

Euler's identity in unification of the fundamental interactions

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Abstract

Euler's identity is the most beautiful equation in mathematics. In this paper Euler's identity will be applied to Physics. It will present new beautiful equations of unification of the fundamental interactions. It will calculate new unity formulas that connect the coupling constants of the fundamental forces. Also it will present new beautiful equations of the Dimensionless unification of atomic physics and cosmology and it will prove that the shape of the Universe is Poincaré dodecahedral space. These equations are applicable for all energy scales.

Keywords

Euler's identity , Hubble constant , Dimensionless unification of the fundamental interactions , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Cosmological parameters , Cosmological constant , Poincaré dodecahedral space

1. Introduction

In physics the mathematical constants appear almost everywhere. In [1] we presented exact and approximate expressions between the Archimedes constant π , the golden ratio ϕ , the Euler's number e and the imaginary number i . While Euler's identity is a direct result of Euler's formula, published in his monumental work of mathematical analysis in 1748, *Introductio in analysin infinitorum*, it is questionable whether the particular concept of linking five fundamental constants in a compact form can be attributed to Euler himself, as he may never have expressed it. Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between the most fundamental numbers in mathematics:

$$e^{i\pi} + 1 = 0$$

All five of the numbers play important and repetitive roles in mathematics. The expression who connects the six basic mathematical constants, the number 0, the number 1, the golden ratio ϕ , the Archimedes constant π , the Euler's number e and the imaginary unit i is:

$$e^{\frac{i\pi}{1+\phi}} + e^{\frac{-i\pi}{1+\phi}} + e^{\frac{i\pi}{\phi}} + e^{\frac{-i\pi}{\phi}} = 0 \quad (1)$$

The fine-structure constant is one of the most fundamental constants of physics. We propose in [2] , [3] and [4] the exact formula for the fine-structure constant α with the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1} = 360 \cdot \phi^{-2} - 2 \cdot \phi^{-3} + (3 \cdot \phi)^{-5} = 137.035999164... \quad (2)$$

Also we propose in [4] , [5] and [6] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π :

$$\alpha^{-1}=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}\cdot \pi\cdot \ln 2=137.035999078\dots \quad (3)$$

In Physics, the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron, from the system of units. We propose in [7] the exact mathematical expressions for the proton to electron mass ratio μ :

$$7\cdot \mu^3=165^3\cdot \ln^{11}10 \Rightarrow \mu=1836.15267392\dots \quad (4)$$

Also was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α :

$$9\cdot \mu-119\cdot \alpha^{-1}=5\cdot (\varphi+42) \quad (5)$$

In [8] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu \cdot \alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot 10^2\cdot \cos(\mu\cdot \alpha^{-1})+13^2=0 \quad (6)$$

The exponential form of this equation is:

$$10^2\cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0 \quad (7)$$

Also this unity formula can also be written in the form:

$$10\cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i \quad (8)$$

It was presented in [9] the mathematical formulas that connects the proton to electron mass ratio μ , the fine-structure constant α , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro's number N_A , the gravitational coupling constant α_G of the electron and the gravitational coupling constant of the proton $\alpha_G(p)$:

$$\alpha_G(p)=\mu^2\cdot \alpha_G$$

$$\alpha=\mu\cdot N_1\cdot \alpha_G$$

$$\alpha\cdot \mu=N_1\cdot \alpha_G(p)$$

$$\alpha^2=N_1^2\cdot \alpha_G\cdot \alpha_G(p)$$

$$4\cdot e^2\cdot \alpha^2\cdot \alpha_G\cdot N_A^2=1 \quad (9)$$

$$\mu^2=4\cdot e^2\cdot \alpha^2\cdot \alpha_G(p)\cdot N_A^2 \quad (10)$$

$$\mu\cdot N_1=4\cdot e^2\cdot \alpha^3\cdot N_A^2 \quad (11)$$

$$4\cdot e^2\cdot \alpha\cdot \mu\cdot \alpha_G^2\cdot N_A^2\cdot N_1=1 \quad (12)$$

$$\mu^3=4\cdot e^2\cdot \alpha\cdot \alpha_G(p)^2\cdot N_A^2\cdot N_1 \quad (13)$$

$$\mu^2=4\cdot e^2\cdot \alpha_G\cdot \alpha_G(p)^2\cdot N_A^2\cdot N_1^2 \quad (14)$$

$$\mu=4\cdot e^2\cdot \alpha\cdot \alpha_G\cdot \alpha_G(p)\cdot N_A^2\cdot N_1 \quad (15)$$

In [10] we presented the recommended value for the strong coupling constant:

$$\alpha_s = \frac{\text{Euler's number}}{\text{Gerford's constant}} = \frac{e}{e^\pi} = e^{1-\pi} = 0,11748\dots \quad (16)$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale.

2. Dimensionless unification of the fundamental interactions

In the papers [11] , [12] , [13] and [14] was presented the unification of the fundamental interactions. We found the unity formulas that connect the strong coupling constant a_s and the weak coupling constant a_w . We reached the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e \cdot a_s = 10^7 \cdot a_w \quad (17)$$

$$a_s^2 = i^{2i} \cdot 10^7 \cdot a_w \quad (18)$$

$$e^n \cdot a_s^2 = 10^7 \cdot a_w \quad (19)$$

We reached the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$a_s \cdot \cos a^{-1} = i^{2i} \quad (20)$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i} \quad (21)$$

$$e^n \cdot a_s \cdot \cos a^{-1} = 1 \quad (22)$$

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \quad (23)$$

We reached the dimensionless unification of the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot a_w \cdot \cos a^{-1} = e \cdot i^{2i} \quad (24)$$

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i} \quad (25)$$

$$10^7 \cdot e^n \cdot a_w \cdot \cos a^{-1} = e \quad (26)$$

$$10^7 \cdot e^n \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \quad (27)$$

We reached the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot a_w \cdot \cos a^{-1} = a_s \quad (28)$$

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot a_s \quad (29)$$

We reached the dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2 = 1 \quad (30)$$

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot NA^2 \quad (31)$$

$$16 \cdot a^2 \cdot a_G \cdot NA^2 = (e^{i/a} + e^{-i/a})^2 \quad (32)$$

We reached the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \quad (33)$$

$$2 \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i} \quad (34)$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i} \quad (35)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA = 1 \quad (36)$$

$$2 \cdot e^{4n} \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot NA^2 = 1 \quad (37)$$

$$e^{4n} \cdot a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot NA^2 = 1 \quad (38)$$

We reached the dimensionless unification of the weak nuclear, the gravitational and electromagnetic interactions:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2 \quad (39)$$

$$4 \cdot 10^{14} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i} \quad (40)$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i} \quad (41)$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = e^2 \quad (42)$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot NA^2 = 1 \quad (43)$$

$$10^{14} \cdot e^{4n} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i} \quad (44)$$

Resulting the unity formula that connect the strong coupling constant a_s , the weak coupling constant a_w , the fine-structure constant a and the gravitational coupling constant $a_G(p)$ for the proton:

$$4 \cdot 10^{14} \cdot NA^2 \cdot a_w^2 \cdot a^2 \cdot a_G(p) = \mu^2 \cdot a_s^2 \quad (45)$$

We reached the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 \quad (46)$$

$$a_s \cdot \cos a^{-1} = 4 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G \quad (47)$$

$$8 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a}) \quad (48)$$

From these expressions resulting the unity formulas that connects the strong coupling constant a_s , the weak coupling constant a_w , the proton to electron mass ratio μ , the fine-structure constant a , the ratio N_1 of electric force to gravitational force between electron and proton, the Avogadro's number NA , the gravitational coupling constant a_G of the electron, the gravitational coupling constant of the proton $a_G(p)$, the strong coupling constant a_s and the weak coupling constant a_w :

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 \quad (49)$$

$$\mu^2 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G(p) \cdot NA^2 \quad (50)$$

$$\mu \cdot N_1 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^3 \cdot NA^2 \quad (51)$$

