Equality hypothesis in Diagonal Argument

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Abstract: In the Diagonal Argument, there are two independent hypotheses, the countability hypothesis and the equality hypothesis, which violate the basic principle of proof by contradiction, so the Diagonal Argument cannot be held. Closer analysis shows that the Diagonal Argument only falsifies the equality hypothesis and has nothing to do with the countability hypothesis. Since the equality hypothesis is inherently wrong and does not need to be falsified at all, the Diagonal Argument actually proves nothing. The transfinite number theory and Continuum hypothesis therefore become meaningless. Once again calls on the education department to suspend the teaching of infinite sets, so as not to mislead people.

Keywords: Diagonal Argument; proof by contradiction; transfinite number theory; Continuum hypothesis; philosophy; logic; education department

1 introduction

Diagonal Argument[1] have had an important impact on the history of mathematics, and even on the history of philosophy and logic. Once the Diagonal Argument is falsified, not only will the history of mathematics be completely rewritten, but many theories of mathematics will also be revised, and it may be a huge shock to the entire academic community.

As is well known, the Diagonal Argument is proved by proof by contradiction.

The form of proof by contradiction is to assume a proposition and then derive a contradiction from the proposition, which proves that the proposition is wrong, that is to say, the contradictory proposition of the proposition is correct. The logical rule is the law of exclusion: either A or not A must be right.

It can be seen from the principle of proof by contradiction that there is only one hypothesis in proof by contradiction, that is, the proposition to be overturned. This is because, if there are two hypotheses, even if a contradiction is derived, how do we know which hypothesis is causing the contradiction?

However, in addition to the countability hypothesis to be overturned, another assumption is implied in the Diagonal Argument, which is the equality hypothesis.

2 Diagonal Argument and infinite matrix

Under the countable assumption we can list the decimals a_1, a_2, a_3……
one by one:

a_1=0.$a_{11}a_{12}a_{13}$....

a_2=0.$a_{21}a_{22}a_{23}$.... (1)

a_3=0.$a_{31}a_{32}a_{33}$....

....

The right ends of the equals form an infinite matrix.

The number of rows in the matrix represents the number of the decimals listed, and the number of columns represents the number of decimal places listed.

Obviously, for any multi-base decimal, such as binary decimal, there are many more rows than columns, that is, the right end of the equals in (1) is an infinite rectangular matrix.

Infinite rectangular matrices have long existed in mathematics and (1) is just one example.

For example, for the cube X-Y-Z, the two coordinates of the x-y plane are divided into n and kn parts, forming $n\times(kn)$ grids. When the constant k=1 and n→∞, the average height value Z of each grid forms an infinite square matrix; When the constant k≠1, an infinite rectangular matrix is formed.

The existence of an infinite rectangular matrix directly proves that the set of natural numbers is not unique [2]: both the number of rows and that of columns of a rectangular matrix are sets of natural numbers, and since they are rectangular matrices, the two sets of natural numbers are not the same.

The fact also directly proves that there is no set of all natural numbers: since the set of natural numbers is not unique, when there are multiple sets of natural numbers, which one is the set of all natural numbers?

In fact, these are fundamental challenges to set theory[2,3], and fully show that set theory needs a revolutionary reconstruction [4].

We will also see below that the existence of a rectangular matrix is acknowledged. I'm going to directly overturn the Diagonal Argument.

The key part of the Diagonal Argument is defining b

b=0.$b_1b_2b_3$... (2)

here.

$b_k \neq a_{kk}$, (k=1,2,3,...) (3)

According to (3), when k=1, $b_1 \neq a_{11}$, we only investigate 1 row and 1 column of the matrix, and the number of row investigated is strictly equal to the number of column; When k=2, $b_2 \neq a_{22}$, we investigate 2 rows and 2 columns of the matrix, and the number of rows investigated is also strictly equal to the number of columns...… In other words, only when the number of rows is exactly equal to the number of columns can the most critical b be obtained in the Diagonal Argument, so the Diagonal Argument is carried out on the premise that the number of rows and columns are exactly equal, which is called the equality hypothesis for the convenience of discussion.

In other words, there are two assumptions in the Diagonal Argument: one is countability assumption and the other is equality assumption.

Obviously, it is not the infinite rectangular matrix shown in (1) that satisfies the equality hypothesis, but an infinite square matrix contained in (1).

Since eq. (3) only considers the diagonal elements in this square matrix. The resulting *b* is different from any decimal in the square matrix and contradicts the equality hypothesis: the number of rows is not equal to the number of columns, and the number of rows is at least one more than the number of columns.

