Energetic Sheaves: Higher Quantization and Symmetry

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Abstract

This document is devoted to understanding and implementing the energy numbers, which were recently explicated very clearly by Emmerson in his recent paper. Through this line of reasoning, it becomes apparent that the algebra defined by the energy numbers are indeed the natural algebra for categorifying quantization. We also develop a notion of symmetric topological vector spaces, and forcing on said spaces motivated by homological mirror symmetry.

Preamble

This document is a bit scattered, as it as actually the combination of two separate documents, which seemed to dovetail quite nicely with one another.

While I have been implored numerous times by Andrius Kulikauskus to avoid jargon and to speak in an informal language, I find myself faced with a quandary. At once, the language I use feels to me quite natural and informal enough (in the sense that the computations are quite "soft"); yet, I hope this writing strikes the appropriate balance between approachable and insightful. As for what is approachable, I cannot say for certain. I take my biases for granted, and so quite naturally the language gap is obfuscated to me.

Acknowledgments

Thank you to Parker Emmerson and Kyle Flynn for the stimulating conversations. Flynn in particular taught me some quant-fin tricks, as well as collaborated with me in translating his highly lucrative algorithm into a formal paper, which remains unpublished. This experience was very illuminating, as it allowed me to apply my knowledge and build confidence, as well as learn much along the way.

Lastly, I am grateful to Allah the one God for giving me life and prosperity. This cannot be understated, even if repeated ad infinitum.

Contents

1 Introduction

Let us begin by establishing a sheaf $\mathcal{S}(\mathcal{N})$ of nuclear spaces, which are such that each is equipped with a foliation with a maximal atlas (see [1]). Write $Fol_{\mathcal{A}}(n)$ for each foliated subspace $n \subset \mathcal{N}$, where we drop $Fol_{\mathcal{A}}$ when it is understood that the space is foliated from context. As a dummy model, consider the quantization map:

$$(n_i \in n) \longmapsto \hat{i}$$

$$\hat{i} = \int_{\tau} \partial^r(i)$$

where r is the rank of the fiber bundle over n_i and τ is the truth value of a proposition about the particle which represents an underlying algebra.

Recall that the tensor product of a nuclear space with a collection of loops $End(\ell_0 \longmapsto \ell_0)$ is an $E_\infty space$ when the looped category of endomorphisms is isomorphic to $Aut(\sum_{\mathbb{I}_{\frac{1}{2}}} \ell_{n \to n})$, where $\mathbb{I}$ is the interval $[-\frac{1}{2}, \frac{1}{2}]$.

In addition to the canonical first and second quantizations, we introduce the kth quantization $Q_k(n_i)$ for every point $\{*\} \in n$ encompassed by a local coordinate system, which is generated by a $Dim(n)$-tuple. This coordinate system gives rise to a parameterized homotopy theory on for every nuclear subspace with the subspace topology, which is parameterized by a smooth embedding $\{n_{\tilde{n}}\}_{\tilde{n} \in n} \hookrightarrow \mathcal{N}$ for every identity arrow $\mathcal{N} \longrightarrow \mathcal{N}$. We define k to be a tuple,

$$k \supseteq \{\int_0^1 \alpha_m, ..., \int_{m-1}^m \alpha_{m+o}\}$$

such that every realization of k is an integral of order $card(k)$. Here,

$$o = codim(im(k))$$

for a map

$$k \longmapsto Q_k(\bullet)$$

. This gives us the classic Haefliger groupoid Γ_q with $q = o$.

In order to describe our quantization, we will explore the notion of a *barycenter* for a system of miniscule co-weights. This idea is inspired by both number-theoretic investigations, as well as models of short-distance gravitational effects. Properly, we use the Picard functor on a character $\mathfrak{c} = char(n_{i_{j \in i}})$. We will write $Pic(j)$ for this map. Essentially, $Pic(j)$ equips a 2-small sub-object out of a small category $\mathscr{C}$ and sends it to the category of spaces equipped with canonical line bundles. This superizes the isotropy groupoid of the point lying in the projection of $\mathscr{C}$. The barycenter of $\mathcal{C}$ is defined to be

$$\frac{\sum_{j=0}^{\infty} \vec{i_j}}{d(Cent(i), j)}$$

where i_j is a uniformizer for the algebra generating the space n_i.