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot \mu \cdot a_G^2 \cdot NA^2 \cdot N_1 \quad (52)$$

$$\mu^3 \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G(p)^2 \cdot NA^2 \cdot N_1 \quad (53)$$

$$\mu \cdot a_s = 4 \cdot 10^{14} \cdot a_w^2 \cdot a_G \cdot G(p)^2 \cdot NA^2 \cdot N_1^2 \quad (54)$$

$$\mu \cdot a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a \cdot a_G \cdot a_G(p) \cdot NA^2 \cdot N_1 \quad (55)$$

These equations are applicable for all energy scales. In [15] and [16] we found the expressions for the gravitational constant:

$$G = (2eaN_A)^{-2} \frac{\hbar c}{m_e^2} \quad (56)$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (57)$$

$$G = (2e^\pi \alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (58)$$

$$G = i^{4i} e^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (59)$$

$$G = (2e^{\pi-1} 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (60)$$

$$G = \alpha_s^2 (2 \cdot 10^7 \alpha_w \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (61)$$

Therefore the formula for the gravitational constant is:

$$G = (2\alpha N_A)^{-2} \cos^2 \alpha^{-1} \frac{\hbar c}{m_e^2} \quad (62)$$

It presented the theoretical value of the Gravitational constant $G=6.67448 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$. This value is very close to the 2018 CODATA recommended value of gravitational constant and two experimental measurements from a research group announced new measurements based on torsion balances. They ended up measuring $6.674184 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$ and $6.674484 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2$ -of-swinging and angular acceleration methods, respectively. In [17] and [18] resulting in the dimensionless unification of atomic physics and cosmology. The gravitational fine structure constant α_g is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3} = \sqrt{\frac{\alpha_G^3}{\alpha^6}} = 1.886837 \times 10^{-61} \quad (63)$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio ϕ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3}\phi^5} \times 10^{-60} = 1,886837 \times 10^{-61} \quad (64)$$

Resulting the unity formula for the gravitational fine-structure constant α_g :

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3} \quad (65)$$

$$\alpha_g = i^{6i} \cdot (2 \cdot \alpha_s \cdot a^2 \cdot N_A)^{-3} \quad (66)$$

$$\alpha_g = i^{6i} \cdot e^3 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot a^3 \cdot N_A)^{-3} \quad (67)$$

$$\alpha_g = (10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot e^{-1} \cdot \alpha_s^{-1} \cdot a^{-1})^3 \quad (68)$$

$$\alpha_g^2 = (10^{14} \cdot \alpha_w^2 \cdot \alpha_G \cdot e^{-2} \cdot \alpha_s^{-2} \cdot a^{-2})^3 \quad (69)$$

$$\alpha_g = 10^{21} \cdot i^{6i} \cdot \alpha_w^3 \cdot \alpha_G^{3/2} \cdot \alpha_s^{-6} \cdot a^{-3} \quad (70)$$

So the unity formula for the gravitational fine-structure constant α_g is:

$$\alpha_g^2 = 10^{42} \cdot i^{12i} \cdot \alpha_w^6 \cdot \alpha_G^3 \cdot \alpha_s^{-12} \cdot a^{-6} \quad (71)$$

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of

the universe. Resulting the dimensionless unification of the atomic physics and the cosmology:

$$|pl^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (72)$$

$$|pl^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (73)$$

$$|pl^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot a_w \cdot a^3 \cdot N_A)^{-6} \quad (74)$$

$$e^6 \cdot a_s^6 \cdot a^6 \cdot |pl^2 \cdot \Lambda = 10^{42} \cdot a_G^3 \cdot a_w^6 \quad (75)$$

$$a_s^{12} \cdot a^6 \cdot |pl^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot a_G^3 \cdot a_w^6 \quad (76)$$

For the cosmological constant Λ equals:

$$\Lambda = \left(2e a^2 N_A \right)^{-6} \frac{c^3}{G \hbar} \quad (77)$$

$$\Lambda = i^{12i} \left(2 a_s a^2 N_A \right)^{-6} \frac{c^3}{G \hbar} \quad (78)$$

$$\Lambda = i^{12i} e^6 \left(2 \cdot 10^7 a_w a^3 N_A \right)^{-6} \frac{c^3}{G \hbar} \quad (79)$$

$$\Lambda = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{e^2 \alpha_s^2 a^2} \right)^3 \frac{c^3}{G \hbar} \quad (80)$$

$$\Lambda = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{a^2 \alpha_s^4} \right)^3 \frac{c^3}{G \hbar} \quad (81)$$

In [19] we found the Equations of the Universe:

$$\frac{\Lambda G \hbar}{c^3} = 10^{42} i^{12i} \left(\frac{\alpha_G \alpha_w^2}{a^2 \alpha_s^4} \right)^3 \quad (82)$$

$$e^{6\pi} \frac{\Lambda G \hbar}{c^3} = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{a^2 \alpha_s^4} \right)^3 \quad (83)$$

For the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2 \varphi^{-5}}{3^3 \pi \varphi^5} \times 10^{-120} \quad (84)$$

In [20] , [21] and [22] we proved that the shape of the Universe is Poincaré dodecahedral space. The assessment of baryonic matter at the current time was assessed by WMAP to be $\Omega_B = 0.044 \pm 0.004$. From the dimensionless unification of the fundamental interactions the density parameter for the normal baryonic matter is:

$$\Omega_B = e^{-n} = i^{2i} = 0.0432 = 4.32\% \quad (85)$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^i + 1 = 0 \quad (86)$$

$$\Omega_B^i = i^2 \quad (87)$$

$$\Omega_B^{2i} = 1 \quad (88)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B = e^{-1} \cdot a_s \quad (89)$$

$$\Omega_B = a_w^{-1} \cdot a_s^2 \cdot 10^{-7} \quad (90)$$

$$\Omega_B = 2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a}) \quad (91)$$

$$\Omega_B = 2 \cdot NA \cdot a_s \cdot a \cdot a_G^{1/2} \quad (92)$$

$$\Omega_B = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (93)$$

$$\Omega_B = 2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2} \quad (94)$$

$$\Omega_B = 10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2} \quad (95)$$

In [23] we presented the solution for the Density Parameter of Dark Energy. The fraction of the effective mass of the universe attributed to dark energy or the cosmological constant is $\Omega_\Lambda = 0.73 \pm 0.04$. With 73% of the influence on the expansion of the universe in this era, dark energy is viewed as the dominant influence on that expansion. From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0.73576 = 73.57\% \quad (96)$$

So apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \quad (97)$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot \cos \alpha^{-1} \quad (98)$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda = e^{i/a} + e^{-i/a} \quad (99)$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (100)$$

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_\Lambda = 2 \cdot i^{2i} \cdot a_s^{-1} \quad (101)$$

$$\Omega_\Lambda = 2 \cdot 10^{-7} a_s \cdot a_w^{-1} \quad (102)$$

$$\Omega_\Lambda = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1} \quad (103)$$

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (104)$$

$$\Omega_\Lambda = 4 \cdot a \cdot a_G^{1/2} \cdot NA \quad (105)$$

$$\Omega\Lambda=i^{8i}\cdot a^{-2}\cdot a_s^{-4}\cdot a_G^{-1}\cdot NA^{-2} \quad (106)$$

$$\Omega\Lambda=10^7\cdot i^{4i}\cdot a^{-1}\cdot a_w^{-1}\cdot a_G^{-1/2}\cdot NA^{-1} \quad (107)$$

$$\Omega\Lambda=8\cdot 10^7\cdot NA^2\cdot a_w\cdot a^2\cdot a_G\cdot a_s^{-1} \quad (108)$$

