold M, along with a restriction*

$$\mathcal{O}_\omega|_x$$

of a a symplectic sheaf to a single stalk x, called the typical stalk.

[1]See [8]

Remark 2.1. *We drop the Sym and simply write ∇ when there is no plausible confusion as to whether the connection is symplectic or not. Symplectic connections always have vanishing torsion [1] and are parallel to the symplectic form.*

Notation 2.1. *We write $Diff_x$ for the set of diffeomorphisms of x.*

Remark 2.2. *We recreate the diagram in [2.4] of [2]:*

$$
\begin{tikzcd}
M_1 \arrow[r, "f_\sharp"] \arrow[d, "\pi_1"'] & M_2 \arrow[d, "\pi_2"] \\
X_1 \arrow[r, "f"'] & X_2
\end{tikzcd}
$$

where $(df_{x_1})^* : T^*_{x_2}X_2 \xrightarrow{\sim} T^*_{x_1}X_1$, such that $f_\sharp|_{T^*_{x_1}}$ is its inverse map.

We use the following equation:

$$\int_{x\sim 0}^{(x\sim 0)\to\infty} \frac{d\varepsilon}{(x)}\delta = \frac{x}{\varepsilon} \tag{2.1}$$

to map every pre-animated object out of x, where x is the kernel for some space $H_n(X)$, which has coherent homotopies at level n. Technically, $x \cong \sum_{i=0}^{\infty} x_i$ is an object in a Lie group, whose etale immersion $(\delta_i(x) = x_i) \hookrightarrow X$ is essentially a projection out of a Noohi topological stack $\mathfrak{X}$.

To measure whether this projection has occurred or not, we attach a binary classifier:

$$B_{\omega\vee\emptyset} : X_{pre} \longrightarrow \Sigma\nabla_X$$

where X is isomorphic to $Diff_x$.

3 Truth values and section bundles

Let $\Gamma_{i\to j}$ be a bundle of sections with filtration $Fil_{[i,j]}$. For each intersection:

$$\hat{i} = (\tilde{i} \in \Gamma_{i\to j}) \cap (i \in [i,j])$$

we have a product

$$(ev_0(\tilde{i} \times i) \times (\tilde{i} \otimes i)) = \tau(i)$$

where ev_0 is evaluation at the zeroth projection.

Notation 3.1. *We use the notation x_{pre} to denote the pullback of the forgetful functor $\mathbb{E}|_x \longrightarrow K$.*

To keep in the spirit of Hancock's "process-oriented mathematics", we will let τ itself be a functor:

$$i_{pre} \longrightarrow i$$

which is dual to the actualization

$$i_{pre} \longrightarrow \mathbb{1}$$

the category of discrete groupoids. If these groupoids are presentable, then they are actually the singleton set containing each x_i.

Notation 3.2. *To denote the actualization, we will write $\tau^\vee(i)$.*

Composition of the $\tau(*)$ and $\tau^\vee(*)$ is commutative, associative, and unital, up to a generic isomorphism with the projection of the tautological bundle, which is taken over a normed Lagrangian gauge space.

Remark 3.1. *It is worth considering these structures as both topological spaces, or as pure categories. The purely categorical perspective is largely Yoneda-based, and relies on a transport of fibrant objects along a chain complex, which form the complete system of weak equivalences of every single-subobject object x_i.*

Remark 3.2. *While the geometric and topological applications of this theory are very interesting, we need not get bogged down into a debate as to whether the "real-world" spacetime realizing these formulae is smooth or discrete. On the categorical level, a morphism may be "smooth" but admit degenerate representative topologies. This is why we prefer here to work with the symplectic spaces. We might be more invested in which Fukaya category we are projecting onto. We shall name here the* relative Fukaya category*, $\mathscr{B}_{\geq 0}$ as a candidate for our purposes. In terms of algebras, these are the absolutely real algebras, whose canonical valuation is given by:*

$$|r| : r \longrightarrow \mathbb{R}_+$$

The underlying philosophy here is that all truth must be inherently realized in order to be measurable.