We will assign n to a positive parity if it lies its canonical loop ℓ_n, in the interval $(0, \frac{1}{2}]$, a negative parity if $-\frac{1}{2} \leq \ell < 0$, and a neutral polarity if $\ell_n \sim 0$. Thus:

$$n = \begin{cases} n^- & \text{if } -\frac{1}{2} \leq \ell_n < 0 \\ n & \text{if } \ell_n \sim 0 \\ n^+ & \text{if } 0 < \ell_n \leq \frac{1}{2} \end{cases}$$

This assignment determines the orientation of $Fol_{\mathcal{A}}(n)$, specifically by mapping the parity of n to the parity of $\mathcal{A}$ via the bijection $n^{\pm} \leftrightarrows \mathcal{A}^{\pm}$. Geometrically, we can think of this as a connection between n and the barycenter of a distinguished triangle in $\mathcal{A}$. We can write ∇ for this connection, and assign a group action to it via the map:

$$\nabla \times \mathfrak{G} \longmapsto \mathfrak{tors}_{\nabla}$$

The image of the map is a torsion class which acts equivariantly on an E_∞ space representing a collection of actions of the holonomy group $Hol_{\Delta}^{\mathscr{G}}$, where $\mathscr{G}$ is the inertia groupoid of the barycenter of $\mathcal{A}$, which is parameterized by i_j. We identify this map here as a modification of the Picard functor. In special cases, when the source of the map is non-commutative, be gain a genuinely quantum system, which is analogous to the system $\hat{P}$ as constructed in [2]. Set

$$\theta(n) \equiv \theta(|n|) = \theta \cdot \mathfrak{tors}_{\nabla}$$

where $|n|$ is the generic realization of n. In our theory, we have a canonical action of $\theta(n)$ on $Cent(\mathcal{A})$, which passes to an action on the etale fiber space $|\mathcal{A}|_{Et}$.

1.1 Action of a Bordism Ring

Let $W \in R_W$ be a bordism. We define here the canonical action

$$R_W \cdot \theta(n) \xrightarrow{can} \theta(\mathcal{EPR})$$

where $\mathcal{EPR}$ is an Einstein-Podolsky-Rosen bridge. For every pair (γ, δ) in $\mathcal{EPR}$, there is a contactomorphism $Cont_{\gamma,\delta} : \gamma \longrightarrow \delta$. The odd actions of the R_W group transform the collection $Cont$ of contactomorphisms into an overtwisted topological space.

Suppose we are given an ensemble of entangled quasi-quanta, $[\hat{q}]$, which admits a degeneration into components:

$$\Theta = \bigcup_p \{\hat{q}_p\}$$

where $\Theta \cong [\theta]$. Then, there is a categorical equivalence between $\theta(n)$ and the action of each individual θ_p on every $\hat{q}_p$. Here, Θ is a monoid with weak equivalences given by the map $\Theta \longmapsto B\Theta$, which classifies the tangent bundle associated with each canonical transformation $\theta(n) : \mathcal{EPR} \longmapsto \mathcal{EPR}'$.

Proposition 1.1. *The appropriate classifying space for the transformation given by $\theta(n)$ is $B\mathbb{E}$, the classifying space of the energy numbers.*

The proof of this proposition will be made manifest as we develop our line of thought. More precisely, for a map $(n \sim e) \longmapsto B\mathbb{K}$, where K is a field whose completion is non-degenerate, and whose composition of additive identity with inverse yields the multiplicative identity, the classifying space $B\mathbb{K}$ is in fact strongly equivalent to $B\mathbb{E}$, such that $B\mathbb{K} \equiv B\mathbb{E}$.