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter $\Omega_D=0.23$. From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D=2\cdot e^{1-n}=2\cdot e\cdot i^{2i}=0.2349=23.49\% \quad (109)$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D=2\cdot a_s \quad (110)$$

$$\Omega_D=2\cdot 10^7\cdot e^{-1}\cdot a_w \quad (111)$$

$$\Omega_D=2\cdot (i^{2i}\cdot 10^7\cdot a_w)^{1/2} \quad (112)$$

$$\Omega_D=4\cdot i^{2i}\cdot (e^{i/a}+e^{-i/a})^{-1} \quad (113)$$

$$\Omega_D=10^7\cdot a_w\cdot (e^{i/a}+e^{-i/a}) \quad (114)$$

$$\Omega_D=4\cdot 10^7\cdot a_w\cdot a\cdot a_G^{1/2}\cdot NA \quad (115)$$

$$\Omega_D=16\cdot 10^7\cdot NA^2\cdot a_w\cdot a^2\cdot a_G\cdot (e^{i/a}+e^{-i/a})^{-1} \quad (116)$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_D=2\cdot e\cdot \Omega_B \quad (117)$$

The relationship between the density parameter of dark energy, dark matter and baryonic matter is:

$$\Omega_D\cdot \Omega\Lambda=4\cdot \Omega_B \quad (118)$$

From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0=\Omega_B+\Omega_D+\Omega\Lambda=e^{-n}+2\cdot e^{1-n}+2\cdot e^{-1}=1.0139 \quad (119)$$

It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere, or some other spherical 3-manifold, such as the Poincaré dodecahedral space, all of which are quotients of the 3-sphere. These results prove that the weather space is finite. In [24] we proposed a possible solution for the Equation of state in cosmology. From the dimensionless unification of the fundamental interactions the state equation w has value:

$$w=-24\cdot e^{-n}=-24\cdot i^{2i}=-1.037134 \quad (120)$$

From Euler's identity resulting the beautiful formulas:

$$e^{-in}+1=0$$

$$(-24^{-1}\cdot w)^i+1=0 \quad (121)$$

$$(-24^{-1}\cdot w)^i=-1 \quad (122)$$

$$(-24^{-1}\cdot w)^i=i^2 \quad (123)$$

In [25] , [26] and [27] we presented the law of the gravitational fine-structure constant α_g followed by ratios of maximum and minimum theoretical values for natural quantities. This theory uses quantum mechanics, cosmology, thermodynamics, and special and general relativity. Length l , time t , speed v and temperature T have the same min/max ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} = \frac{t_{min}}{t_{max}} = \frac{v_{min}}{v_{max}} = \frac{T_{min}}{T_{max}} \quad (124)$$

Energy E , mass M , action A , momentum P and entropy S have another min/max ratio, which is the square of α_g :

$$\alpha_g^2 = \frac{E_{min}}{E_{max}} = \frac{M_{min}}{M_{max}} = \frac{A_{min}}{A_{max}} = \frac{P_{min}}{P_{max}} = \frac{S_{min}}{S_{max}} \quad (125)$$

Force F has min/max ratio which is α_g^4 :

$$\alpha_g^4 = \frac{F_{min}}{F_{max}} \quad (126)$$

Mass density has min/max ratio which is α_g^5 :

$$\alpha_g^5 = \frac{\rho_{min}}{\rho_{max}} \quad (127)$$

Length l has the max/min ratio which is:

$$\alpha_g = \frac{l_{min}}{l_{max}} \quad (128)$$

In [28] , [29] , [30] , [31] and [32] we presented the Dimensionless theory of everything. A smallest length in nature thus implies that there is no way to define exact boundaries of objects or elementary particles. Perhaps for the minimum distance l_{min} apply:

$$l_{min} = 2 \cdot e \cdot l_p \quad (129)$$

$$l_{min} = 2 \cdot e^{\pi} \cdot a_s \cdot l_p \quad (130)$$

From expressions apply:

$$\cos \alpha^{-1} = \frac{2l_p}{l_{min}} \quad (131)$$

For the Bohr radius a_0 apply:

$$a_0 = N_A \cdot l_{min} \quad (132)$$

$$a_0 = 2 \cdot e \cdot N_A \cdot l_p \quad (133)$$

$$a_0 = 2 \cdot e^{\pi} \cdot a_s \cdot N_A \cdot l_p \quad (134)$$

The maximum distance l_{max} corresponds to the distance of the universe $l_{max} = \alpha_g^{-1} \cdot l_{min} = 4.657 \times 10^{26}$ m. In [33] and [34] we presented the New Large Number Hypothesis of the universe. The diameter of the observable universe will be calculated to be equal to the product of the ratio of electric force to gravitational force between electron and proton on the reduced Compton wavelength of the electron:

$$2 \cdot RU = N_1 \cdot \tilde{\lambda}_c \quad (135)$$

So apply the expression:

$$RU = e \cdot a \cdot N_1 \cdot N_A \cdot l_p \quad (136)$$

The expressions for the radius of the observable universe are:

$$R_U = \frac{\alpha N_1}{2} \alpha_0 = \frac{N_1}{2\alpha} r_e = \frac{1}{2\mu\alpha_G} r_e = \frac{m_{pl}^2 r_e}{2m_e m_p} = \frac{\hbar c r_e}{2G m_e m_p} = \frac{\alpha \hbar}{2G m_e^2 m_p} \quad (137)$$

We Found the value of the radius of the universe $R_U=4.38 \times 10^{26}$ m. The expressions for the radius of the observable universe are:

$$T_U = \frac{R_U}{c} = \frac{N_1 r_e}{2\alpha c} = \frac{r_e}{2\mu\alpha_G c} = \frac{\alpha N_1 \alpha_0}{2c} = \frac{\alpha \hbar}{2c G m_e^2 m_p} = \frac{\hbar r_e}{2G m_e m_p} \quad (138)$$

The expressions for the gravitational constant are:

$$G = \frac{\hbar c r_e}{2m_e m_p} \frac{1}{R_U} \quad (139)$$

$$G = \frac{\alpha \hbar}{2m_e^2 m_p} \frac{1}{R_U} \quad (140)$$

$$G = \frac{\alpha \hbar}{2c m_e^2 m_p} \frac{1}{T_U} \quad (141)$$

$$G = \frac{\hbar r_e}{2m_e m_p} \frac{1}{T_U} \quad (142)$$

In [35] we found a mass relation for fundamental masses:

$$M_n = a^{-1} \cdot a_g^{(2-n)/3} \cdot m_e \quad (143)$$

$$M_n = a_g^{-n/3} \cdot M_{\min} \quad (144)$$

$$n=0,1,2,3,4,5,6$$

For the minimum mass M_{\min} apply:

$$M_{\min} = \frac{m_{pl}^2}{M_{\max}} = \alpha_g m_{pl} = \frac{\alpha_G}{\alpha^3} m_e = \frac{\sqrt[3]{\alpha_g^2}}{\alpha} m_e \quad (145)$$

$$M_{\min} = (2 \cdot e \cdot N_A)^{-2} \cdot a^{-1} \cdot m_e = 4.06578 \times 10^{-69} \text{ kg} \quad (146)$$

The expressions for the mass of the observable universe M_U are:

$$M_U = a^{-1} \cdot a_g^{-4/3} \cdot m_e = a^3 \cdot a_G^{-2} \cdot m_e = (2 \cdot e \cdot a^2 \cdot N_A)^2 \cdot N_1 \cdot m_p = \mu \cdot a \cdot N_1^2 \cdot m_p \quad (147)$$

For the value of the mass of the observable universe M_U apply $M_U=1.153482 \times 10^{53}$ kg. The expressions who calculate the number of protons in the observable universe are:

$$N_{Edd} = \frac{M_U}{m_p} = \mu \alpha N_1^2 = \frac{N_1}{\alpha_g^{2/3}} = (2e\alpha^2 N_A)^2 N_1 = \left(\frac{r_e}{l_{pl}}\right)^2 N_1 = 6.9 \times 10^{79} \quad (148)$$