We will prove the smoothness of the map $0 \longrightarrow 1$ now.

Proof. Let $x = 1$ and put $y \in (0, 1)$. Then we, can have $x - y^{z \in \mathbb{Z}}$, and we can take the limit $z \to \infty$ and prove that there is a rational limit for which the series converges. Since every z is no greater than ∞, then we can always select a finite partion of z which admits an infinite number of finite decompositions, etcetera. Thus, the map $0 \longrightarrow 1$ is C^∞-smooth over $\mathbb{R}$, and since it is classically invertible (bijective on objects), its inverse is also smooth. $\square$

Example 3.1. *It is worth considering, rather than strict smooth manifolds, any old-fashioned topological space $\mathscr{T}$. Suppose it is the discrete space $|\mathbb{N}|$. Then, we have a discontinuity at the punctured point $|z|$.*

Axiom 3.1. *$|z|$ is a regular cardinal.*

This axiom non-trivially states that $z \leq \infty$ restricts to the strict filtration $Fil_z \;\; z < \infty$.

4 Spectral Operators

In this section, we present the minimal technical jargon in order to construct a synthetic spectral theory. The desiderata for such a theory are the ability to perform soft computations on $\mathscr{B}_{\geq 0}$, compatibility of syntax with pre-existing motivic constructions, and the production of exact formulae for computing the homotopy types of slice categories of a generic topological space. Suppose for a moment that we are given two objects α and β (roughly analogous to particles), and we are free to index them with subobject identifiers of our choosing.

Define

$$\alpha_i \circledast \beta_j := \int_i^j \alpha \bigwedge_{j-i} \beta$$

where the right-hand-side of the above equation is a smooth sum of smash products of k-tuples for $i < j$.

Using the field $\mathbb{E}$ of energy numbers (as defined by Emmerson[2]), set

$$\mathbb{E}^{\infty}(\alpha_i \circledast \beta_j) = \mathbb{E}^{\infty}_{i \to j}(\alpha, \beta)$$

$$= \sum_{i+j}^{\infty} (\alpha \wedge \beta) \cong \perp\mathbb{E} \to \top\mathbb{E}$$

$$\cong \mathfrak{k}$$

where $i \to j$ is a structure-preserving map homomorphic to a group completion, $\mathfrak{k}$ is a sheaf, and $\alpha_i \times \beta_j$ forms a presheaf of symmetric spectra.

Emmerson recently gave us his Σ-adic algebra:

$$\Sigma_{\mathbb{E}} = \Big(\sum_{n=v_{\mathbb{E}}}^{(\top\mathbb{E})\sim\infty} a_n \star a^n \Big) \hookrightarrow DM_{Eff}$$

which is an alternative way of counting the automorphisms of $[0,1] \subseteq \mathbb{1}$. Here, the star operator is an operator which endows each actualization $(a_n \vee a^n) \twoheadrightarrow \mathbb{R}_+$ with a transitive relationship ρ such that $\rho^{\infty}((a_n) \vee (a^n)) \hookrightarrow |a_n| \vee |a^n|$ is a 1-cell in $\mathbb{1}$.

Remark 4.1. *We can reword the above into a map:*

$$(\gamma_i \rho \gamma_j) \longrightarrow \nu(\gamma_j)$$

for all section bundles $\Gamma_{i \to j}$.

The Σ-adic algebra is a technique for producing an internal object in the category of energetic sheaves with an exact representation as a normed Lagrangian submanifold. Substituting $\circledast$ for the relationship ρ gives us the smash product of all independent α, β as indexed by a tautological line bundle $\mathcal{L}_{taut}$.

[2] See [3]

In Schreiber's notation:

$$\forall \ell_n \in \mathcal{L}_{taut} \;\; \exists f : \ell_n \mapsto \mathfrak{w}_m$$

where $m - n$ vanishes whence the the section $\gamma \ni (n \leftrightarrow m)$ is Ricci flat.

