A New Approach to Unification Part 3: Deducting gravity physics

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Abstract

In a series of 4 papers an approach to a unified physics is presented. In part 1 the foundation of such an approach is given. In part 2 it was shown how particle physics follows. In this 3^{rd} part gravitational physics will be derived. In part 4 open fundamental questions of actual physics are answered and the concept of a new cosmology is introduced.

Contents

Basic equations found in part 1.

To simplify quoting, here are some results from part 1:

In the assigned spacetime, a 6d Minkowski space, it holds

- a 6d Klein-Gordon equation without mass term

$$\partial^\alpha \partial_\alpha \phi = 0 \text{ with } \alpha = 1, 2 \ldots 6 \text{ and the 6d wavefunction } \phi. \tag{1}$$

- a Lagrangian $\mathcal{L}6_{KG}$ based on the 6d Klein-Gordon equation

$$\mathcal{L}6_{KG} = ((\partial_\alpha - igA_\alpha)\,\Phi)^+ \,((\partial^\alpha - igA_\alpha)\,\Phi) - \mathcal{L}6_B. \tag{2}$$

with $\Phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$ (T means transposed), where the ϕ_i are solutions of the Klein-Gordon equation. The A_α are representing the boson field describing the one force in 6d. The free boson field part $\mathcal{L}6_B$ describes interaction of the bosons. The A_α and $\mathcal{L}6_B$ do not matter in the following.

- With the 6d coordinates $x6^\alpha = x6^\alpha(T, u, v, x)$ (x stands for x^1, x^2, x^3 coordinates occurring also in 4d and u,v for coordinates not accessible for a 4d observer) and the Jacobi determinant $J(T, u, v, x)$ the 6d action integral S can be written as

$$S = \int dx^3 dT \int dudv J(T, u, v, x)\mathcal{L}6(T, u, v, x).$$

The inner integral

$$\hat{\mathcal{L}4} = \int_{x,t} dudv J(T, u, v, x)\mathcal{L}6(T, u, v, x). \tag{3}$$

is called non-interpretable Lagrangian.

The following is very much based on the principles set out in part 1 of the series. You can find it at http://viXra.org/abs/2312.0062

1 Introduction

Having shown in part 2 that UR allows deducting particle physics to be a universal approach the same procedure must lead to gravitation.

Newton understood gravity, which he introduced, as a property of (heavy) mass, as a force causing material bodies to attract each other. It is a long-range force that does not spread out in time, but acts simultaneously at all distances.

The concept is not compatible with the special theory of relativity, e.g. because of their limited propagation speed. All trials to introduce gravity into special relativity did fail. [1]

Einstein overcame the problem and founded 1915 with its general relativity (GR) the modern theories of gravitation. It describes the behavior of classical particles in curved spaces. The Einstein field equations connect the energy-momentum tensor to the curvature of spacetime. Up to now solutions exist only for a few geometries. They are formulated as a tensor equation in a spacetime with Riemannian metric what makes them automatically background independent.[2]

Solving the Einstein equation gives the metric coefficients of the spacetime depending on the regarded energy-momentum tensor.

All attempts to formulate quantum physics with tensors did fail. Especially the different meaning of time in the background-dependent and -independent theories generates problems.

Also the further development of gravity physics – as for example in the Einstein-Cartan theory that regards besides curvature also torsion or in the Brans-Dicke theory that generates a variable (effective) gravitational constant by introducing an additional scalar field – is built upon the same fundamental assumptions.

Striking is the similarity of results given by GR with its curved space with those of Newton's gravity theory formulated in a Euclidean space. Question is whether there is an inner relation between the two theories.

The best known solution of GR is given for spherical symmetry by Schwarzschild. It made it possible to overcome the shortcomings of Newton's celestial mechanics. The elimination of the error in the perihelion rotation of the planets and in the diffraction of light in the gravitational field of the sun served to confirm the validity of GR.

Also in cosmology, particularly for the large scales considerations, GR is important. This topic will be covered in the next part of the series.

GR can be seen as a way to find the metric for a given energy-momentum tensor. As there are originally no regulations for the energy-momentum tensor for certain specifications physically problematic metrics can arise. This is shown impressively by Gödel's proof, that in GR there only can exist spatial and no temporal coordinates. To show this he created an exact solution of the Einstein field equations with a special energy-momentum tensor (dust universe) and showed that in this universe time traveling would be possible. Analyzing the consequences he came to his conclusion.[3] There are other solutions e.g. the Kerr metric that under special conditions produce structures ("naked singularities") in which causality is breached. To avoid such phenomena destroying our understanding of the world based on time and causality additional conditions as the Chronology Protection Conjecture of Hawking or the Cosmic Censorship hypotheses of Penrose have been introduced.

2 Gravitaty with UR

It is an advantage of UR in the calculation of gravity that the energy-momentum tensor and the 4d metric are derived together from a single source.

That starting with the same flat 6d space and using the same procedure can give particle and gravity physics is due to the symmetry of the flat 6d space expressed in its adapted coordinates (see part 1 of the series). This means Cartesian coordinates if the space has translational symmetry and spherical coordinates if the space e.g. has spherical symmetry. The coordinates of the 6d space are propagated to 4d.

What are the differences in explaining gravitation between UR and the other approaches:

1. In 6d there is no mass and the space is flat so there is nothing comparable with 4d gravity. While the inner forces in 4d reflect the inner force in 6d gravity is a mere 4d issue. As particle mass it occurs as a compensatory measure with the dimensional transition.

   The fact that gravitation has a fully different origin as the external forces may be used to explain its weak strength compared to these. This difference is a problem for theories that want to justify the different forces in a uniform way.

2. Our spacetime is a 4d bubble expanding with the 6d assigned spacetime in the complex extension of the 6d Euclidean space. In contrast to the expanding universe in GR for UR however there exists the Euclidean 6d space. Its invariant structure gives a framework that can serve as a stage on which the 4d events happen. In the adapted coordinates a joint description of particle and gravity physics is possible. Only in a 4d perspective spacetime is dependent on the distribution of mass.

3. For the actual theories the 4d spacetime has to be Minkowskian or Riemannian. It is difficult to define a spacetime fulfilling both qualifications. In UR the 4d spacetime is not genuine. 4d physics is just a picture of 6d physics adequate to our abilities. This means that not the 4d spacetime has to fulfill both qualifications but the picture we get of the 6d one. With other words, as the aim of 4d physics is to replicate the 6d one as good as possible this can demand in one case a flat 4d spacetime and in the other a curved one. So a compromise between the seemingly incompatible preconditions is possible.

Further conditions must be met by the 6d specifications of UR to lead to gravity in 4d:

1. There must exist symmetries of the 6d Euclidean space generating non-interpretable Lagrangians $\hat{\mathcal{L}}4$ to which no interpretable 4d Lagrangian formulated in a Minkowski

space can be adapted, that it needs the elements of the metric tensor of a curved 4d spacetime as additional adaption parameters to achieve it.

2. UR at least in 6d is a quantum field theory. Deducting gravitation will generate 4d quantum gravitation. In order to enable comparisons with the results of GR, it must be possible to derive a classic version which has to hold for entities corresponding to the classical particles described by GR (or Newton's gravity law).
3. Gravitation acts on the mass of particles. The 6d Lagrangian knows neither mass nor particles. To generate gravitation the transition to 4d so mandatory has to generate the mass of a planet particle – as it does in a 6d spacetime with translational symmetry – but also that of a central particle.
4. The 6d space is flat. The transition to 4d has to generate a curved spacetime with a curvature creating the correct force.