For the value of the age of the universe apply $T_U=1.46 \times 10^{18}$ s. The gamma rhythm is a pattern of neuronal oscillations whose frequency ranges from 25 Hz to 100 Hz although 40 Hz is typical. Gamma frequency oscillations are present during wakefulness and REM sleep. The time quantum in the brain t_b , the smallest unit of time that related to the 40 Hz oscillation of the gamma rate:

$$\frac{t_B}{t_{pl}} = \sqrt[3]{\alpha_g^2} \quad (149)$$

For the minimum distance l_{min} apply $l_{min}=2 \cdot e \cdot l_{pl}$. So for the minimum time t_{min} apply:

$$t_{min} = \frac{l_{min}}{c} = \frac{2el_{pl}}{c} = 2et_{pl} \quad (150)$$

From expressions apply:

$$\cos \alpha^{-1} = \frac{2t_{pl}}{t_{min}} \quad (151)$$

In the papers [36] was presented the theoretical value for the Hubble Constant. The formulas for the Hubble Constant are:

$$H_0 = c \sqrt{\frac{e}{6} \Lambda} \quad (152)$$

$$H_0 = \frac{\alpha_g}{t_{pl}} \sqrt{\frac{e}{6}} \quad (153)$$

These equations calculate the theoretical value of the Hubble Constant $H_0=2.36 \times 10^{-18} \text{ s}^{-1}=72.69 \text{ (km/s)/Mpc}$. Also apply the expression:

$$\frac{G\hbar H_0^2}{c^5} = \frac{e}{6} \alpha_g^2 \quad (154)$$

$$\frac{G\hbar H_0^2}{c^5} = \frac{1}{6e^5 (2\alpha^2 N_A)^6} \quad (155)$$

$$\frac{G\hbar H_0^2}{c^5} = \frac{e}{48 (e^\pi \alpha_s \alpha^2 N_A)^3} \quad (156)$$

$$\frac{G\hbar H_0^2}{c^5} = \frac{10^{42}}{6e^5} \left(\frac{\alpha_w^2 \alpha_G}{\alpha_s^2 \alpha^2} \right)^3 \quad (157)$$

$$\frac{6G\hbar H_0^2}{ec^5} = \left(\frac{10^{14} \alpha_w^2 \alpha_G}{e^{2\pi} \alpha^2 \alpha_s^4} \right)^3 \quad (158)$$

$$6e^{6\pi} \frac{G\hbar H_0^2}{c^5} = e \left(\frac{10^{14} \alpha_w^2 \alpha_G}{\alpha^2 \alpha_s^4} \right)^3 \quad (159)$$

$$6e^{5\pi} \frac{G\hbar H_0^2}{c^5} = \frac{1}{\alpha_s^{11}} \left(\frac{10^{14} \alpha_w^2 \alpha_G}{\alpha^2} \right)^3 \quad (160)$$

The Equations of the Universe are:

$$e^{6\pi} \frac{\Lambda G \hbar}{c^3} = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3 \quad (161)$$

$$6e^{5\pi} \frac{G \hbar H_0^2}{c^5} = 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{11}} \quad (162)$$

$$e^{7\pi} \frac{G \hbar \Lambda^2}{c H_0^2} = 6 \cdot 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{13}} \quad (163)$$

3. Euler's identity in unification of the fundamental interactions

In [10] we presented the recommended value for the strong coupling constant $\alpha_s = \alpha_s(M_Z) = e^{-n} = 0.11748\dots$. This value is the current world average value for the coupling evaluated at the Z-boson mass scale. It fits perfectly in the measurement of the strong coupling constant of the European organization for nuclear research (CERN). Also for the value of the strong coupling constant we have the equivalent expressions:

$$\alpha_s = \alpha_s(M_Z) = e \cdot e^{-n} = e \cdot i^{2i} = i^{-2i/n} \cdot i^{2i} = i^{2i - (2i/n)} = i^{2i(n-1)/n} \quad (164)$$

So apply the expressions:

$$e^n \cdot \alpha_s = e \quad (165)$$

$$e^n = e \cdot \alpha_s^{-1} \quad (166)$$

$$e^{-n} = e^{-1} \cdot \alpha_s \quad (167)$$

From Euler's identity and the equation $e^n = e \cdot \alpha_s^{-1}$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(e \cdot \alpha_s^{-1})^i + 1 = 0 \quad (168)$$

$$e^i \cdot \alpha_s^{-i} + 1 = 0 \quad (169)$$

$$e^i \cdot \alpha_s^{-i} = -1 \quad (170)$$

$$e^i \cdot \alpha_s^{-i} = i^2 \quad (171)$$

$$e^i = i^2 \cdot \alpha_s^i \quad (172)$$

$$e^i = -\alpha_s^i \quad (173)$$

$$e^i + \alpha_s^i = 0 \quad (174)$$

Also from Euler's identity and the equation $e^{-n} = e^{-1} \cdot \alpha_s$ resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$(e^{-1} \cdot \alpha_s)^i + 1 = 0 \quad (175)$$

$$e^{-i} \cdot \alpha_s^i + 1 = 0 \quad (176)$$

$$e^{-i} \cdot \alpha_s^i = -1 \quad (177)$$

$$e^{-i} \cdot \alpha_s^i = i^2 \quad (178)$$

$$a_s^i = -e^i \quad (179)$$

$$a_s^i + e^i = 0 \quad (180)$$

$$e^i \cdot a_s^{-i} = i^2 \quad (181)$$

$$a_s^i = i^2 \cdot e^i \quad (182)$$

From the dimensionless unification of the strong nuclear and the weak nuclear interactions equals:

$$e^n \cdot a_s^2 = 10^7 \cdot a_w \quad (183)$$

From Euler's identity and the equation $e^n \cdot a_s^2 = 10^7 \cdot a_w$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i + 1 = 0 \quad (184)$$

$$10^{7i} \cdot a_s^{-2i} \cdot a_w^i + 1 = 0 \quad (185)$$

$$10^{7i} \cdot a_s^{-2i} \cdot a_w^i = -1 \quad (186)$$

$$10^{7i} \cdot a_s^{-2i} \cdot a_w^i = i^2 \quad (187)$$

$$a_s^{2i} = i^2 \cdot 10^{7i} \cdot a_w^i \quad (188)$$

Also from Euler's identity resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i + 1 = 0 \quad (189)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} + 1 = 0 \quad (189)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} = -1 \quad (190)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} = i^2 \quad (191)$$

$$a_s^{2i} = i^2 \cdot 10^{7i} \cdot a_w^i \quad (192)$$

From the dimensionless unification of the strong nuclear and the electromagnetic interactions equals:

$$e^n \cdot a_s \cdot \cos a^{-1} = 1 \quad (193)$$

From Euler's identity and the equation $e^n \cdot a_s \cdot \cos a^{-1} = 1$ resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$(a_s \cdot \cos a^{-1})^i + 1 = 0 \quad (194)$$

$$a_s^i \cdot (\cos a^{-1})^i + 1 = 0 \quad (195)$$

$$a_s^i \cdot (\cos a^{-1})^i = -1 \quad (196)$$

$$a_s^i \cdot (\cos a^{-1})^i = i^2 \quad (197)$$

$$a_s^i = i^2 \cdot (\cos a^{-1})^{-i} \quad (198)$$

$$a_s^i = -(\cos a^{-1})^{-i} \quad (199)$$

$$a_s^i + (\cos a^{-1})^{-i} = 0 \quad (200)$$

The figure 1 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius e^π .

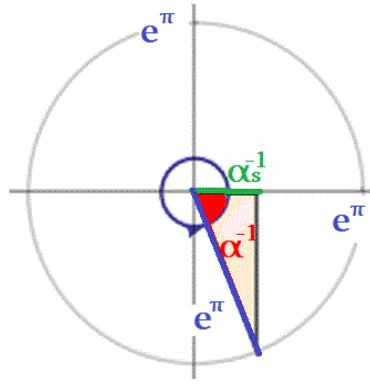


Figure 1. Geometric representation of the unification of the strong nuclear and the electromagnetic interactions.

