Fine-Structure Constant and 5-D Space

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Abstract. Conductivity quantization and 5-D thermal field theory for bosons in odd dimensions are employed. A relationship between thermal mass and the fine-structure constant was derived. The calculations demonstrated that the numerical value of the fine-structure constant obtained for the dimension d=5 aligns with the experimental value of the fine-structure constant. This could imply that weak interactions occur in a five-dimensional space.

Key words: Conductance Quantization ◦ Thermal Field Theory ◦ Fine-Structure Constant ◦ Bosonic Field Dimension

1 Introduction

In various works on theoretical physics, the question of the dimension of the interaction space is a recurring theme. For instance, in thermal field theory, solutions and approaches are provided for the cases of 3-D, 5-D, and 7-D [1]. Additionally, for dark matter fermions, a dimension of 5-D is proposed [2]. Consequently, the question arises as to whether it is practically feasible to determine the dimension of the weak interaction space based on existing constants. To address this issue, the dimensionless fine-structure constant is employed.

2 Quantum Models

2.1 Conductance Quantization

The conductance quantum [3], denoted by the symbol G_0, is the quantized unit of electrical conductance. It is defined by the elementary charge g and Planck constant h as:

$$G_0 = \frac{2g^2}{h} \tag{1}$$

It arises when measuring the conductance of a quantum point contact.

2.2 The Fine-Structure Constant and Bosonic Mass

The relationship between quantum conductivity and bosons is primarily seen in the context of superconductivity. Superconductors have zero electrical resistance and expel magnetic fields, and their behavior is strongly associated with the formation of Cooper pairs, which involve interactions between electrons and lattice vibrations (phonons). Here's how quantum conductivity and bosons, particularly phonons, are related in the context of superconductivity [4].
Let's consider the conductance quantum as an equivalent bosonic thermal field momentum:

$$G_0 = P \tag{2}$$

The Equation (2) can we written in the following form:

$$\frac{2g^2}{h} = \frac{m}{\pi}c \tag{3}$$

where m is a unitless mass, and c is a light speed.
The Equation (3) can be transformed into the following equation where left and right parts are unitless:

$$\frac{2\pi g^2}{hc} = m \tag{4}$$

The left side is the unitless fine-structure constant:

$$\alpha = \frac{2\pi g^2}{hc} \tag{5}$$

Finally, it can be written as follows:

$$\alpha = m \tag{6}$$

2.3 Thermal Field Theory

In thermal field theory, for the bosonic case, the gap equations in dimensions d = 3 and d=5 are given respectively by [1]

$$-m_{th} = 2\log(1 - e^{-m_{th}}) \tag{7}$$

$$-\frac{1}{6}m_{th}^3 = Li_3(e^{-m_{th}}) + m_{th}Li_2(e^{-m_{th}}) \tag{8}$$

where function Li_a is defined as:

$$Li_a(z) = \sum_{n=1}^{\infty}\frac{z^n}{n^a} \tag{9}$$

and m_{th} is the thermal mass with well-known solution:

$$m_{th}^{(d=3)} = 2\log\left(\frac{1+\sqrt{5}}{2}\right) \approx 0.96242 \tag{10}$$

$$m_{th}^{(d=5)} \approx 1.174310 \pm 1.198082i \tag{11}$$

2.4 Comparative Analysis and Calculations

Let's substitute the corresponding expression for thermal mass into Formula (6), taking into account the dimension of space and the value of the fine-structure constant:

$$m = \left(\frac{m_{th}^{(d=3)}}{\pi}\right)^3 \tag{12}$$

$$m = \left[\frac{Re\left(m_{th}^{(d=5)}\right)}{\pi}\right]^5 \tag{13}$$

where $Re\left(m_{th}^{(d=5)}\right)$ is a real part of the solution (11).

The experimental value and results of calculations are given in the Table Table 1. Experimental value and results of the fine-structure constant α calculations .

Table 1. Experimental value and results of the fine-structure constant α calculations

Source of value	Fine-structure constant α
Experimental [3]	≈ 0.00729735
Calculated for d = 5	≈ 0.00729733
Calculated for d = 3	≈ 0.028712803

The result for d=5 satisfies the experimental data with error less than $2.1 \cdot 10^{-4}$ %. The discrepancy between the calculated and experimental data falls within the rounding accuracy of the computer calculations.
This could imply that weak interactions occur in a five-dimensional space, d=5.

3 How Can Continuous Objects Be Observed In Lower Dimensional Space as Quantum Effects?

Let's say a thin ring moves in three-dimensional space and intersects two-dimensional space (plane). Let the observer have access to observations only in two-dimensional space. And what does the observer see?
Suddenly, out of nowhere, a “particle” appears, which then splits into two ones. Next, two “particles” move along an elliptical (this depends on the angle of inclination of the ring in relation to the observation plane) orbit, which are then connected into one “particle”.
For an observer, this process looks as if two “particles” appeared out of nowhere from one point in space, moving along an elliptical orbit, and “annihilated” at another point in space (Figure1 and Figure2).

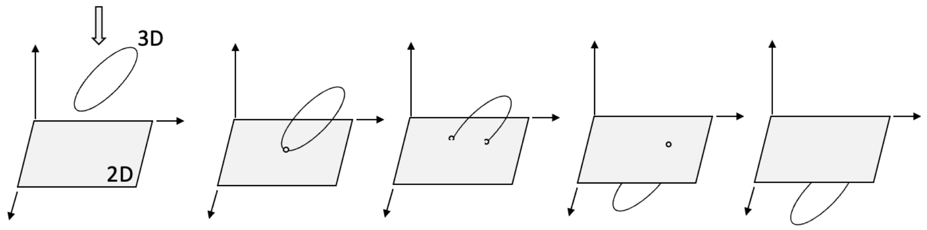

Figure1. The object “ring” is moving in 3D space

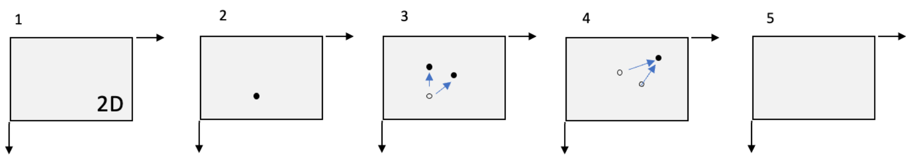

Figure2. An observer observes the intersection of a ring with a 2D plane.
1. There are no "particles"
2. Out of nowhere (“out of vacuum”) one “particle” appears
3. The “particle” split into two “particles”
4. Two “particles” flew along an elliptical trajectory and united again into one “particle”
5. "Particle" falls into "vacuum"

4 Conclusion

This paper presents one of the possible ways to determine the dimension of the space of weak interactions based on experimental data on the weak interaction constant and models of 5-D thermal field theory. This might suggest that weak interactions take place in a five-dimensional space.

If weak interactions do take place in a five-dimensional space, it could prompt a reevaluation of theoretical quantum mechanical models and potentially challenge certain theories. As illustrated in the graphical example of a three-dimensional ring intersecting a two-dimensional plane, "continuous" objects can induce "discontinuous" quantum mechanical effects in lower-dimensional observable spaces.

It is likely that the development of more complete models (in terms of the spatial dimension) could easily explain phenomena such as the almost instantaneous rate of quantum entanglement, or how objects can be "generated by a vacuum" and disappear within it.

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References

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