Gravitation is characterized by the absence of inner forces and spin what means confining the view to effects of geometry. These restrictions are also used with UR when describing gravity. They make analytical calculations possible and allow demonstrating the necessity of a curved spacetime.

Neglecting effects generated by spin means starting with the Lagrangian of the 6d Klein-Gordon equation $\mathcal{L}6_{KG}$ of equation (2). Without Yang-Mills fields this Lagrangian decomposes in a sum over four equal Lagrangians generating four equal Klein-Gordon equations in curvilinear coordinates. It suffices to consider one. So the action integral describing gravity in UR becomes

$$S = \int (dx)^3 \, dT du dv \sqrt{-g} g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi. \tag{4}$$

$g^{\mu\nu}$ is the inverse metric tensor, g the determinant of the metric tensor $g_{\mu\nu}$[†]

This integral of action is the UR pendant to Hilbert's integral of action. The one generates a homogenous linear equation of motion, the other the inhomogeneous tensoral Einstein equation.

The main reason for the simple structure UR provides, is that what is described by the energy-momentum tensor in GR does not act as an external source in UR but is inherent in the equation and emerges only in the transition to 4d.

As a consequence of omitting the Yang-Mills fields in 6d in 4d all nongravitational forces vanish. So all binding forces for the wave functions usually used to describe particles are zero.

To examine only gravity it has no effect whether single particles or groups of particles are considered (neglecting gravitational forces in the groups). To get however a better fit to 4d reality where the nongravitational forces are always active the loose particles virtually are to be combined to larger units. The composition of these virtual units is to correspond with that of particles like e.g. galactic nebula particles in which all of the internal nongravitational forces are saturated so that they cannot be affected by external nongravitational forces.

The energy of the missing internal forces in the virtual units according to Einstein's principle can be added to their mass. As this mass usually is not known and has to be determined by experiment this has no effect.

The so constructed units in spite of their artificiality are to be called particles.

The 6d metric tensor occurring in the Lagrangian cannot be chosen arbitrarily. It has to respect the Euclidean or pseudo Euclidean structure of the 6d space or spacetime what means that its curvature tensor has to be zero.

With respect to the various symmetries of the tensor in a n-dimensional space these are $\frac{1}{12}n^2\left(n^2-1\right)$ second order nonlinear partial differential equations for the metric coefficients.[4] In 6d 105 equations result.

Among the possible solutions of this system of equations (in the assigned spacetime) there are all pseudo Euclidean four-dimensional metrics (with coefficients independent

[†]As metric in gravitation theory is a common quantity the Jacobean determinant is here written by the equal expression $\sqrt{-g}$.

of u and v) expanded by "1"-coefficients for the missing two diagonal elements and "0"-coefficients elsewhere.

Easier than checking for a given metric the 105 equations is introducing suitable coordinates in the 6d space and calculating the resulting metric. As transformations of coordinates do not change curvature the found metric will be (pseudo) Euclidean.

3 From 6d to 4d with spherical symmetry

The necessity to use a curved 4d spacetime is to be shown exemplary with a symmetry of the 6d space that allows comparing results with those of the Schwarzschild solution of GR.

Starting point is a sphere in the Euclidean 6d space. The adapted coordinates for this symmetry are spherical coordinates with radius r_6 and the angles α, β, γ, δ and θ defining the direction of the radius. This means

$$\begin{aligned} u &= r_6 \sin(\delta) \sin(\theta) \sin(\gamma) \sin(\alpha)\, cos(\beta)\,, \\ v &= r_6 \sin(\delta) \sin(\theta) \sin(\gamma) \sin(\alpha) \sin(\beta)\,, \\ y &= r_6 \sin(\delta) \sin(\theta) \sin(\gamma) \cos(\alpha)\,, \\ x &= r_6 \sin(\delta) \sin(\theta) \cos(\gamma)\,, \\ z &= r_6 \sin(\delta) \cos(\theta)\,, \\ w &= r_6 \cos(\delta)\,. \end{aligned} \tag{5}$$

From this it follows the metric

$$ds^2 = dr_6^2 + r_6^2 d\delta^2 + r_6^2 \sin(\delta)^2 A \tag{6}$$

with

$$A = d\theta^2 + \sin(\theta)^2 d\gamma^2 + \sin(\theta)^2 \sin(\gamma)^2 d\alpha^2 + \sin(\theta)^2 \sin(\gamma)^2 \sin(\alpha)^2 d\beta^2 \tag{7}$$

As was formulated when defining the adapted coordinates, the transition to the assigned spacetime is made by introducing time as a Cartesian coordinate and maintaining the original symmetry for the remaining spatial coordinates.

This is achieved by introducing two new variables $Z = r_6 \cos(\delta)$ and $r_5 = r_6 \sin(\theta)$. With $\delta = \arctan(\frac{r_5}{Z})$ and $r_6 = \sqrt{r_5^2 + Z^2}$ we find $r_6 d\delta = \frac{r_5 dZ - Z dr_5}{\sqrt{Z^2 + r_5^2}}$ and $dr_6 = \frac{Z dZ + r_5 dr_5}{\sqrt{Z^2 + r_5^2}}$. Inserting these expressions into equation (6) gives the metric $ds^2 = dZ^2 + dr_5^2 + r_5^2 A$. That is the appropriate form to introduce the time variable $dT = idZ$.

So finally we get the metric of the assigned spacetime

$$ds^2 = -dT^2 + dr_5^2 + r_5^2 A. \tag{8}$$

As for translational symmetry the variable T appears only in the differential dT and not in the metric coefficients. Time T results from integration of the differentials. So the meaning of time is identical in spacetimes with translational and spherical symmetry.

3.1 Non-interpretable Lagrangian

The adapted spatial coordinates to a five-dimensional sphere are

$$\begin{aligned} u &= r_5 \sin(\theta) \sin(\gamma) \sin(\alpha)\, cos(\beta)\,, \\ v &= r_5 \sin(\theta) \sin(\gamma) \sin(\alpha) \sin(\beta)\,, \\ y &= r_5 \sin(\theta) \sin(\gamma) \cos(\alpha)\,, \\ x &= r_5 \sin(\theta) \cos(\gamma)\,, \\ z &= r_5 \cos(\theta) \end{aligned} \tag{9}$$

Adding the squares of the differentials of these coordinates and introducing the square of the time-like differential we get the metric

$$ds^2 = -dT^2 + dr_5^2 + r_5^2 A. \tag{10}$$

As next step the equation of motion following from equation (4) in these coordinates has to be found and solved.