1.2 Energy Numbers

Emmerson [3] recently published a beautifully concise and invigorating paper on his energy numbers, lucidly defining their behavior and axioms. The crucial innovation of the energy numbers ($\mathbb{E}$) is the prescription of an element $v_{\mathbb{E}}$, which is analogous to zero, but which represents an unobserved or non-appreciable physical state. Among the many axioms listed by Emmerson, the central axiom is

Axiom 1.1. $\forall \alpha \in \mathbb{E} \quad \alpha \cdot \alpha^{-1} = 1_{\mathbb{E}}$

which holds even for $v_{\mathbb{E}}$. The intended effect of introducing such a constant is to reduce the number of axioms for the field $\mathbb{E}$ by eliminating the special exemption given to $0 \in \mathbb{R}$, thus aligning the energy numbers more closely with physical reality than the reals, while simultaneously affording a simpler and more consistent set of axioms.

In this new regime, where $v_{\mathbb{E}}$ plays the role of both the additive identity and absorbing element, we are given the multiplicative identity $\mu_{\mathbb{E}}$, which reduces to a potent algebraic variable α whence "existential contributory effects are observed," and to $v_{\mathbb{E}}$ otherwise.[1]

It turns out that the algebra of energy numbers is the appropriate one for describing quantization. We propose here the following axiom:

Axiom 1.2. $\forall \alpha \in \mathbb{E} \qquad (Cr_\alpha)^{-1} = v_{\mathbb{E}}$

Where $Cr_\alpha : \{\} \longrightarrow \alpha$ is the creation map on α, and where its inverse is isomorphic to the canonical annihilation of α in a ring $(R \ni v_{\mathbb{E}}) \subseteq \mathbb{E}$.

The full extent of the consistency of $\mathbb{E}$, as well as a complete bestiary of its properties, are not yet known. We will present a number of conjectures and applications relating to this field in both the present paper, and forthcoming future works. What is known at present is that the algebra

$$A_{\mathbb{E}}$$

is best thought of as a ($\mathbb{Z}_2$-graded) super-algebra. That is to say, we have:

$$v_{\mathbb{E}} \cong 0^+$$

and

$$v_{\mathbb{E}}^{-1} \cong 0^-$$

such that the operation reduces to a superposition $v_{\mathbb{E}}^{\pm}$ of an annihilation/creation pair.

Axiom 1.3. $v_{\mathbb{E}}^{\pm} \implies ((v_{\mathbb{E}} \vee v_{\mathbb{E}}^{-1}) \cong (0^+ \vee 0^-))$

Using wheel theory [4] we impose the following axioms:

Axiom 1.4. $\frac{\alpha}{v_{\mathbb{E}}^{\pm}} = \infty$

and

Axiom 1.5. $v_{\mathbb{E}}^{\pm\infty} \leftrightarrow \mathbb{1}$

We will make some remarks about the category $\mathbb{1}$ later. For now, we will simply establish that it is the unital ∞-cosmos of monads. Using para-consistent logic, we propose two additional equivalences

Axiom 1.6. $\mathbb{1} = \int_{\mathbb{E}} 1_{\mathbb{E}} = \{\forall e \in \mathbb{E} | \exists \alpha : e \longrightarrow 1\}$

where $1 \in \mathbb{1}$ is a unital object, and

Axiom 1.7. $0^{\pm} = ((0 \oplus_{\mathbb{E}} \varepsilon) \vee (0 \ominus_{\mathbb{E}} \varepsilon)) = 0 \pm \varepsilon = (0 \sim \varepsilon)$

where

$$0^{\pm} = (v_{\mathbb{E}} \otimes_{\mathbb{R}} \pm(e \simeq 1)) \underset{-}{\overset{+}{\leftrightarrows}} 0$$

and where 0 is a two-sided nilpotent ideal.

Notice that the Galois realization of $\mathbb{E}$ is a connection

$$\nabla : \mathbb{E} \longmapsto \mathbb{R}$$

which produces bonafide measurable physical values in a controlled parameter space.