The Diagonal Argument disproves the equality hypothesis, does it prove that the decimals are uncountable?

To prove that the decimals are uncountable, one must prove that *b* is not in the rectangular matrix shown by (1), which Cantor apparently did not do. He actually just proved:

if k=1, $b \neq a_1$

If k=2, $b \neq a_1$ and $b \neq a_2$.

.....

That is, when the number of decimal places *k* is arbitrarily large, the diagonal can only guarantee that *b* is not equal to *k* decimals among 2^k decimals.The above situation is obviously still true when $k \to \infty$. In other words, Cantor cannot guarantee that *b* is different from any of the decimals in (1).

Therefore, the Diagonal Argument only negates the equality hypothesis in the two hypotheses, and does not negate the most critical countability hypothesis, that is, the Diagonal Argument is not prove that the real numbers are uncountable.

3 The non-uniqueness of the set of natural numbers and the independence of the two hypotheses

In fact, the countability hypothesis can only infer that rows and natural numbers are one-to-one correspondences. As mentioned earlier, since the set of natural numbers is not unique, it is possible whether the number of columns is less than or equal to or more than the number of rows, which does not contradict the countability hypothesis. That is, the equality hypothesis does not follow from the countability hypothesis, which are two independent assumptions.

In proof by contradiction, there are two independent hypotheses, the result of which is, of course, absurd.

Of course, if we insist that the set of natural numbers is unique regardless of the facts, then the equality hypothesis naturally holds, and so does the Diagonal Argument.

The question is, science is nothing more than a system of concepts used to describe facts. Science cannot disrespect facts, otherwise it cannot be called science.

Cantor's theorem also proves that the set of natural numbers N cannot correspond to its power set P(N), and if the set of natural numbers is not unique, it cannot exclude the possibility that P(N) corresponds to a larger set of natural numbers different from N, that is, Cantor's theorem also does not prove that a binary decimal, which corresponds to P(N) one by one, are uncountable.

Cantor's earlier proof that real numbers are uncountable, based on the nested intervals theorem [5], also essentially assumes that the set of natural numbers is unique.

Since the diagonal argument does not prove that the real numbers are uncountable, the entire Diagonal Argument should actually be rewritten as: Assuming that the number of rows and that of columns are equal, the obtained (1) is an infinite square matrix, but *b* defined by (2) and (3) is not in the square matrix, forming a contradiction. So, the Diagonal Argument disproves the equality hypothesis.

It can be seen here that the actual Diagonal Argument has nothing to do with countably uncountable, either in the beginning or in the end.

It seems like the following: Cantor made a non-existent dummy (equality hypothesis) next to the real person (countability hypothesis). He knocked the dummy down with a punch, and then claimed to have knocked down the real person!

There were people cheering, and one of them was named Russell, and the other was named Hilbert.

In fact, since (1) is a rectangular matrix, it does not conform to the equality hypothesis in the first place, so the Diagonal Argument merely overturns a hypothesis that does not need to be overridden. In other words, the Diagonal Argument proves nothing.

However, the history of mathematics and even the history of human thought has gone through a big detour that does not need to be taken at all

In a word, Cantor made a terrible mistake by treating an infinite rectangular matrix as an infinite square matrix.

The mistakes are not difficult to find, but the strange thing is that they have not been discovered and corrected for a long time, which makes people wonder whether human thinking ability has been highly degraded.

Since there are errors in the supposedly most rigorous field of mathematics, not to mention in other areas of humanity, like religion and ideology. People's actions are controlled by their thoughts. No wonder there are unwise conflicts and even wars everywhere in the world.

4 Conclusions

In the Diagonal Argument, there are two independent hypotheses, the countability hypothesis and the equality hypothesis, which violate the basic principle of proof by contradiction, so the Diagonal Argument cannot be held.

Closer analysis shows that the Diagonal Argument only falsifies the equality hypothesis and has nothing to do with the countability hypothesis.

Since the equality hypothesis is inherently wrong and does not need to be falsified at all, the Diagonal Argument actually proves nothing.

As for the subsequent so-called transfinite number theory[6], Continuum hypothesis and so on of course, has become a tree without root and meaningless, but it has spent a lot of energy of generations of mathematicians, not only wasting a lot of time, but also confusing the normal thinking ability of humans, which can be said to be harmful rather than beneficial, and must be corrected as soon as possible.

Math needs to be saved!

Once again calls on the education department to suspend the teaching of infinite sets, so as not to mislead people.

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