The Lagrangian of the metric is given by

$$\mathcal{L}6 = \frac{\partial\phi^\star}{\partial T}\frac{\partial\phi}{\partial T} - \frac{\partial\phi^\star}{\partial r_5}\frac{\partial\phi}{\partial r_5} - \frac{1}{r_5^2}\left\{\frac{\partial\phi^\star}{\partial\theta}\frac{\partial\phi}{\partial\theta} + \frac{1}{\sin(\theta)^2}\frac{\partial\phi^\star}{\partial\gamma}\frac{\partial\phi}{\partial\gamma} + \right.$$
$$\left.\frac{1}{\sin(\theta)^2\sin(\gamma)^2}\frac{\partial\phi^\star}{\partial\alpha}\frac{\partial\Phi}{\partial\alpha} + \frac{1}{\sin(\theta)^2\sin(\gamma)^2\sin(\alpha)^2}\frac{\partial\phi^\star}{\partial\beta}\frac{\partial\phi}{\partial\beta}\right\}, \tag{11}$$

and the Jacobean determinant by

$$J = r_5^4\sin(\theta)^3\sin(\gamma)^2\sin(\alpha). \tag{12}$$

Using Hamilton's principle we find the equation of motion

$$A_1\left(r_5^4\frac{\partial^2\phi}{\partial T^2} - \frac{\partial}{\partial r_5}\left(r_5^4\frac{\partial\phi}{\partial r_5}\right)\right) - r_5^2\left\{A_2\frac{\partial}{\partial\theta}\left(\sin(\theta)^3\frac{\partial\phi}{\partial\theta}\right) + \right.$$
$$\left.A_3\frac{\partial}{\partial\gamma}\left(\sin(\gamma)^2\frac{\partial\phi}{\partial\gamma}\right) + A_4\left(\frac{\partial}{\partial\alpha}\sin(\alpha)\frac{\partial\phi}{\partial\alpha}\right) + A_5\frac{\partial^2\phi}{\partial\beta^2}\right\} = 0 \tag{13}$$

with

$$\begin{aligned} A_1 &= \sin(\theta)^3\sin(\gamma)^2\sin(\alpha), & A_2 &= \sin(\gamma)^2\sin(\alpha), \\ A_3 &= \sin(\theta)\sin(\alpha), & A_4 &= \sin(\theta) \\ A_5 &= \sin(\theta)\sin(\alpha)^{-1}. \end{aligned}$$

This equation can be separated in a part depending on r_5 and T and one depending on the angular variables. Using

$$\phi = \Theta(T)\,G(r_5)\,F(\theta,\gamma,\alpha,\beta) \text{ with } \Theta(T) = e^{ikT} \tag{14}$$

the radial dependence is given by

$$\frac{\partial^2 G}{\partial r_5^2} + \frac{4}{r_5}\frac{\partial G}{\partial r_5} + \left(k^2 + \frac{p}{r_5^2}\right)G = 0\,. \tag{15}$$

p is the constant of separation.

Solutions of this equation are Bessel functions J and Y of transcendent order.[5] They can be combined to something similar to spherical Bessel functions of the third kind

$$G_p(kr_5) = \frac{J_{\frac{1}{2}\sqrt{9-4p}}(kr_5) + iY_{\frac{1}{2}\sqrt{9-4p}}(kr_5)}{(kr_5)^{\frac{3}{2}}}\,. \tag{16}$$

For large r_5 they converge to $G_0(kr_5)$, the solutions with $p = 0$:

$$G_0(kr_5) = \frac{e^{ikr_5}}{(kr_5)^2} + i\frac{e^{ikr_5}}{(kr_5)^3}\,. \tag{17}$$

Then $F(\theta,\gamma,\alpha,\beta) = \text{const}$ for all $\theta,\gamma,\alpha,\beta$ is a solution of the angular dependence of equation (13). This means that ϕ is isotropic and that the angular dependence in the action integral is introduced only by the Jacobean. Integration over its angle variables then can be executed giving $\int\frac{J}{r_5^4}F_0F_0^* d\theta d\gamma d\alpha d\beta = \text{const}$. Since the Lagrangian is only defined up to an arbitrary multiplicative factor without loss of generality, the constant can be assumed to be 1.

Thus we obtain for the action integral

$$S = \int \frac{\partial \Theta^\star}{\partial T}\frac{\partial \Theta}{\partial T} dT \int r_5^4 G_0 G_0^* dr_5 - \int \Theta^\star \Theta dT \int r_5^4 \frac{\partial G_0^\star}{\partial r_5}\frac{\partial G_0}{\partial r_5} dr_5 \ . \tag{18}$$

To gain the non-interpretable Lagrangian, integration in the action integral over the two variables not occurring in 4d must be executed what means introducing new spatial variables in which these two variables occur explicitly.

Maintaining the demands of symmetry this is achieved by transferring the original spherical symmetry to the in four dimensions remaining three spatial variables what means choosing spherical coordinates for them. Instead of integrating over u and v integration can be done over the associated polar coordinates as they also span the u,v-plain.

Introducing the new coordinates

$$\begin{aligned} r_3 &= r_5\sqrt{1-\sin(\theta)^2\sin(\gamma)^2\sin(\alpha)^2}, \\ \rho &= r_5 \sin(\theta)^2 \sin(\gamma)^2 \sin(\alpha)^2 \ , \\ \tan(\bar{\theta}) &= \tan(\theta)\cos(\gamma)\sqrt{1+\tan(\gamma)^2\cos(\alpha)^2} \\ \tan(\bar{\alpha}) &= \tan(\beta) \ , \\ \tan(\bar{\gamma}) &= \tan(\gamma)\cos(\alpha) \ . \end{aligned} \tag{19}$$

describing two spheres in three and two dimensions in the spatial part of the assigned spacetime we get

$$\begin{aligned} &x = r_3 \sin(\bar{\theta})\cos(\bar{\gamma}) \ , y = r_3 \sin(\bar{\theta})\sin(\bar{\gamma}) \ , z = r_3\cos(\bar{\theta}) \ , \\ &u = \rho\cos(\bar{\alpha}) \ , v = \rho\sin(\bar{\alpha}) \ . \end{aligned} \tag{20}$$

It also holds $r_3^2 + \rho^2 = r_5^2$.

In the new variables the Jacobi determinant becomes $\rho r_3^2 \sin(\bar{\theta})$. For the isotropic solution also in the new variables the angular dependence enters the action integral only by the Jacobi determinant.

So integration over the angular variables can be executed. Besides an unessential constant factor the spatial part of the action integral is transformed in $\int r_5^4 dr_5 = \int r_3^2 dr_3 \int \rho d\rho$. Using instead of ρ the integration variable $\zeta^2 = r_3^2 + \rho^2$ we finally find the non-interpretable Lagrangian

$$\hat{\mathcal{L}}4 = \frac{1}{k^2}\frac{\partial \Theta}{\partial T}\frac{\partial \Theta^\star}{\partial T}\frac{1}{(kr_3)^2}\left(1+\frac{1}{2(kr_3)^2}\right) - \frac{\Theta\Theta^\star}{(kr_3)^2}\left(1+\frac{3}{2(kr_3)^2}+\frac{3}{(kr_3)^4}\right). \tag{21}$$

It must be mentioned that r_3 is a 6d variable.

3.2 Interpretable Lagrangian

Next a Lagrangian formulated with 4d variables must be found giving the same contribution in the 6d action integral as $\hat{\mathcal{L}}4$. As the non-interpretable Lagrangian given in equation (21) is isotropic the interpretable Lagrangian with spherical symmetry looked for has to have this property as well.

3.2.1 Impossibility of adaption in a Euclidean space

First, it is to be shown that with spherical symmetry of the 6d Euclidean space, there is no 4d Lagrangian that can be adapted.