[1] Ibid, page 6

1.2.1 Acyclic Energy Groups and Hamiltonians

Let G be an acyclic group. Denote by $G_{\mathbb{E}}$ the image of the map

$$(G \ni e_{\mathbb{R}}) \cong 0 \longmapsto \hat{G} \ni v_{\mathbb{E}}$$

where $v_{\mathbb{E}}$ is the exact center of Emmerson's algebra. We will prove the following fact about $G_{\mathbb{E}}$.

Proposition 1.2. *For some character $i \in G_{\mathbb{E}}$, there is a miniscule co-character j such that i_j^{∞} is isomorphic to $1 \in \mathbb{R}$.*

Proof. Assume $\mathbb{E}$ is a quasi-category furnished with fibrations, cofibrations, and weak equivalences, such that it is a homotopy category.

Construct an exact sequence

$$1 \xrightarrow{sSet} i \xrightarrow{\infty|Set_{Fin}} i_j \xrightarrow{\infty} 1$$

where 1 is the zero object. Note that the map factors through i_j. To show that this is a miniscule cocharacter, let $Rep(i_j) \neq Sp(1)$, where $1 \cdot e = Cent(\mathbb{E}) = Cent(\mathbb{1})$. □

Remark 1.1. *By "miniscule," we mean specifically that the π-weighted filtration of the space $Sp(i_j)$ is homeomorphic to some ε-module, where ε is not representative of $Sp(\mathbb{E}) \supset Sp(i_j)$. This roughly means that the energy Hamiltonian may be written*

$$\mathfrak{H}_{\mathbb{E}} \simeq \left(\left(\frac{\partial^n \sum_{j=0}^{\infty} i_j}{\partial^n Avg(i_j)}\right) \otimes_{\mathbb{E}} \mathfrak{h}\right)$$

where

$$Avg(\bullet) = \frac{\sum_i^n \bullet_i}{n}$$

and where $\bullet$ denotes Dirac's monopole and $\mathfrak{h} = \hbar_{\mathbb{E}}$ is Emmerson's reduced Planck constant.

Setting $n = \infty$ gives us $\mathfrak{H}_{\mathbb{E}} = c^2 = 1e$ where e is a unit in $\mathbb{E}^{\times}$.

2 Energization

We propose a map analogous to the complexification functor

$$(\mathbb{K} \neq \mathbb{C}) \longmapsto \mathbb{C}$$

which we shall call "energization." It is written

$$(\mathbb{K} \neq \mathbb{E}) \longmapsto \mathbb{E}$$

This functor is contravariant to the geometric realization functor $||\mathbb{K}|| \longmapsto Sp(\mathbb{K})$. In the case where $\mathbb{K} = \mathbb{E}$, the functor $|| \cdot ||$ is forgetful, losing the logical paraconsistent logical information about the unit $v_{\mathbb{E}}$. That is to say, $0^{\pm}$ is sent to the bare zero object $0 \longrightarrow \mathfrak{h} \longrightarrow 1 \longrightarrow c^2 \longrightarrow 0$, and a select odd homotopy type of the map is killed off, transforming the ambient space into either a fivebrane or a ninebrane structure.

3 Forcing and Symmetry

Let $\mathcal{R}$ be a ring with identity. Let $\mathcal{T}$ be a topological space. Define a stable bicategory $\mathcal{C}_2$ whose objects are pairs (r, k), where r is an element of $\mathcal{R}$ and k is a tuple consisting of local coordinates over $\mathcal{T}$.

Since the category of rings is presentable, a ring $\mathcal{R}$ is manifestly a set; ergo, the notion of forcing is applicable. For a ring $\mathcal{R}$, a forcing notion is an injection $\mathcal{R} \xrightarrow{\aleph} \mathcal{R}^+ \cong \mathcal{R} \Vdash \mathcal{R}^+$ into an overring induced by the map $\aleph$. However, it is not exactly clear what the appropriate forcing notion for a topological space is. A good hunch is that, for a topological space $\mathcal{T}$, endowed with the Euclidean topology, the map $\mathcal{T}^n \xrightarrow{q} \mathcal{T}^m$ is isomorphic to an immersion of a space of dimension n into one of dimension $m > n$, with a fiber of codimension q.