From the dimensionless unification of the strong nuclear and the electromagnetic interactions equals:

$$e^\pi \cdot \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \quad (201)$$

So from Euler's identity resulting the beautiful formulas:

$$e^{-in} + 1 = 0$$

$$[2^{-1} \cdot \alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha})]^i + 1 = 0 \quad (202)$$

$$2^{-i} \cdot \alpha_s^i \cdot (e^{i/\alpha} + e^{-i/\alpha})^i + 1 = 0 \quad (203)$$

$$2^{-i} \cdot \alpha_s^i \cdot (e^{i/\alpha} + e^{-i/\alpha})^i = -1 \quad (204)$$

$$2^{-i} \cdot \alpha_s^i \cdot (e^{i/\alpha} + e^{-i/\alpha})^i = i^2 \quad (205)$$

$$\alpha_s^i \cdot (e^{i/\alpha} + e^{-i/\alpha})^i = 2^i \cdot i^2 \quad (206)$$

$$\alpha_s^i = 2^i \cdot i^2 \cdot (e^{i/\alpha} + e^{-i/\alpha})^{-i} \quad (207)$$

$$\alpha_s^i = -2^i \cdot (e^{i/\alpha} + e^{-i/\alpha})^{-i} \quad (208)$$

$$\alpha_s^i + 2^i \cdot (e^{i/\alpha} + e^{-i/\alpha})^{-i} = 0 \quad (209)$$

From the dimensionless unification of the weak nuclear and the electromagnetic interactions equals:

$$10^7 \cdot e^\pi \cdot \alpha_w \cdot \cos \alpha^{-1} = e \quad (210)$$

$$10^7 \cdot e^\pi \cdot \alpha_w \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot e \quad (211)$$

The figure 2 below shows the angle in α^{-1} radians. The rotation vector moves in a circle of radius 10^7 .

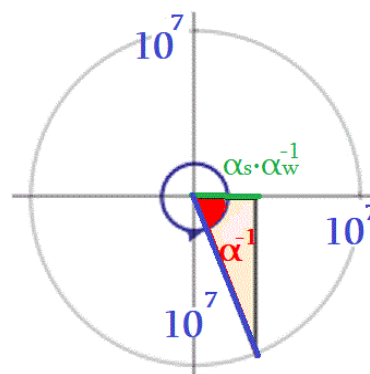


Figure 2. Geometric representation of the unification of the strong, the weak and the electromagnetic interactions.

So from Euler's identity resulting the beautiful formulas:

$$e^{-in}+1=0$$

$$(10^7 \cdot e^{-1} \cdot a_w \cdot \text{cosa}^{-1})^i+1=0 \quad (212)$$

$$10^{7i} \cdot e^{-i} \cdot a_w^i \cdot (\text{cosa}^{-1})^i+1=0 \quad (213)$$

$$10^{7i} \cdot e^{-i} \cdot a_w^i \cdot (\text{cosa}^{-1})^i=-1 \quad (214)$$

$$10^{7i} \cdot e^{-i} \cdot a_w^i \cdot (\text{cosa}^{-1})^i=i^2 \quad (215)$$

$$10^{7i} \cdot a_w^i \cdot (\text{cosa}^{-1})^i=i^2 \cdot e^i \quad (216)$$

$$10^{7i} \cdot a_w^i \cdot (\text{cosa}^{-1})^i=-e^i \quad (217)$$

$$10^{7i} \cdot a_w^i \cdot (\text{cosa}^{-1})^i+e^i=0 \quad (218)$$

The figure 3 below shows the angle in a^{-1} radians. The rotation vector moves in a circle of radius NA^{-1} .

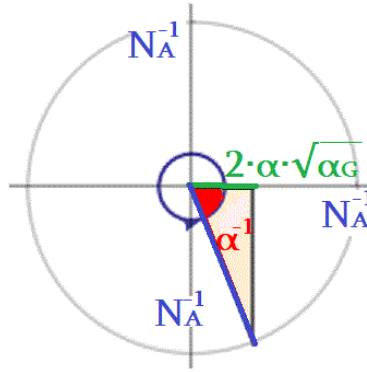


Figure 3. Geometric representation of the unification of the gravitational and the electromagnetic interactions.

From the dimensionless unification of the gravitational and the electromagnetic force equals:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2=1 \quad (219)$$

So from Euler's identity resulting the beautiful formulas:

$$e^{in}+1=0$$

$$e^{in}+4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2=0 \quad (220)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2=-e^{in} \quad (221)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2=i^2 \cdot e^{in} \quad (222)$$

$$4 \cdot a^2 \cdot a_G \cdot NA^2=i^2 \cdot e^{in-2} \quad (223)$$

From Euler's identity, the equation $4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2=1$ and the equation $e^n \cdot a_s=e$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$e^n \cdot a_s=e$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2=1$$

$$e^{in}+4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2=0 \quad (224)$$

$$(e \cdot a s^{-1})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (225)$$

$$e^i \cdot a s^{-i} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (226)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = -e^i \cdot a s^{-i} \quad (227)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = i^2 \cdot e^i \cdot a s^{-i} \quad (228)$$

$$4 \cdot e^{2-i} \cdot a^2 \cdot a_G \cdot N A^2 = i^2 \cdot a s^{-i} \quad (229)$$

$$4 \cdot e^{2-i} \cdot a^2 \cdot a_G \cdot a s^i \cdot N A^2 = i^2 \quad (230)$$

From Euler's identity, the equations $4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 1$ and $e^n \cdot a s^2 = 10^7 \cdot a_w$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^n \cdot a s^2 = 10^7 \cdot a_w$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 1$$

$$e^{in} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0$$

$$(10^7 \cdot a s^{-2} \cdot a_w)^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (231)$$

$$(10^{-7} \cdot a s^2 \cdot a_w^{-1})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (232)$$

$$10^{-7i} \cdot a s^{2i} \cdot a_w^{-i} + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (233)$$

$$(10^7 \cdot a s^{-2} \cdot a_w)^i = -4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 \quad (234)$$

$$(10^{-7} \cdot a s^2 \cdot a_w^{-1})^i = 4 \cdot i^2 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 \quad (235)$$

$$10^{-7i} \cdot a s^{2i} \cdot a_w^{-i} = -4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 \quad (236)$$

$$10^{-7i} \cdot a s^{2i} \cdot a_w^{-i} = 4 \cdot i^2 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 \quad (237)$$

$$a s^{2i} = 4 \cdot i^2 \cdot 10^{7i} \cdot e^2 \cdot a^2 \cdot a_G \cdot a_w^i \cdot N A^2 \quad (238)$$

Also from Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^n \cdot a s = e$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 1$$

$$(e^{-1} \cdot a s)^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (239)$$

$$(a_w^{-1} \cdot a s^2 \cdot 10^{-7})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (240)$$

$$[2^{-1} \cdot a s \cdot (e^{i/a} + e^{-i/a})]^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (241)$$

$$(2 \cdot N A \cdot a s \cdot a \cdot a_G^{1/2})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (242)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})]^i + 14 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (243)$$

$$(2 \cdot 10^7 \cdot N A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (244)$$

$$(10^{-7} \cdot a_g^{1/3} \cdot a s^2 \cdot a \cdot a_w^{-1} \cdot a_G^{-1/2})^i + 4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N A^2 = 0 \quad (245)$$

From the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions equals:

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1 \quad (246)$$

So from Euler's identity resulting the beautiful formulas:

$$e^{-in}+1=0$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$e^{in}+2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (247)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=-e^{in} \quad (248)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=i^2 \cdot e^{in} \quad (249)$$

$$2 \cdot e^{n(1-i)} \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=i^2 \quad (250)$$

From Euler's identity, the equations $2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$ and $e^n \cdot a_s=e$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$(e \cdot a_s^{-1})^i+2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (251)$$

$$e^i \cdot a_s^{-i}+2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (252)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=-e^i \cdot a_s^{-i} \quad (253)$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=i^2 \cdot e^i \cdot a_s^{-i} \quad (254)$$

$$2 \cdot e^{n-i} \cdot a_s^{1-i} \cdot a \cdot aG^{1/2} \cdot NA=i^2 \quad (255)$$

Also from Euler's identity, the equations $2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$ and $e^n \cdot a_s^2=10^7 \cdot a_w$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$e^n \cdot a_s^2=10^7 \cdot a_w$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i+2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (256)$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i+2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (257)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i}+2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (258)$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i=-2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (259)$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i=2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (260)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i}=-2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (261)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i}=2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (262)$$

$$a_s^{2i}=2 \cdot i^2 \cdot e^n \cdot 10^{7i} \cdot a_s \cdot a \cdot aG^{1/2} \cdot a_w^i \cdot NA \quad (263)$$