Due to the transfer of symmetries in UR, the spatial part of the 4d spacetime also has spherical symmetry. The adapted coordinates in a Minkowski spacetime then generate a diagonal metric tensor. Replacing $\bar{\theta}$ by the new variable $\mu = \cos\left(\bar{\theta}\right)$ and introducing the radius r of the 3d spatial part the diagonal elements are given by $\left\{1, -1, -r^2\left(1-\mu^2\right)^{-1}, -r^2\left(1-\mu^2\right)\right\}$ and it holds $\sqrt{-g} = r^2$.

With the 4d wave function ψ the test-Lagrangian is given by

$$\mathcal{L}4 = \frac{\partial\psi}{\partial T}\frac{\partial\psi^\star}{\partial T} - \frac{\partial\psi}{\partial r}\frac{\partial\psi^\star}{\partial r} - \frac{1-\mu^2}{r^2}\frac{\partial\psi}{\partial\mu}\frac{\partial\psi^\star}{\partial\mu} - \frac{1}{r^2\left(1-\mu^2\right)}\frac{\partial\psi}{\partial\bar{\psi}}\frac{\partial\psi^\star}{\partial\bar{\phi}} - a\psi\psi^\star. \tag{22}$$

Dabei ist

$$a = 1/\lambda_C^2 = m_0^2c^2/\hbar^2 \tag{23}$$

the mass term generated in the transition from 6d to 4d with translational symmetry as shown in part 2 of the series equation (12).

The resulting equation of motion is

$$r^2\frac{\partial^2\psi}{\partial T^2} - \frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) - \frac{\partial}{\partial\mu}\left(\left(1-\mu^2\right)\frac{\partial\psi}{\partial\mu}\right) - \frac{1}{\left(1-\mu^2\right)}\frac{\partial^2\psi}{\partial\bar{\gamma}^2} + ar^2\psi = 0\ . \tag{24}$$

As it was shown above an isotropic solution in 6d demands an isotropic solution in 4d. For this isotropic solution it holds $\frac{\partial\psi}{\partial\mu} = 0$ and $\frac{\partial\bar{\gamma}}{\partial\mu} = 0$

Separated by a product ansatz

$$\psi_i = \Theta\left(T\right)W\left(r\right) \tag{25}$$

equation (24) gives the ordinary differential equations

$$\frac{\partial^2\Theta}{\partial T^2} = -k^2. \tag{26}$$

and

$$-\left(k^2-a\right)W - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial W}{\partial r}\right) = 0\ . \tag{27}$$

Introducing in the last equation the new constant

$$\kappa^2 = k^2 - a \tag{28}$$

and the new variable

$$\hat{r} = \kappa r \tag{29}$$

it becomes

$$\frac{1}{\hat{r}^2}\frac{\partial}{\partial\hat{r}}\left(\hat{r}^2\frac{\partial W}{\partial\hat{r}}\right) + W = 0. \tag{30}$$

If k is chosen equal to the value found in equation (14) this is the same time dependency as in the 6d equation of motion.

Solutions are a complex exponential function

$$\Theta = e^{ikT} \tag{31}$$

and the spherical Bessel function of the third kind [6] zero order

$$W = h_0\left(\kappa r\right) = \frac{e^{i\kappa r}}{\kappa r}\ . \tag{32}$$

Inserting this solution in the isotropic part of the test-Lagrangian (21) gives

$$\mathcal{L}4 = -\frac{2k^2-a}{r^2(k^2-a)} - \frac{1}{r^4(k^2-a)}.$$

This test-Lagrangian has not the form of the non-interpretable Lagrangian (21). In particular, the occurrence of the mass term a in the expression makes it unusable. Also no terms with powers greater than $1/r^4$ are possible. So it cannot be used as a 4d transfer function.

Thus it is shown that for spherical symmetry in 6d there is no adaptable Lagrangian in a 4d Minkowski space.

3.2.2 Adaption in a curved spacetime

Here the basic idea of UR comes into play. It allows using a more flexible test-Lagrangian enabling adaption. That is e.g a Lagrangian in a spacetime with a pseudo Riemannian metric.

For the required spherical symmetry we then find the metric

$$\left\{ g_{11}(r), -g_{22}(r), -\frac{r^2}{1-\mu^2}, -r^2\left(1-\mu^2\right) \right\} \quad \text{with} \quad \sqrt{-g} = r^2\sqrt{-g_{11}g_{22}} \tag{33}$$

By symmetry considerations the metric coefficients only occur for the temporal and radial components.[7] g_{11} and g_{22} are two unknown functions depending on r and T. The structure of the non-interpretable Lagrangian suggests an approach with coefficients depending only on r.

This gives the interpretable test-Lagrangian

$$\mathcal{L}4 = g^{\nu\lambda}\partial_\nu\psi\partial_\lambda\psi^* - a\psi\psi^* \tag{34}$$

and the action integral

$$S = \int r^2\sqrt{-g_{11}g_{22}}\mathcal{L}4 dT dr d\mu d\bar{\gamma} \; . \tag{35}$$

The metric coefficients g_{11}, g_{22} are the additional adaption parameters needed.

Introducing the modified metric coefficient

$$A = \sqrt{-g^{11}/g^{22}} \tag{36}$$

the equation of motion becomes

$$\begin{aligned} &\frac{r^2}{A}\frac{\partial^2\psi}{\partial T^2} - \frac{\partial}{\partial r}\left(r^2 A\frac{\partial\psi}{\partial r}\right) + \sqrt{-g_{11}g_{22}}r^2 a\psi \\ -\sqrt{-g_{11}g_{22}}&\left[\frac{\partial}{\partial\mu}\left(\left(1-\mu^2\right)\frac{\partial\psi}{\partial\mu}\right) + \frac{1}{\left(1-\mu^2\right)}\frac{\partial^2\psi}{\partial\bar{\gamma}^2}\right] = 0 \; . \end{aligned} \tag{37}$$

For the isotropic solution the second line is zero.

Adaption to $\hat{\mathcal{L}}4$ means finding metric coefficients that give for the isotropic part of the differential equation (37) a solution which inserted in the action integral (equation (35)) together with the metric coefficients of the Jacobian determinant therein gives formally the same expression as $\hat{\mathcal{L}}4$ of equation (21).

This procedure would not be feasible with completely unknown metric coefficients, but here it helps that the solution of the equation of motion of the sought Lagrangian must be a function of Hilbert space.

Some general considerations valid for arbitrary symmetries can be made: As in the gravitational approximation in 6d there is no force the assumption can be made that in great distance of the center of symmetry the influence of the symmetry on the wave function vanishes and the Lagrangian together with the radial part of the Jacobian is that of a plane wave.

Plane waves are the limiting case of a wave function in Hilbert space. A slower decay of the wave function is not allowed as such functions are not normalizable. A stronger decay as e.g. exponential on the other hand characterizes a bound state and implies a force.

A general wave function in the assigned spacetime with all Young-Mills field being zero therefore must be described by a power series in $1/r$ (r here means the distance from the center of symmetry) multiplied with a phase factor whose differential is also a power series in $1/r$. Equation (21) shows this behavior.