Combining these two notions, we obtain a refinement of naked forcing; let us call this notion *symmetric forcing.* This is described by the functor

$$\mathfrak{Walls} : A \xrightarrow{\Vdash} B \qquad B = A \bigcup_i \beta_i$$

where $\beta_{min(i)} \rhd A^\circ$, and A° is the group completion of A.

Suppose we are given an overcategory $\mathcal{C}_2^+ \supset \mathcal{C}_2$, such that $((\mathcal{C}^+, \mathcal{C})/\Vdash) = [\aleph]$. Then, we effectively have an exit path

$$\mathcal{EP}_\aleph : SC \longrightarrow FC$$

from the category of second-countable spaces to the category of first countable spaces.

Axiom 3.1. *The above map is weight-preserving; i.e., the minimal cardinality for a base in $\mathcal{T}^n$ is the same in its image.*

We impose the above axiom so that the coarseness/granularity of the space is preserved across the forcing. This ensures that all of the internal homs remain intact.

Definition 3.1. *A symmetric topological space, $\mathcal{T}_{Sym}$, is a (separable) homogenous topological space $\mathcal{T}^d$ whose pointwise reflection $\Sigma(\mathcal{T}_*)$ has parameterization $\dot{\wp}$ if and only if the representative point $* \in \mathcal{T}$ has parameterization $\wp$.*

Let Sm/k denote the category of smooth, separable schemes of finite type over $Spec(k)$. Denote the class of maps $Sm/k \longmapsto (\mathcal{C}_2)$ by $\underline{Hom}_k$. We want to consider the subclass of split maps $\underline{Hom}_k \rightrightarrows \underline{Hom}_{k,\dot{k}}$ consisting of morphisms of finite type to the birational geometric representation of $\underline{Hom}_k$.

Proposition 3.1. *$\underline{Hom}_{k,\dot{k}}$ is homeomorphic to $\Sigma(\mathcal{T}_*)$.*

Proof. We have $\Sigma(\mathcal{T}_*) : * \in \mathcal{T} \longrightarrow \dot{*} \in \dot{\mathcal{T}} \cong Cr_{\dot{*}} \cong Cr_{\dot{k}} \cong \{\} \mapsto Id_{\dot{e}}$, thus giving us the desired homeomorphism. $\square$

Let $\overset{\Gamma}{\nabla}(k,d) = \gamma_\nabla$ be a generalized connection[2] $p \xrightarrow{\gamma_\nabla} q$. Let $\mathcal{T}_{Disc}$ be a discrete topological space. Then:

Axiom 3.2. *Every $\gamma_\nabla \in \mathcal{T}_{Disc}$ is an exit path.*

It suffices to show that γ_∇ is the creation map on q, and that every discrete topological space is stratified. Thus, $\mathcal{T}_{Disc} \cong Strat^*_{\mathcal{T}}$, and every full fibration (i.e., non-quasi fibration) is an exit path $\mathcal{EP}_{pq} : p \longmapsto \dot{p} \cong q$. Note that $\mathcal{T}_{Disc}$ is not a totally contractible space; thus, it is ≥ -2-truncated.

Denote by $C(\mathcal{T}_{Disc})$ the smallest category containing every $(p,q) \in \mathcal{T}_{Disc}$. This is the minimal contraction of a disconnected topological space.

Proposition 3.2. *If $C(\mathcal{T}_{Disc})$ is symmetric, then every exit path it contains is a forcing notion $\mathcal{EP} \Vdash (p,q)$.*

Proof. Suppose $\mathcal{T}_{Disc} \cong Rep(\mathbb{Q})$. Then, given a smooth path γ_∇ of length ℓ. Then, we have $(exp(\ell))^{-k}$ for all C^k. Thus, these values are transcendental over $\mathcal{T}_{Disc}$, and they force a new space, $\bar{\mathcal{T}}_{Disc}$, such that p lies in the first and q lies in the second. □

Remark 3.1. *In the above, the extension $\mathbb{Q} \cup [\mathbb{K}]$ is the Cauchy co-completion of $\mathbb{Q}$, sending every $x \in Rep(\mathbb{Q})$ to a uniform space $\mathcal{U}_x$. Uniform spaces are highly "symmetric" objects; they are necessarily complete, and for every such space $\mathcal{U}_\bullet$, there is a "main diagonal," $\bullet \times \bullet \longmapsto \Delta$. Further, these spaces are very amenable, as they may be translated into "topological vector spaces."*

4 Constructible Symmetries

Let $T \rightrightarrows (s,t) \in T$ be a topological vector space sending every functor in $End(T)$ to a presentable pair. Then, there exists a main diagonal $\Delta \in T$ as described above.