Also from Euler's identity, the equations $2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$ and $4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2=1$ resulting the beautiful formulas:

$$e^{in}+1=0$$

$$4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2 = 1$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA = 1$$

$$(2 \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA)^i + 4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2 = 0 \quad (264)$$

$$4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2 = -(2 \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA)^i \quad (265)$$

$$4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2 = i^2 \cdot (2 \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA)^i \quad (266)$$

$$4 \cdot e^2 \cdot a^2 \cdot aG \cdot NA^2 = 2^i \cdot i^2 \cdot a_s^i \cdot a^i \cdot aG^{1/2i} \cdot NA^i \quad (267)$$

From the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions equals:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot NA^2 \quad (268)$$

From Euler's identity, the equations $a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot NA^2$ and $e^n \cdot a_s = e$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^n \cdot a_s = e$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 1$$

$$e^{in} + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 0 \quad (269)$$

$$(e^{-1} \cdot a_s)^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 0 \quad (270)$$

$$e^{-i} \cdot a_s^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 0 \quad (271)$$

$$e^{-i} \cdot a_s^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 \quad (272)$$

$$e^{-i} \cdot a_s^i = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 \quad (273)$$

$$a_s^{i+2} = 4 \cdot i^2 \cdot e^i \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot NA^2 \quad (274)$$

From Euler's identity, the equations $a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot NA^2$ and $e^n \cdot a_s^2 = 10^7 \cdot a_w$ resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$e^n \cdot a_s^2 = 10^7 \cdot a_w$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 1$$

$$e^{in} + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 0$$

$$(10^7 \cdot a_s^{-2} \cdot a_w)^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 0 \quad (275)$$

$$(10^{-7} \cdot a_s^2 \cdot a_w^{-1})^i + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 0 \quad (276)$$

$$10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} + 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = 0 \quad (277)$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = -10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} \quad (278)$$

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 = i^2 \cdot 10^{-7i} \cdot a_s^{2i} \cdot a_w^{-i} \quad (279)$$

$$a_s^{2(i+1)} = 4 \cdot i^2 \cdot 10^{14+7i} \cdot a_w^{2+i} \cdot a^2 \cdot aG \cdot NA^2 \quad (280)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in}+1=0$$

$$2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=1$$

$$(e^{-1} \cdot a_s)^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0$$

$$(aw^{-1} \cdot a_s^2 \cdot 10^{-7})^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (281)$$

$$[2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})]^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (282)$$

$$(2 \cdot NA \cdot a_s \cdot a \cdot aG^{1/2})^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (283)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})]^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (284)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (285)$$

$$(10^{-7} \cdot ag^{1/3} \cdot a_s^2 \cdot a \cdot aw^{-1} \cdot aG^{-1/2})^i + 2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA=0 \quad (286)$$

So resulting the formulas:

$$(e^{-1} \cdot a_s)^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (287)$$

$$(aw^{-1} \cdot a_s^2 \cdot 10^{-7})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (288)$$

$$[2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})]^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (289)$$

$$(2 \cdot NA \cdot a_s \cdot a \cdot aG^{1/2})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (290)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})]^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (291)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (292)$$

$$(10^{-7} \cdot ag^{1/3} \cdot a_s^2 \cdot a \cdot aw^{-1} \cdot aG^{-1/2})^i = -2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (293)$$

$$(10^{-7} \cdot ag^{1/3} \cdot a_s^2 \cdot a \cdot aw^{-1} \cdot aG^{-1/2})^i = 2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (294)$$

$$10^{-7i} \cdot ag^{1/3i} \cdot a_s^{2i} \cdot a^i \cdot aw^{-i} \cdot aG^{-1/2i} = 2 \cdot i^2 \cdot e^n \cdot a_s \cdot a \cdot aG^{1/2} \cdot NA \quad (295)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in}+1=0$$

$$4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=1$$

$$(e \cdot a_s^{-1})^i + 4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=0 \quad (296)$$

$$(aw \cdot a_s^{-2} \cdot 10^7)^i + 4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=0 \quad (297)$$

$$[2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=0 \quad (298)$$

$$[(2 \cdot NA \cdot a_s \cdot a \cdot aG^{1/2})^{-1}]^i + 4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=0 \quad (299)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=0 \quad (300)$$

$$[(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^{-1}]^i + 4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=0 \quad (301)$$

$$(10^7 \cdot ag^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot aw \cdot aG^{1/2})^i + 4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2=0 \quad (302)$$

So resulting the formulas:

$$(e \cdot a_s^{-1})^i = -4 \cdot 10^{14} \cdot aw^2 \cdot a^2 \cdot aG \cdot a_s^{-2} \cdot NA^2 \quad (303)$$

$$(a_w \cdot a_s^{-2} \cdot 10^7)^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (304)$$

$$[2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (305)$$

$$[(2 \cdot NA \cdot a_s \cdot a \cdot a_G^{1/2})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (306)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (307)$$

$$[(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^{-1}]^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (308)$$

$$(10^7 \cdot a_g^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot a_w \cdot a_G^{1/2})^i = -4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (309)$$

So resulting the formulas:

$$e^i \cdot a_s^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (310)$$

$$a_w^i \cdot a_s^{-2i} \cdot 10^{7i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (311)$$

$$2^i \cdot a_s^{-i} \cdot (e^{i/a} + e^{-i/a})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (312)$$

$$(2 \cdot NA \cdot a_s \cdot a \cdot a_G^{1/2})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (313)$$

$$2^{-i} \cdot e^{-i} \cdot 10^{7i} \cdot a_w^i \cdot (e^{i/a} + e^{-i/a})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (314)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2})^{-i} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (315)$$

$$10^{7i} \cdot a_g^{-i/3} \cdot a_s^{-2i} \cdot a^{-i} \cdot a_w^i \cdot a_G^{i/2} = 4 \cdot i^2 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot a_s^{-2} \cdot NA^2 \quad (316)$$

4. The cosine of angle in a^{-1} radians

The fine-structure constant is one of the most fundamental constants of physics. It describes the strength of the force of electromagnetism between elementary particles in what is known as the standard model of particle physics. For the cosine of angle in a^{-1} radians equals:

$$\cos a^{-1} = i^{2i} \cdot a_s^{-1} \quad (317)$$

$$\cos a^{-1} = e \cdot 10^{-7} \cdot i^{2i} \cdot a_w^{-1} \quad (318)$$

$$\cos a^{-1} = 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (319)$$

$$\cos a^{-1} = 2 \cdot a \cdot a_G^{1/2} \cdot NA \quad (320)$$

$$\cos a^{-1} = 2^{-1} \cdot i^{8i} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot NA^{-2} \quad (321)$$

$$\cos a^{-1} = 2^{-1} \cdot 10^7 \cdot i^{4i} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot NA^{-1} \quad (322)$$

$$\cos a^{-1} = e^{-n} \cdot a_s^{-1} \quad (323)$$

$$\cos a^{-1} = 10^{-7} \cdot e^{-1-n} \cdot a_w^{-1} \quad (324)$$

$$\cos a^{-1} = 10^{-7} \cdot a_s \cdot a_w^{-1} \quad (325)$$

$$\cos a^{-1} = 2 \cdot a \cdot a_G^{1/2} \cdot NA \quad (326)$$

$$\cos a^{-1} = 2^{-1} \cdot e^{-4n} \cdot a^{-2} \cdot a_s^{-4} \cdot a_G^{-1} \cdot NA^{-2} \quad (327)$$

$$\cos a^{-1} = 2^{-1} \cdot 10^7 \cdot e^{-2n} \cdot a^{-1} \cdot a_w^{-1} \cdot a_G^{-1/2} \cdot NA^{-1} \quad (328)$$

$$\cos a^{-1} = 4 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G \cdot a_s^{-1} \quad (329)$$

The figure 4 below shows the geometric representation of the solution for the Density Parameter of Dark Energy.