As thus a general $\hat{\mathcal{L}}4$ is represented by a power series in $1/r$ this has to hold also for the adapting $\mathcal{L}4$, here given by equation (34). It is achieved when all summands, their factors and the derivatives of the phases in $\mathcal{L}4$ are given by power series in $1/r$ what means that metric coefficients are described by power series in $1/r$.

Coming back to the spherical symmetry. It seems that for the isotropic part of equation (37)

$$\frac{r^2}{A}\frac{\partial^2\psi}{\partial T^2}-\frac{\partial}{\partial r}\left(r^2 A\frac{\partial\psi}{\partial r}\right)+\sqrt{-g_{11}g_{22}}r^2 a\psi=0 \tag{38}$$

with metric coefficients given by a power series in $1/r$ no formulated solution is available. Therefore such a solution is looked for with a power series ansatz.

The Klein-Gordon equation (38) has two independent parameters A and $\sqrt{-g_{11}g_{22}}$. To simplify the examination firstly two simplified equations, the "S-type" with $\sqrt{-g_{11}g_{22}}=1$ and the "N-type" with $A=1$ are to be considered. The general solution is discussed later.

3.2.3 S-type adaption

Motivation for the denomination S-type is the similarity of the found metric with that of the external Schwarzschild solution in GR.

Multiplying the simplified equation (38) with A/r^2, executing the differentiation of the first term as given in equation (26), using the parameter κ given in equation (28) and the variable $\hat{r}$ given in equation (29) we find

$$\frac{A}{\hat{r}^2}\frac{d}{d\hat{r}}\left(A\hat{r}^2\frac{dW}{d\hat{r}}\right)+\left(1-\frac{a}{\kappa^2}(A-1)\right)W=0\ . \tag{39}$$

Since, as we will see, there is a close relationship between this differential equation and the differential equation of the Coulomb wave functions [8], its asymptotic solution for large r should is taken as a template for the solution we are looking for.

Introducing the approach

$$W=z(r)\exp iq(r) \tag{40}$$

this means for the different terms of the formula specific power series

$$\begin{aligned} A &= 1+r_1/\hat{r}+r_2/\hat{r}^2+\ldots \\ q &= k_0\hat{r}+k_1\ln\hat{r}+\sigma_0 \\ z &= z_1/\hat{r}+z_2/\hat{r}^2+z_3/\hat{r}^3+\ldots\ . \end{aligned} \tag{41}$$

σ_0 is a phase not depending on $\hat{r}$ needed to make the argument of the exponent dimensionless.

The coefficients in A and q are real, those in z might be complex. It is notable that q is not a series but has only two terms.

Implementing these expressions in equation (39) gives a power series whose coefficients must be zero. For the first powers after a lengthy calculation it results

$$\begin{aligned} & z_1=\text{arbitrary} \\ & z_2=ik_0z_1\left(\left(r_1^2-r_2\right)\left(1+\frac{a}{2\kappa^2}\right)-\frac{a^2r_1^2}{8\kappa^4}+\frac{ak_0r_1i}{4\kappa^2}\right) \\ & k_0^2=1 \\ & k_1=-r_1k_0\left(1+\frac{a}{2\kappa^2}\right)\ . \end{aligned} \tag{42}$$

It can be seen that z_1 is the free multiplicative constant and that $k_0=1$ and $k_0=-1$ generate the two independent solutions conjugate-complex to each other of the homogeneous second order differential equation. The equation for k_1 shows that for large $\hat{r}$ gravity and the logarithmic phase are related.

Next step is showing to which extent the interpretable Lagrangian $\mathcal{L}4$ given by equation (34) formulated with the found functions can be adapted to the non-interpretable Lagrangian $\hat{\mathcal{L}}4$ of equation (21).

Implementing $\Theta(T)=e^{ikT}$ of equation (14) in the $\hat{\mathcal{L}}4$ of equation (21) with

$$\Theta\Theta^\star=1\ \text{ and }\ \frac{1}{k^2}\frac{d\Theta}{dT}\frac{d\Theta^\star}{dT}=1$$

the terms quadratic in $1/r$ cancel each other. Only terms of fourth and sixth order in $1/r$ remain. This means that the different terms in $\mathcal{L}4$ up to third order must cancel each other and the sum over the terms of fourth order must become $1\,(kr)^4$.

This is just what results if Θ and the functions found in equation (42) are implemented in $\mathcal{L}4$. As necessary for an adaption all terms up to third order vanish and it remains $z_1 z_1^* / \hat{r}^4$.

By an appropriate formulation of the found "0" the interpretable Lagrangian can be written as

$$\mathcal{L}4 = \frac{1}{k^2}\frac{d\Theta}{dT}\frac{d\Theta^\star}{dT}\frac{1}{(kr)^2}\left(1+\frac{1}{2\,(kr)^4}\right)$$

$$-\Theta\Theta^\star\left(\frac{1}{(kr)^2}+\frac{1}{2\,(kr)^4}+\frac{z_1 z_1^\star}{(kr)^4}\frac{k^4}{\kappa^4}\right) \tag{43}$$

Setting in a last step

$$z_1 z_1^\star = \frac{\kappa^4}{k^4} \tag{44}$$

we see that $\mathcal{L}4$ and $\hat{\mathcal{L}}4$ are formally equal up to fourth order, if the 6d variable r_3 is replaced by the 4d variable r. As mentioned this is possible as it happens for dummy variables in an integral.

This shows that the Lagrangian of the S-type approach for large distances of the center can be adapted to $\hat{\mathcal{L}}4$ and that the 4d observer to describe the spherical symmetry of 6d space must introduce a curved space that can be assumed as pseudo-Riemannian.

3.2.4 N-type adaption

Although the Lagrangian given in equation (38) is formulated in a spacetime with Riemannian metric in the N-type approach because of the demand $A = 1$ it seems to act in a Minkowski spacetime. $f(r) = \sqrt{-g_{11}g_{22}}$ enters the equation as a force field. The "N" in the denomination is to show the similarity to Newton's gravity theory.

It can be seen that this approach must be an approximation as the force field $f(r)$ cannot act on particles without rest mass.

As the metric coefficients in the last chapter $f(r)$ can be written as a power series in the dimensionless variable $\hat{r}$. With

$$f(r) = 1 + f_1/\hat{r} + f_2/\hat{r}^2 + \dots \tag{45}$$

the equivalent to equation (39) becomes

$$\frac{1}{\hat{r}^2}\frac{\partial}{\partial \hat{r}}\left(\hat{r}^2\frac{\partial W}{\partial \hat{r}}\right) + \left(1 - \frac{a}{\kappa^2}(f-1)\right)W = 0. \tag{46}$$

With the new variable $w = W\hat{r}$ it follows

$$\frac{d^2 w}{d\hat{r}^2} + \left(1 - \frac{a}{\kappa^2}\left(\frac{f_1}{\hat{r}} + \frac{f_2}{\hat{r}^2} + \dots\right)\right)w = 0. \tag{47}$$

Setting $af_1/\kappa^2 = 2\eta$ and f_2 and all coefficients of higher powers to zero, Coulomb wave functions [8] for zero angular momentum $H_0(\eta, \hat{r})$ and $G_0(\eta, \hat{r})$ solve this equation.

In general the two solutions cannot be combined like spherical Bessel functions of first and second order to those of third order. But this is possible for their asymptotic expansions for large $\hat{r}$.