Let

$$tr(\mathfrak{k}) = \sum_{i=0}^{j} \delta_i(x) \hookrightarrow \Delta_i(X)$$

where $\Delta_i(X) = (\Delta \subset X)/T_i M$ for an immersed submanifold M of X. Note that if $tr(\mathfrak{k})$ has codimension zero, then it is exact, so the condition

$$codim(tr(\mathfrak{k})) = 0$$

implies exactness; however, this is not an only if statement, as we need only let

$$max(Dim(\delta_i(X))) = dim(\Delta_i(X))$$

in order for the inclusion to be exact.

Remark 4.2. *Note that the codimension of the immersion is the dimension of the free loop space obtained by sending each ℓ_n to some $\Omega\ell_n$. See [4] for information on extending this construction.*

Suppose we have two cotangent bundles, and we relate them by:

$$T^*\alpha_i \circledast T^*\beta_j$$

Then, we perform our ordinary computation, but rather than receiving the usual geometric data, we obtain a parameterization $\zeta(M)$ of spectra $\mathcal{S}$, such that

$$\zeta^*(M)\zeta(M) = \mathcal{S} \longrightarrow \{*\} \sim 0$$

is the contraction of our total space to a point. This gives us an orbifold $\mathcal{S}_{orb} : \mathscr{O}rb_* Rep(\mathcal{S})$. The homotopy type of $\mathcal{S}_{orb}$ is given by calculating $\pi_n(\zeta(M))$ for $n \to \infty$.

Thus, we have

$$H_{soft}(\mathcal{S}_{orb}) = \underset{n\to\infty}{colim} \prod_n M$$
$$= N(i,\gamma)\; (\mathfrak{w}_i \in \mathbb{E}|_{H_n(M)}) \times \gamma \in \Gamma_{v_{\mathbb{E}} \to n}$$

which reduces to a Calabi quasi-morphism (in the sense of Fukaya, et al. [5]) $n \xrightarrow{\sim} \infty$, which is a Riehl-Verity isofibration. Essentially, the homotopy type of slice category $\mathscr{T}_{/n}$ is determined uniquely by the chosen class of "good reductions," or Bousfield localizations to singletons. In the categorical framework, these singletons are objects, and they are singular in the sense that they contain just a single subobject: $s \longmapsto o_s$.

4.1 A_∞ algebras

Recollection 4.1. *An A_∞ algebra A over k is given by the following data:*

- *a $\mathbb{Z}$-graded free k-module A*
- *a codifferential on the cofree algebra $T(A[1]) = \oplus_{n\geq 1} A[1]^{\otimes n}$*

and we refer the reader to [9] for more information.

Definition 4.1. *An A_∞ category is a large A_∞ algebra isomorphic to $End(\oplus_{X\in Ob(C)} X)$.*

Let $\mathscr{A}$ be an A_∞ algebra. We define the map $\mathscr{A} \longmapsto X_{dg}$ to a dg-manifold by taking the evaluation $ev_X : (X_i \oplus Y_j) \longrightarrow X$ for $0 \leq i < j \leq n$. Then, n is the formal colimit

$$\underset{n}{colim} \ \frac{d_{j-i}\varepsilon}{k\delta} k$$
$$= d^* \varepsilon$$

obtained by modding the class of codifferentials by a quasi-linear representative of the cotangent bundle. Thus, one has:

$$[T(A[1])]/\tilde{\mathfrak{w}}_\varepsilon = d^* \varepsilon$$

and we use the usual cup product to obtain the Bressler-Soibelman holonomy object, $hol_{BS}(X)$, which acts on stalks of the sheaf $\mathcal{O}_\omega$ via the formula:

$$hol^\bullet_{BS}(X) = \mathcal{O}_\omega|_\bullet \times d_{j\cup i}\varepsilon$$

where ω is a symplectic form on X.