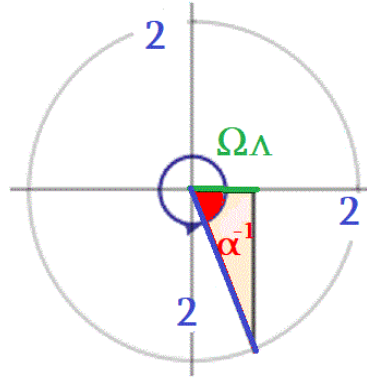


Figure 4. Geometric representation of the solution for the Density Parameter of Dark Energy.

The figure 5 and 6 below shows the geometric representation of the unification of the fundamental interactions.

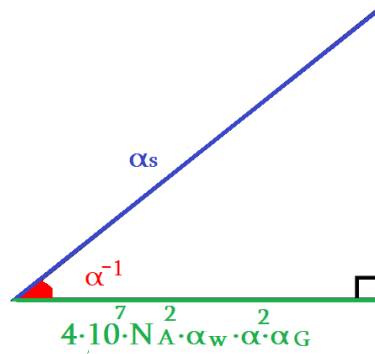


Figure 5. Geometric representation of the unification of the fundamental interactions.

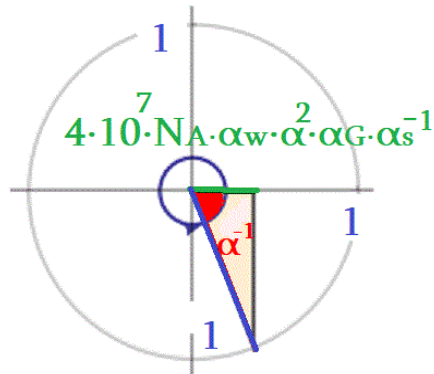


Figure 6. The unification of the fundamental interactions.

Also for the cosine of angle in α^{-1} radians equals:

$$\cos \alpha^{-1} = \frac{L_H^2}{2R_d^2} \quad (330)$$

$$\cos \alpha^{-1} = \frac{2l_{pl}}{l_{min}} \quad (331)$$

$$\cos \alpha^{-1} = \frac{2N_A l_{pl}}{\alpha_0} \quad (332)$$

$$\cos \alpha^{-1} = \frac{2t_{pl}}{t_{min}} \quad (333)$$

$$\cos \alpha^{-1} = \frac{\alpha N_1 N_A l_{pl}}{R_U} \quad (334)$$

$$\cos \alpha^{-1} = \frac{\Lambda c^2}{6H_0} \quad (335)$$

Also for the cosine of angle in α^{-1} radians equals:

$$\cos \alpha^{-1} = \frac{\Omega_B}{\alpha_s} \quad (336)$$

$$\cos \alpha^{-1} = \frac{\Omega_\Lambda}{2} \quad (337)$$

$$\cos \alpha^{-1} = \frac{2\Omega_D^{-1}}{e^\pi} \quad (338)$$

$$\cos \alpha^{-1} = \frac{2\Omega_B}{\Omega_D} \quad (339)$$

So resulting the formulas:

$$\frac{L_H^2}{(2R_d)^2} = \frac{l_{pl}}{l_{min}} \quad (340)$$

$$\left(\frac{L_H}{2R_d} \right)^2 = \frac{l_{pl}}{l_{min}} \quad (341)$$

5. Gelfond's constant in unification of the fundamental interactions

Gelfond's constant, in mathematics, is the number e^π , e raised to the power π . Like e and π , this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond–Schneider theorem, noting that $e^\pi = (e^{i\pi})^{-i} = (-1)^{-i} = i^{-2i}$. From the Dimensionless unification of the fundamental interactions resulting the expressions for the Gelfond's constant:

$$e^\pi = (e^{i\pi})^{-i}$$

$$e^\pi = (-1)^{-i}$$

$$e^\pi = i^{-2i}$$

$$e^\pi = e \cdot a_s^{-1} \quad (342)$$

$$e^\pi = 10^7 \cdot a_w \cdot a_s^{-2} \quad (343)$$

$$e^n = 2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (344)$$

$$e^n = (2 \cdot NA \cdot a_s \cdot a \cdot aG^{1/2})^{-1} \quad (345)$$

$$e^n = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (346)$$

$$e^n = (2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot aG^{1/2})^{-1} \quad (347)$$

$$e^n = 10^7 \cdot a_g^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot a_w \cdot aG^{1/2} \quad (348)$$

So resulting the formulas:

$$a_g^{1/3} \cdot a_s^2 \cdot a \cdot e^n = 10^7 \cdot a_w \cdot aG^{1/2} \quad (349)$$

$$a_g^2 \cdot a_s^{12} \cdot a^6 \cdot e^{6n} = 10^{42} \cdot a_w^6 \cdot aG^3 \quad (350)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(e^n)^i + 1 = 0$$

$$(e \cdot a_s^{-1})^i + 1 = 0 \quad (351)$$

$$(a_w \cdot a_s^{-2} \cdot 10^7)^i + 1 = 0 \quad (352)$$

$$[2 \cdot a_s^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 1 = 0 \quad (353)$$

$$[(2 \cdot NA \cdot a_s \cdot a \cdot aG^{1/2})^{-1}]^i + 1 = 0 \quad (354)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1}]^i + 1 = 0 \quad (355)$$

$$[(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot aG^{1/2})^{-1}]^i + 1 = 0 \quad (356)$$

$$(10^7 \cdot a_g^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot a_w \cdot aG^{1/2})^i + 1 = 0 \quad (357)$$

Also from the Dimensionless unification of the fundamental interactions resulting the expressions:

$$e^{-n} = e^{-1} \cdot a_s \quad (358)$$

$$e^{-n} = a_w^{-1} \cdot a_s^2 \cdot 10^{-7} \quad (359)$$

$$e^{-n} = 2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a}) \quad (360)$$

$$e^{-n} = 2 \cdot NA \cdot a_s \cdot a \cdot aG^{1/2} \quad (361)$$

$$e^{-n} = 2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) \quad (362)$$

$$e^{-n} = 2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot a_w \cdot a \cdot aG^{1/2} \quad (363)$$

$$e^{-n} = 10^{-7} \cdot a_g^{1/3} \cdot a_s^2 \cdot a \cdot a_w^{-1} \cdot aG^{-1/2} \quad (364)$$

From Euler's identity resulting the beautiful formulas:

$$(e^{-1} \cdot a_s)^i + 1 = 0 \quad (365)$$

$$(a_w^{-1} \cdot a_s^2 \cdot 10^{-7})^i + 1 = 0 \quad (366)$$

$$[2^{-1} \cdot a_s \cdot (e^{i/a} + e^{-i/a})]^i + 1 = 0 \quad (367)$$

$$(2 \cdot NA \cdot as \cdot a \cdot aG^{1/2})^i + 1 = 0 \quad (368)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})]^i + 1 = 0 \quad (369)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^i + 1 = 0 \quad (370)$$

$$(10^{-7} \cdot ag^{1/3} \cdot as^2 \cdot a \cdot aw^{-1} \cdot aG^{-1/2})^i + 1 = 0 \quad (371)$$

Also from Euler's identity resulting the beautiful formulas:

$$(aw \cdot as^{-2} \cdot 10^7)^i = i^2 \quad (372)$$

$$[2 \cdot as^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = i^2 \quad (373)$$

$$(2 \cdot NA \cdot as \cdot a \cdot aG^{1/2})^i = i^2 \quad (374)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})^{-1}]^i = i^2 \quad (375)$$

$$[(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^{-1}]^i = i^2 \quad (376)$$

$$(10^7 \cdot ag^{-1/3} \cdot as^{-2} \cdot a^{-1} \cdot aw \cdot aG^{1/2})^i = i^2 \quad (377)$$