Then it holds

$$w(\hat{r}) = (G_0(\eta, \hat{r}) + iF_0(\eta, \hat{r})) = (f_0 + ig_0)\,e^{i\Theta_0} \tag{48}$$

with

$$f_0 + ig_0 = 1 + \frac{i\eta\,(i\eta+1)}{1!\,(2i\hat{r})} + \frac{i\eta\,(i\eta+1)^2\,(i\eta+2)}{2!\,(2i\hat{r})^2} + \dots \tag{49}$$

and

$$\theta_0 = \hat{r} - \eta \ln 2\hat{r} + \sigma_0. \tag{50}$$

σ_0 is introduced for the same reason as in equation (41).

Reintroducing W this gives

$$W = \left(\frac{z_1}{\hat{r}} + \frac{z_1 \eta \left(i\eta + 1\right)}{2\hat{r}^2} \dots\right) e^{i\Theta_0}. \tag{51}$$

The same procedure as in the last chapter can be used to show that this function allows an adaption to $\hat{\mathcal{L}}4$ as well.

This shows that also the N-type solution is able to describe the 4d manifestations of the spherical 6d symmetry.

3.2.5 Comparing S- and N-type solution

The main difference between S- and N-type solution consists in the second terms of the phases q of equation (42) and θ_0 of equation (50). In the N-type solution it is proportional to the mass term a, what means that it vanishes for massless particles, whereas in the S-type solution an additional term not dependent on a occurs so that also massless particles are affected.

The otherwise narrow connection between the two solutions can be shown by a coordinate transformation. Starting point is equation (39) of the S-type solution. It is assumed that the power series of the metric has only terms up to first order in $1/\hat{r}$.

With $A = 1 + \frac{r_1}{\hat{r}}$ and the new variable $\rho = \frac{r_1}{\ln\left(1+\frac{r_1}{\hat{r}}\right)}$ it becomes

$$\frac{1}{\rho^2}\frac{\partial}{\partial\rho}\left(\rho^2\frac{\partial W}{\partial\rho}\right) + \frac{\hat{r}^4}{\rho^4}\left(1 - \frac{\rho}{\hat{r}}\frac{a r_1}{\kappa^2 \rho}\right) W = 0. \tag{52}$$

Developing $1/\hat{r}$ in a power series of $1/\rho$ to elimimate $\hat{r}$ gives

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2\frac{dW}{d\rho}\right) + \left(1 - \frac{r_1}{\rho}\left(2 + \frac{a}{\kappa^2}\right) + \frac{r_1^2}{\rho^2}\left(\frac{11}{6} + \frac{3a}{2\kappa^2}\right) + \dots\right) W = 0\ . \tag{53}$$

The relation between ρ and $\hat{r}$ is given by

$$\rho = \hat{r}\left(1 + \frac{r_1}{2\hat{r}} - \frac{r_1^2}{12\hat{r}^2} + \dots\right) \text{ or } \rho \approx \hat{r} + \frac{r_1}{2}. \tag{54}$$

This means that as long as $a \gg k^2$ the two solutions are essentially equivalent if f_1 and r_1 are equal, $\hat{r} \gg r_1$ and if effects of quadratic and higher order in $1/\hat{r}$ can be neglected.

The fact that both S-type and N-type solution allow adapting a 4d Lagrangian to $\hat{\mathcal{L}}4$ shows an inner connection of Newton's theory of gravity and the external Schwarzschild solution of GR. It is no happenstance but a consequence of being almost equivalent solutions of a common equation. As well it does not mean that Newton's theory is valid only as a limit of GR. Besides the (small) difference occurring when taking the angular part of equation (37) into account (see below) in the mentioned domain the two solutions are equal.

4 Effects of gravitation on particles

The previous considerations were used to derive the 4d effects of spherical symmetry in 6d. We did find metric coefficients modifying the Klein-Gordon equation. In the following effects of these changes on particles as e.g. planets, that are subject to them will be examined. Their movement refers to the secondary effects described at the end of section 4.1.4. For these particles the requirement of isotropy does not hold. In order to fully describe their motion their torque has to be taken into account. This means that a Klein-Gordon equation of the type given in equation (37) with angular dependence must be used.

Separating this equation by a product ansatz

$$\psi = \Theta(T)\, P(\mu)\, Q(\bar{\gamma})\, W(r) \quad \text{with} \quad \Theta(T) = e^{ikT} \tag{55}$$

we get the ordinary differential equations

$$\frac{\partial^2 Q}{\partial \bar{\gamma}^2} = -m^2 Q \tag{56}$$

$$\frac{\partial}{\partial \mu}\left(\left(1-\mu^2\right)\frac{\partial P}{\partial \mu}\right) - \frac{m^2}{1-\mu^2} P = -l\,(l+1)\, P \tag{57}$$

and, using the variable given in equation (29),

$$\frac{1}{\hat{r}^2}\frac{\partial}{\partial \hat{r}}\left(\hat{r}^2 \frac{\partial W}{\partial \hat{r}}\right) + \left(1 - \frac{l\,(l+1)}{\hat{r}^2}\right) W = 0. \tag{58}$$

Solutions are complex exponential functions

$$Q = Q_m = e^{im\bar{\gamma}} \ (m \text{ integer}), \tag{59}$$

Legendre functions [6]

$$P = P_l^m = \sqrt{\frac{(2l+1)\,(1+|m|)!}{2\,(l+|m|)!}}\left(1-\mu^2\right)^{\frac{|m|}{2}} \frac{d^{|m|} P_l(\mu)}{d\mu^{|m|}} \tag{60}$$

with Legendre polynomials $P_l(\mu) = \frac{1}{2^l l!}\frac{d^l\left(\mu^2-1\right)^l}{d\mu^l}$ $(l = 0, 1 \ldots, |m| \leq l)$
and spherical Bessel functions of the third kind [9]

$$W = h_l(\kappa r) \qquad \text{with} \qquad h_l(x) = x^l \left(\frac{1}{x}\frac{d}{dx}\right)^l \frac{e^{ix}}{x}\ . \tag{61}$$

The dependence on the angular variables μ and $\bar{\gamma}$ often is expressed together by spherical harmonics $Y_{l,m}(\mu, \bar{\gamma}) = P_l^m Q_m$.

It is not yet obvious how gravitation arises out of these equations found in the last section. To show this at first the particle aspect of the theory is to be examined what should give besides others the results found by the Schwarzschild solution of GR. In section 8 it will be considered to what extent the quantum theoretical character of the equations can give additional information.

Without any understanding of the meaning of a wave function in a theory of gravitation the Klein-Gordon equation equation was widely used in GR. Its separability in different coordinate systems served to find most known metrics like that of Schwarzschild, de Sitter or Kerr.[10, 11, 12, 13] In [12] Carter writes: "We shall be led to impose the ... condition that the ... Schrodinger equation (in our nomenclature the Klein-Gordon equation) is separable not because there is any good physical reason for doing so but because it leads to a very simple algebraical form for the metric."

This strictly formal use of the wave function differs fully from the approach followed here. For UR the wave function also in describing gravity is of physical relevance.

Interpreting the S-type solution of equation (38) two limiting cases can be distinguished: slowly moving particles with rest mass and massless particles moving by nature with the speed of light. For the N-type solution only the first limit is of interest. Therefore in a first step both solutions are examined for slowly moving particles with rest mass and then the general solution for massless particles.