Definition 4.1 (Constructibility)**.** *We call a function $f(T)$ on T* construcible *if, for every germ $g \sim Diff_{(s,t)} \cong ([s,g,t])/\simeq$, there is a sheaf O_g in T supporting the action of a flow coefficient on (s,t).*

Paths can also be constructible. Consider a constructible exit path $\mathcal{EP}_{Const}$: $s \longmapsto t$. This path consists of the following data:

1. A coequalizer $g \in [s,t]$ in $\underline{Hom}(s,t)$.

2. A map of sheaves $O_g \longrightarrow O_g$ containing the class $[g]$ of germs.

When we are working with a real vector space, we always have $\Delta \subset \underline{Hom}(s,t)$ giving us at least one trivial symmetry: $(s,t) \longrightarrow (\dot{s},\dot{t})$ given by fixing a point as the origin and sending each point x to a point $\dot{x} = -(x,x)$.

[2]Preconnection

Definition 4.2 (Compactness)**.** *A subspace $S \subset T$ is said to be* compact *if, for all $x_i \in S$, the inverse limit $\underset{0 \longleftarrow i}{lim}\ x_i$ sends a point to itself.*

This gives us the trivial identity $Id_0 = Id_x \equiv Id_{x_0}$. We also have the identity $Id_{\dot{x}} = d(x, \dot{x}) = ((f(x) = 0) \sim \infty)$, sending each outcome of the distance function on a pair $(x, \dot{x})$ to a rational point at infinity, which is identified as the zero for a polynomial equation on x.

Lemma 4.1. *The above identities are constructible symmetric functors.*

We will call an identity function $Id_{x_i} : x_i \longmapsto x$ *local* if $d(x, x_i) < \varepsilon$ for some small enough ε, and *global* otherwise.

Definition 4.3. *A* local-to-global correspondence *is a map $H_n(x_i) \longmapsto H_{n+1}(x)$ which acts on the sheaf of germs of x, such that there is an induced forcing notion $x_i \Vdash (x \longmapsto \dot{x} \in [x_i])$.*

We shall call the image of the above map a *mirror image* if it is etale.

Example 4.1. *Let $T \in \underline{Hom}(Top, Vec)$ be a topological vector space. Denote the class of open elements in each by $\mathfrak{o} \in Top$ and $\mathfrak{p} \in Vec$ respectively. The Koszul nerve $\mathcal{N}^1_{Kosz} : \mathfrak{o} \leftrightarrows \mathfrak{p}$ defines the equivalence class $(Shv_{\mathfrak{o},\mathfrak{p}})/ \simeq$, with every mirror image embedded into a constructible class. It suffices to show that the maps $\mathfrak{o} \longmapsto \dot{\mathfrak{o}}$ and $\mathfrak{p} \longmapsto \dot{\mathfrak{p}}$ send open kernels to open images, and thus the nerve is etale.*

Remark 4.1. *If a nerve of a constructible topological vector space is etale, then there will be a bijection between diffeomorphisms and opens:*

$$Diff_{T_{et}} \cong \Omega T$$

where $\Omega \equiv [\ell_*]$ is the subcategory of (based) loops in T.

4.1 Constructible modules

Let $\mathcal{N}_{Mod}$ be a module acting on a nerve. Let $\phi_{\mathcal{N}} : \mathcal{N}_{Mod} \longrightarrow \mathcal{N}_{Mod}$ be a display map. We say the module is a *constructible module* if there is a loop group ΩG which supports the sheaf O_g as described above. By "supports," we mean that the support of the class of flow functions $[f(x) \longrightarrow \dot{x}]$ are supported on ΩG.