Proposition 4.1. *The above equation is weakly equivalent to $Pic(\mathscr{A}) : \mathscr{A} \longrightarrow \mathcal{L}_{taut} \otimes_{\mathbb{Z}} \int \mathfrak{w}$.*

Proof. Recall that the Picard functor takes as its input some object at a desired level of abstraction (e.g. ind-schemes, varieties, and less typically, stacks), and outputs a fiber bundle with a distinguished line $\mathcal{L}_{taut}$.

Because this functor is representable (at least for the naive Fukaya category), we obtain a family of geodesible subspaces of the space $\mathscr{T}$ generated by $\mathscr{A}$. Thus, we may attach to each geodesic, a Lie algebra $\mathfrak{g} : \mathfrak{w} \hookrightarrow \mathscr{T}|_{\mathfrak{g}}$ such that each $\mathfrak{w}$ is the barycenter of a Lagrangian submanifold of $\mathscr{T}$. □

Proposition 4.2. *If $\mathscr{T}$ is a complex (oriented or otherwise) manifold, then, as per [10] there is a Betti realization functor*

$$SH(k) \longrightarrow SH^{top}$$

where k is the ring containing $\mathfrak{w}_0$.

Remark 4.3. *The above proposition is analogous to the map* $\widetilde{Ham}_k \longrightarrow Ham_k$ *as established in the preamble. When the chosen field* k *is equal to* $\mathbb{E}$*, we obtain a restricted specialization:*

$$k|_{k'} : (z \in k) \rightsquigarrow |z|$$

for $|z|$ *a preferred realization. In this formula,* $k \cong k'_{pre}$*, and the pre-quantum line bundle* Bun_k *is a dg-ind-scheme for a totally real manifold.*

Letting $|\bullet| : \bullet \longrightarrow \mathbb{1}$ be the actualization used in [Sect. 3], we obtain a short exact sequence:

$$0 \longrightarrow k \longrightarrow Bun_k \longrightarrow k \longrightarrow k' \longrightarrow 0$$

which is isomorphic to $z \longrightarrow B\zeta(z)$ and weakly equivalent to the interval $(-\infty, \infty)$, the closure of which is the push-out of the following Cartesian square:

$$\begin{tikzcd} (j-i) \arrow[r] \arrow[d] & n^- \arrow[d] \\ n^+ \arrow[r] & n^\pm \sim \pm\infty \end{tikzcd}$$

where n^+ is the bosonic sector and n^- is fermionic. We have:

$$n^\pm = \sqrt{(\downarrow n)^2 + (\uparrow n)^2}$$

where n is given by one of three Pauli matrices: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, or $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ denoted respectively by σ_x, σ_y, and σ_z.

By letting $\sigma_x = T_i^* x$ and $\sigma_y = T_j^* y$, one obtains $\sigma_x \cdot \sigma_y = \sigma_z \star tr(\mathfrak{k})$. This can be seen by decomposing $tr(\mathfrak{k})$ into comodules and taking the cup product:

$$((\mathfrak{k}_i \vee \mathfrak{k}_j \vee \mathfrak{k}_k) \bigwedge \mathfrak{k}_{ijk}) \vee \frac{1}{d\varepsilon}$$

which gives us the stabilizer for a representative groupoid $\mathscr{G}_{\mathbb{1}}$.

Remark 4.4. *An embedding* $\mathscr{G}_{\mathbb{1}} \overset{\sim}{\hookrightarrow} *$ *is, in general, not flat. Thus, we can "unstraighten" the point by taking the inverse* $* \wedge \mathscr{G}_{\mathbb{1}}^{-1}$*. This gives us a lossless transfer from Euclidean space to the class of all topological spaces, by first factoring through a topological stack, giving us the sequence:*

$$\mathscr{G}_{\mathbb{1}} \longrightarrow * \longrightarrow (* \wedge \mathscr{G}_{\mathbb{1}}^{-1}) \longrightarrow \mathfrak{X} \longrightarrow \xi$$

where $\xi \in \mathfrak{X}$ *is a one-object category. This category is a presentable character whose representation is a point.*

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