The metric coefficients in UR are given by power series in $1/r$. Weinberg describes some metrics introduced by Eddington and Robertson that also use higher powers of $1/r$ and discusses their implications.[14] He shows that possible effects are so small that they are completely masked by the effects of GR and up to now not yet found. Therefore in the following the power series in UR are cut for all terms with order higher than $1/r$.

4.1 Slowly moving particles with rest mass

Knowing that

1. the Klein-Gordon equation describes systems in which particles and antiparticles occur simultaneously,
2. in the non-relativistic limit the Klein-Gordon equation degenerates into two Schrödinger equations one for particles and one for antiparticles,
3. our solar system is built by particles only and
4. the speed of most particles with rest mass in the solar system is small compared to the velocity of light

it seems natural not to use the Klein-Gordon equation in describing particles with rest mass but to replace it by a Schrödinger equation.

The transition from the Klein-Gordon to the Schrödinger equation is achieved by eliminating the mass term in the Klein-Gordon equation by means of a unitary transform.

Non-relativistic physics is characterized by the approximation that in all areas where the wave function is measurably distinct from zero for any function Θ occurring in the system the relation $|\hbar\partial_t\Theta| \ll |m_0c^2\Theta|$ holds.[15]

This means introducing by

$$\psi = Xe^{-i\alpha t} \quad \text{with} \quad \alpha = \frac{m_0c^2}{\hbar} \quad \text{and} \quad T = ct \tag{62}$$

a modified wave function X and neglecting all small terms in the differential operator. It follows

$$\begin{aligned}\partial_T\partial_T\psi &= \frac{1}{c^2}\frac{e^{-i\alpha t}}{c^2}\left(\partial_t\partial_t - 2\alpha i\partial_t - \alpha^2\right)X \\ &\approx \frac{e^{-i\alpha t}}{c^2}\left(-2\alpha i\partial_t - \alpha^2\right)X.\end{aligned} \tag{63}$$

With this approximation, introducing the mass term $a = m_0^2c^2/\hbar^2$ of equation (23) and using for the angular part the result of equation (57) we get instead of the Klein-Gordon the Schrödinger equations

$$\begin{aligned}i\hbar\frac{\partial X_S}{\partial t} = -\frac{\hbar^2 A}{2m_0r^2}\left[\frac{\partial}{\partial r}\left(r^2A\frac{\partial X_S}{\partial r}\right) - l\left(l+1\right)\right] + \\ \frac{m_0c^2}{2}\left(A-1\right)X_S\end{aligned} \tag{64}$$

and

$$\begin{aligned}i\hbar\frac{\partial X_N}{\partial t} = -\frac{\hbar^2}{2m_0r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial X_N}{\partial r}\right) - f(r)l\left(l+1\right)\right] + \\ \frac{m_0c^2}{2}\left(f\left(r\right)-1\right)X_N\end{aligned} \tag{65}$$

The indices S and N stand for the S- and N-type solutions.

It can be seen that both solutions have a modified mass term that can be interpreted as a potential. The solutions differ in the expression of kinetic energy and by the multiplicative factor A or $f(r)$ in the angular parts.

Separation of the Schrödinger equations with an ansatz

$$X = e^{-i\frac{E}{\hbar}t}R(r) \tag{66}$$

gives the radial parts

$$\frac{A}{r^2}\frac{d}{dr}\left(Ar^2\frac{dR_S}{dr}\right) - \left(\hat{\kappa}^2 + A\frac{l\left(l+1\right)}{r^2} - \frac{2}{r_Br}\right)R_S = 0 \tag{67}$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_N}{dr}\right) - \left(\hat{\kappa}^2 + f\frac{l\left(l+1\right)}{r^2} - \frac{2}{r_Br}\right)R_N = 0 \tag{68}$$

with

$$\hat{\kappa}^2 = -\frac{2m_0}{\hbar^2}E \quad \text{and} \quad r_B = \frac{2\hbar^2}{m_0^2 c^2 r_S} \ . \tag{69}$$

As demonstrated above it is natural assuming f_1 and r_1 to be equal. So in the last terms $r_S = -f_1/\kappa = -r_1/\kappa$ is introduced.

Applying the same transformation as used to show the connection between S- and N-solution of the Klein-Gordon equation to equation (67) leads in first order of r_S/r to an equation with the structure of equation (68):

$$\frac{1}{\rho^2}\frac{d}{d\rho}\left(\rho^2 \frac{dR_S}{d\rho}\right) - \left[\hat{\kappa}^2 + \frac{l(l+1)}{\rho^2}(1-\epsilon) - \frac{2}{\rho r_B}\right] R_S = 0 \ . \tag{70}$$

This shows that for $r \gg r_S$ the S- and N-type solutions are identical besides the term $\epsilon = \frac{3r_S}{r_B l(l+1)}$. Implementing the result we find in equation (82) this gives $\epsilon \approx \frac{3r_S}{\rho} \ll 1$.

The parameter $\hat{\kappa}^2 = -\frac{2m_0}{\hbar^2}E$ is clarifying the different types of solutions for positive or negative energy. For $E > 0$ we get $\hat{\kappa}^2$ negative, $\hat{\kappa}$ imaginary and the solutions are spherical waves. For $E < 0$ $\hat{\kappa}^2$ is positive and the solutions are bound states.

Setting in the Schrödinger equations (67) $A = 1$ or in (68) $f = 1$ they formally coincide with the Schrödinger equation of a hydrogen atom in center of mass coordinates. r_B is the equivalent of the Bohr radius introduced there.

The eigenfunctions R of the simplified Schrödinger equations for $E < 0$ are Laguerre functions [16]

$$R = R_{n,l}(x) = x^l e^{-\frac{x}{2}} L_{n-l-1}^{2l+1}(x) \tag{71}$$

with $x = \frac{2r}{nr_B}$, $n = 1, 2\ldots$ and $l < n$. $L_{n-l-1}^{2l+1}(x)$ are associated Laguerre polynomials.

Formally the full equations (67) and (68) have the same structure as the simplified equations. The small terms r_s/r in A and f or the ϵ in equation (70) however destroy the integrity of the angular momentum term so that Laguerre functions are only an approximate solution. Nevertheless because of the smallness of the distortion - it generates, as will be shown, perihelion rotation - they describe the behavior of particles with rest mass quite well.

The relativistic Klein-Gordon equations comprise the square of energy as it describes at a time particles and antiparticles. As this expression is always positive no binding force exists. In the non-relativistic Schrödinger equations energy enters linearly so that it can be also negative. Negative energies generate bound states what requires a binding force. With this force of attraction finally for particles with rest mass we have found gravitation.

4.1.1 Elementary particles in a gravitational field

The Schrödinger equation commonly is used to describe the behavior of elementary particles. So a first test of the here found Schrödinger equation of gravitation is to examine whether it can correctly characterize the performance of elementary particles in a gravity field.

Such an experiment was conducted at the ILL[†]. In order to avoid distortions by the much larger electromagnetic force the experiment was accomplished with neutrons. It could be shown that the behavior of a neutron in the gravitational field on the surface of the earth agrees fully with the predictions calculated with the Schrödinger equation.[17].

This result can be understood as a confirmation showing the validity of the approach taken.