From Euler's identity resulting the beautiful formulas:

$$e^{in} + 1 = 0$$

$$(e^{-1} \cdot as)^i = i^2$$

$$(aw^{-1} \cdot as^2 \cdot 10^{-7})^i = i^2 \quad (378)$$

$$[2^{-1} \cdot as \cdot (e^{i/a} + e^{-i/a})]^i = i^2 \quad (379)$$

$$(2 \cdot NA \cdot as \cdot a \cdot aG^{1/2})^i = i^2 \quad (380)$$

$$[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot aw \cdot (e^{i/a} + e^{-i/a})]^i = i^2 \quad (381)$$

$$(2 \cdot 10^7 \cdot NA \cdot e^{-1} \cdot aw \cdot a \cdot aG^{1/2})^i = i^2 \quad (382)$$

$$(10^{-7} \cdot ag^{1/3} \cdot as^2 \cdot a \cdot aw^{-1} \cdot aG^{-1/2})^i = i^2 \quad (383)$$

$$10^{-7i} \cdot ag^{i/3} \cdot as^{2i} \cdot a^i \cdot aw^{-i} \cdot aG^{-i/2} = i^2 \quad (384)$$

$$ag^{i/3} \cdot as^{2i} \cdot a^i = 10^{7i} \cdot i^2 \cdot aw^i \cdot aG^{i/2} \quad (385)$$

$$ag^{2i} \cdot as^{12i} \cdot a^{6i} = 10^{7i} \cdot i^2 \cdot aw^i \cdot aG^{i/2} \quad (386)$$

Archimedes constant π appears in many types in all fields of mathematics and physics. It is found in many types of trigonometry and geometry, especially in terms of circles, ellipses or spheres. It is also found in various types from other disciplines, such as Cosmology, numbers, Statistics, fractals, thermodynamics, engineering, and electromagnetism. Also Archimedes constant π appears in the cosmological constant, Heisenberg's uncertainty principle, Einstein's field equation of general relativity, Coulomb's law for the electric force in vacuum, Magnetic permeability of free space, Period of a simple pendulum with small amplitude, Kepler's third law of planetary motion, the buckling formula, etc. From the Dimensionless unification of the fundamental interactions Archimedes constant π equals:

$$\pi = \ln(e \cdot as^{-1}) \quad (387)$$

$$\pi = \ln(10^7 \cdot aw \cdot as^{-2}) \quad (388)$$

$$\pi = \ln[2 \cdot as^{-1} \cdot (e^{i/a} + e^{-i/a})^{-1}] \quad (389)$$

$$\pi = -\ln(2 \cdot N_A \cdot a_s \cdot a \cdot a_G^{1/2}) \quad (390)$$

$$\pi = \ln[2^{-1} \cdot e^{-1} \cdot 10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a})^{-1}] \quad (391)$$

$$\pi = -\ln(2 \cdot 10^7 \cdot N_A \cdot e^{-1} \cdot a_w \cdot a \cdot a_G^{1/2}) \quad (392)$$

$$\pi = \ln(10^7 \cdot a_G^{-1/3} \cdot a_s^{-2} \cdot a^{-1} \cdot a_w \cdot a_G^{1/2}) \quad (393)$$

6. Conclusions

It presented the dimensionless unification of the fundamental interactions. From the most beautiful equation in mathematics Euler's identity it presented new beautiful equations of unification of the fundamental interactions. We calculated new unity formulas that connect the coupling constants of the fundamental forces.

We reached the dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e \cdot a_s = 10^7 \cdot a_w$$

$$e^n \cdot a_s^2 = 10^7 \cdot a_w$$

The imaginary dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$a_s^2 = i^{2i} \cdot 10^7 \cdot a_w$$

$$a_s^{2i} = i^2 \cdot 10^{7i} \cdot a_w^i$$

We reached the dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$e^n \cdot a_s \cdot \cos a^{-1} = 1$$

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2$$

The imaginary dimensionless unification of the strong nuclear and the electromagnetic interactions:

$$a_s \cdot \cos a^{-1} = i^{2i}$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

$$a_s^i + (\cos a^{-1})^{-i} = 0$$

$$a_s^i + 2^i \cdot (e^{i/a} + e^{-i/a})^{-i} = 0$$

We reached the dimensionless unification of the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot e^n \cdot a_w \cdot \cos a^{-1} = e$$

$$10^7 \cdot e^n \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e$$

The imaginary dimensionless unification of the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot a_w \cdot \cos a^{-1} = e \cdot i^{2i}$$

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot e \cdot i^{2i}$$

$$10^{7i} \cdot a_w^i \cdot (\cos a^{-1})^i + e^i = 0$$

We reached the dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot a_w \cdot \cos a^{-1} = a_s$$

The imaginary dimensionless unification of the strong nuclear, the weak nuclear and the electromagnetic interactions:

$$10^7 \cdot a_w \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot a_s$$

We reached the dimensionless unification of the gravitational and the electromagnetic interactions:

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot NA^2 = 1$$

$$a^{-2} \cdot \cos^2 a^{-1} = 4 \cdot a_G \cdot NA^2$$

The imaginary dimensionless unification of the gravitational and the electromagnetic interactions:

$$16 \cdot a^2 \cdot a_G \cdot NA^2 = (e^{i/a} + e^{-i/a})^2$$

$$4 \cdot a^2 \cdot a_G \cdot NA^2 = i^2 \cdot e^{in-2}$$

We reached the dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interactions:

$$2 \cdot e^n \cdot a_s \cdot a \cdot a_G^{1/2} \cdot NA = 1$$

$$2 \cdot e^{4n} \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot NA^2 = 1$$

$$e^{4n} \cdot a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot NA^2 = 1$$

The imaginary dimensionless unification of the strong nuclear, the gravitational and the electromagnetic interaction

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i}$$

$$2 \cdot a^2 \cdot \cos a^{-1} \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i}$$

$$a^2 \cdot (e^{i/a} + e^{-i/a}) \cdot a_s^4 \cdot a_G \cdot NA^2 = i^{8i}$$

$$4 \cdot e^{2-i} \cdot a^2 \cdot a_G \cdot a_s^i \cdot NA^2 = i^2$$

We reached the dimensionless unification of the weak nuclear, the gravitational and electromagnetic interactions:

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = e^2$$

$$4 \cdot 10^{14} \cdot e^{2n} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot NA^2 = 1$$

The imaginary dimensionless unification of the weak nuclear, the gravitational and electromagnetic interactions:

$$4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2 = i^{4i} \cdot e^2$$

$$4 \cdot 10^{14} \cdot a^2 \cdot \cos^2 a^{-1} \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i}$$

$$10^{14} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i}$$

$$10^{14} \cdot e^{4n} \cdot a^2 \cdot (e^{i/a} + e^{-i/a})^2 \cdot a_w^2 \cdot a_G \cdot NA^2 = i^{8i}$$

We reached the dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$a_s^2 = 4 \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2$$

$$a_s \cdot \cos a^{-1} = 4 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G$$

The imaginary dimensionless unification of the strong nuclear, the weak nuclear, the gravitational and the electromagnetic interactions:

$$8 \cdot 10^7 \cdot NA^2 \cdot a_w \cdot a^2 \cdot a_G = a_s \cdot (e^{i/a} + e^{-i/a})$$

$$a_s^{2i} = 2 \cdot i^2 \cdot e^n \cdot 10^{7i} \cdot a_s \cdot a \cdot a_G^{1/2} \cdot a_w^i \cdot NA$$

$$a_s^{i+2} = 4 \cdot i^2 \cdot e^i \cdot 10^{14} \cdot a_w^2 \cdot a^2 \cdot a_G \cdot NA^2$$

$$a_s^{2(i+1)} = 4 \cdot i^2 \cdot 10^{14+7i} \cdot a_w^{2+i} \cdot a^2 \cdot a_G \cdot NA^2$$

The Equations of the Universe are:

$$e^{6\pi} \frac{\Lambda G \hbar}{c^3} = 10^{42} \left(\frac{\alpha_G \alpha_w^2}{\alpha^2 \alpha_s^4} \right)^3$$

$$6e^{5\pi} \frac{G \hbar H_0^2}{c^5} = 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{11}}$$

$$e^{7\pi} \frac{G \hbar \Lambda^2}{c H_0^2} = 6 \cdot 10^{42} \frac{\alpha_w^6 \alpha_G^3}{\alpha^6 \alpha_s^{13}}$$

All these equations are applicable for all energy scales.

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