The map $\phi_{\mathcal{N}}$ can be thought of as a map of graphs, $\Gamma_{\mathscr{G}} \longrightarrow \Gamma'_{\mathscr{G}}$, where $\mathscr{G} = \mathscr{G}_x$ is the isotropy group of x. When, and only when, a mirror image exists for each x, then $\mathscr{G}$ is a groupoid. We have

$$\phi_{\mathcal{N}} \Vdash \mathcal{U}_x$$

for some uniform space $\mathcal{U}_x$. Complete[3] neighborhoods of $\mathcal{U}_x$ are called "entourages," and are denoted by $\mathcal{E}$. The span of every $\mathcal{E}$ is written:

$$inf(\mathcal{E}, \mathcal{E}) \longleftarrow \Delta \longrightarrow sup(\mathcal{E}, \mathcal{E})$$

[3] Not necessarily co-complete

where the infimum is defined to be a quantity smaller than ε, and the supremum is equivalent to $\varepsilon \wedge \infty$. This forms a compass $\Omega^{\varepsilon\wedge\infty}_{<\varepsilon}$. Let us add the following fact:

Theorem 4.1. *$Rep(\Delta)$ is a symmetric topological space*

The proof of which the reader is left to verify.

5 Universals

Let $P(x)$ be a property of x. We shall call this property universal if it holds for every flat embedding $x^\flat \hookrightarrow \bullet$, and quasi-universal if it holds almost everywhere. Universality is always with respect, of course, to some Grothendieck universe $\mathcal{V}$.

Example 5.1. *A universal property for $\mathbb{N}$ is that for all $n \in \mathbb{N}$, $n \equiv_1 0$. That is to say, n mod 1 is equal to 0.*

Example 5.2. *A quasi-universal property for $\mathbb{R}$ is that r is nonzero $\forall r$.*

Put another way, a property is universal if, for a fiber spectrum x_b of x, and for every loop $\ell : x_b \longmapsto x_{b'}$, then $b = b'$. This leaves x invariant with respect to transport in some space of homotopy types. That is to say, for a spectral sequence containing x_b, its E_∞ page should resemble the nth page, for $n \ll \infty$.

Put yet another way, for some pair of evaluation maps $ev_0 : x_b \hookrightarrow x$ and $ev_1 : x_b \hookrightarrow x$, the canonical valuation $\mathfrak{o}(x)$ is the same in both.

Theorem 5.1. *It is the universal property of a symmetric topological space that the functor*

$$x \longmapsto \dot{x}$$

has an inverse.

Example 5.3. *Given a foliation $\mathcal{F}_x$, leaving x fixed, if every leave is diffeotopic to any other, then each leave has a universal property containing its elements. This is a symmetric universal property; every leaf is non-empty (i.e., there is a bijection of leaves of elements onto non-empty sets of propositions, or facts) becomes the dual.*

Let $P(x)$ be a proposition about x. We say $P(x)$ is an *analogy* if, for every $P \simeq P'$, there is a $P(x) \simeq P(x')$. We say that the items on either side of the isomorphism are "analogous." An analogy is complete if it contains its own closure, or in other words if the group completion for $G \ni (x \vee x')$ is given by the analogy. For a complete analogy, we write $\bar{A}(P, P')$. In a complete analogy, every x' is given the same universal property as every x.

6 Nervous Energy and Quantization

Let $\mathcal{N}_{\hat{Q}}^{[n]}$ be a dendrite, as constructed in [5] and [6]. We shall call this the "quantum dendrite," and, for every $n \in [n]$, we have a nerve $\hat{Q} \xrightarrow{\sim} \{\hat{q}_\alpha, \hat{q}_\beta\}$, which is essentially a quasi-fibration. The appropriate choice of groupoid for tensoring with the quantum dendrite is Θ, as constructed in section [1.1]. Furthermore, for each $\hat{q}_\bullet$, we have the group

$$Hol_{q_\bullet}^{\Theta} \ni \ell_{q_\bullet}$$

upon which the gauge-invariant holonomy of $q_\bullet \longrightarrow ||q_\bullet||_{\mathcal{N}}$ is supported, where $||\cdot||_{\mathcal{N}}$ is the nervous realization constructed in [5].

Theorem 6.1. *There is an equivalence between the quantization constructed in [sec. 1], and the map* $\mathcal{N}_{\hat{Q}}^{k} \longmapsto \mathbb{Z}/k$.

This fact is deduced by the rational equivalence $(\hat{Q})/\mathbb{Q} \simeq \mathbb{Z}$. We can appeal to physical motivation by considering brane-wrapping around a charged instanton located at $Sing(\mathbb{L}^k)$, which is realized as the contraction of the worldvolume of an $\mathcal{A}d\mathcal{S}_5$ brane. It is hereby conjectured to be homologous to gaugino condensation at the mesoscopic scale.

Consider a smooth manifold $\mathcal{M}$ with a discretization $\delta : (\mathcal{M} \xrightarrow{C^\infty} \mathcal{M}) \longmapsto (\mathcal{M} \xrightarrow{C^1} \mathcal{M})$. Denote the left-hand-side of the functor by E and the right-hand-side by F. Note that E is contractible to a point, while F is not. However, for a topological point $\{*\} \in \mathcal{M}$, there is a flat embedding

$$\{*\}^\flat : \{*\} \overset{k=1}{\hookrightarrow} \mathcal{M}$$

corresponding to the $C^k = C^1$-discretized immersion of a quantum of energy into a topological space, which we will assume here to be a topological vector space for simplicity.

Assuming there is an isomorphism $\mathcal{M} \cong \mathcal{U}_\delta$, such that the manifold is a uniform space, then there is a metric on $\mathcal{M}$ which is quasi-forgotten by the map to $\mathcal{U}_q$ for a quantum of q. That is to say, if the discretization is made exact by the relation $q \sim e\{*\}$, then the map $\delta \longrightarrow q$ is surjective.

Furthermore, the curl of $\mathcal{M}$ is determined uniquely by a motivic spectrum $MU_*(\Omega^k)$, which is given by the underlying spin structure of $\mathcal{M}$, which can be obtained via reduction by killing its third homotopy type, $\pi_3(\mathcal{M})$. With this in mind, we deduce that $MU_*(\Omega^k)$ has at least a fivebrane structure.

A Future Work

In the sequel, we consider the element i_j in further detail, or in the words of Emmerson, we shall "deprogram" it. We will consider spaces based at j and their motivic spectra. We will attempt to make connections to Quillen's "Rational Homotopy Theory," and to the work of Sati and Schreiber.

It would be worthwhile to attempt to energize the Pauli matrices, and perhaps develop a notion of forcing for the energy numbers. We will consider the case where $\hat{Q}$ is a tadpole, and study the null object induced by its cancellation. We may equate this with a vacuum

$$\not{\hat{Q}} = \mu_0^\dagger$$

which has a positive third Betti number. Specifically, the derived cohomology of $\hat{Q}$ may be shown to be topologically homologous to the synthetic homology of the class $[(\mu_0^\dagger)^{-1}]$. This is part of an ambitious plan to develop a spectral theory of the holes generated by the annihilation of particles, which are effectively physical instantiations of exciton-polaritons. We may employ the notion of compasses as previously developed in order to describe foliations of the ambient space of these exotic particle pairs, as well as (hopefully) characterize (one manifestation of) quantum gravity as interactions between the strong nuclear force and mesoscopic gravitational lensing at the site of these particles. The polarization of an exciton may be explained as a manifestation of spontaneous breaking of the electroweak symmetry of the gauge group $G_{\Sigma\hat{q}}$ consisting of polygamously entangled quasi-quanta transitioning from individual pure states to a collective mixed state.

With respect to the energy numbers themselves, we may take further steps to develop a truly quantum algebra of energy numbers, and develop them from a model-theoretic angle.

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