4.1.2 Planet orbits and perihelion rotation of classical particles

To give results for UR comparable to those of GR it must be formulated for classical particles. So we need to find out how wave function and classical particles relate to each other.

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This can be achieved going back to the early days of quantum mechanics. Then a formalism was introduced determining how mechanical quantities must be “translated“ to give quantum physical quantities. This translation scheme shows that a classical potential enters formally unaltered the Schrödinger equation in position representation. In turn this means that a potential in a Schrödinger equation expressed with the position operator formally unaltered describes the potential of a classical Hamilton equation. This relation offers the easiest way to calculate trajectories of particles in UR.

How can the Schrödinger equation describe the behavior of such large structures as planets with their huge amount of particles? In the gravitational approximation the interaction between the individual particles is neglected and only their interaction with the central gravitational force is taken into account. The Schrödinger equation can be understood as describing one representative particle. All others as the whole planet show the same behavior.

GR tells us that particles under the influence of gravitation move along of geodetic lines. The Hamilton equation with the re-translated potential describes the trajectories of particles enabling a comparison with the results of UR.

The effective potential in the radial parts of the Schrödinger equations (67) respective (68) simplified by setting $A = 1$ resp. $f = 1$ is given by

$$V_{eff} = \frac{\hbar^2}{2m_0}\left(\hat{\kappa}^2 + \frac{l\,(l+1)}{r^2} - \frac{2}{r_B r}\right). \tag{72}$$

This expression interpreted as a classical potential is equal to that of Newton's celestial mechanics if the parameter r_S introduced in equation (69) is identical with the Schwarzschild radius

$$r_S = \frac{2GM}{c^2} \tag{73}$$

(G gravitational constant, M mass of the central star) and if as usual $\hbar^2 l(l+1)$ is replaced by the square of the classical angular momentum.

This can be shown by retranslating the simplified Schrödinger equations using the effective potential (72) giving the original Hamilton function

$$\frac{p_r^2}{2m_0} + \frac{L^2}{2m_0 r^2} - \frac{Gm_0 M}{r} = E$$

where p_r stands for the radial momentum.

Without simplifications the found Schrödinger equation of gravitation is equivalent to the Einstein equation in spaces with spherical symmetry. This can be seen comparing the trajectories for massive particles found with the Schrödinger equation with the geodetic lines of the external Schwarzschild solution.

Equations (67) respective (68) describe the effective potentials

$$V_{eff} = \frac{\hbar^2}{2m_0}\left(\hat{\kappa}^2 + \left(1 - \frac{r_S}{r}\right)\frac{l\,(l+1)}{r^2} - \frac{2}{r_B r}\right). \tag{74}$$

This potential (also with $\hbar^2 l(l+1)$ replaced by the square of the classical angular momentum) generates the same trajectories (with perihelion rotation) as found with a lengthy calculation for the Schwarzschild solution.[18]

The metric coefficient in the term $\frac{\hbar^2 A}{r^2}\frac{d}{dr}\left(Ar^2\frac{dR_S}{dr}\right)$ in equation (67) results as shown in equation (70) in a negligibly small change of the radius of a planet by $r_B/2$.

4.1.3 Barycentric coordinate time

A problem with local time arises in GR because of the different clock rates at different locations in the solar system. This would mean that time runs with different speed at the positions of the different planets, which contradicts observations.

To overcome the problem in 1991 the barycentric coordinate time was introduced, a uniform variable of time for all heavy bodies in the solar system.[19] It can be understood as the time at the position of the barycenter of a solar system without mass. This common time for the whole solar system allows a correct description of the movement of the various bodies.

The barycentric coordinate time is the time of UR. It is the time used in the Schrödinger equation of gravity.

The time deducted from 6d does not only hold in our solar system but holds universally. This is shown, for example, in the fact that the behavior of the galaxies is described quite well by Newton's theory of gravity with its uniform time.

The necessity of the barycentric coordinate time is a strong endorsement of the UR approach.

4.1.4 Black holes

Black Holes often are understood as a consequence of the pole of the $1/r$ part in the metric of GR. They are interpreted as huge mass accumulation concentrated in a volume smaller than its Schwarzschild radius. For a quantum field theory of gravitation as UR a pole in the metric coefficients must not mean a pole in the wave function. Furthermore the metric coefficients are given by series in $1/r$ with unknown zeros and poles.

Although gravitation as deducted here only holds for large radii, UR offers an explanation for the effects found in cosmology attributed to black holes. To do this we need to take a closer look at the Schrödinger equation of gravity.

In Newtons celestial mechanics the orbits of the planets around the sun are calculated in center of mass coordinates, what in line with the observations means that the sun is moving under the influence of the planets. The point around which planets and sun orbit is the barycenter. Only if one mass is infinitely large the center of rotation lies in the center of its mass.

That differs from GR, where spherical symmetry around the sun is assumed.

With UR, we have found that the center of a spherical symmetry in 6d generates in 4d an immovable attractor characterized by a radial force for large r proportional to $-1/r^2$. This immovability causes that all massive particles in the solar system are moving around a fix center, given by $r = 0$ in the Schrödinger equation, the barycenter. The radius in the Schrödinger equation of gravitation describes the distance from this center and not the distance between center and center of two particles as in center of mass coordinates.

As we have seen an immobile center corresponds to a huge mass. If this mass furthermore is invisible it often is interpreted as a black whole.

Attractors result from a feature of 6d space. Their strength expressed in the metric coefficients is defined essentially by a parameter with the unit of a mass. Its value is not given by the symmetry. By measurement it has exactly the strength to stabilize the surrounding arrangement of matter. (For the generation of mass and gravity see also section cosmology in part 4 of the series.)

It is helpful to consider separately the universal effects derived from the 6d symmetry and the secondary effects arising due to the gravity of the mass generated in 4d.

Universal is the attractor and a unique time. Their action allows explaining celestial structures.

By the transition to 4d matter is generated that also exerts gravity which generates secondary effects (see section 5). Over time the matter is not homogeneous but forms individual bodies. These bodies interact with each other according to the same law of force as the attractor. Each of these bodies generates its own local metric but this does not influence in the larger scale the original one.

The strength of the attractor is not given by the theory. So the value of its mass can be understood as an adaption parameter selectable to explain the compensatory effects of 4d physics.

4.2 Particles without rest mass

Some effects to proof the validity of GR are related to the behavior of particles without rest mass – what usually means light. In the following it is demonstrated that also

UR describes these effects – light deflection, Shapiro effect, redshift and time dilation – correctly.

4.2.1 Light deflection

To describe the deflection of light moving in the gravitational field e.g. of a star the relativistic equation (37) has to be used. No rest mass means $a = 0$ what gives

$$\frac{r^2}{A}\frac{\partial^2\psi}{\partial T^2} - \frac{\partial}{\partial r}\left(r^2 A\frac{\partial\psi}{\partial r}\right) + l(l+1)\psi = 0. \tag{75}$$

Separating equation (75) with an ansatz

$$\psi = e^{-i\frac{E}{\hbar}}R(r) \tag{76}$$

gives a radial part

$$\frac{A}{r^2}\frac{d}{dr}\left(Ar^2\frac{dR}{dr}\right) + \left(\frac{E^2}{\hbar^2c^2} - A\frac{l\,(l+1)}{r^2}\right)R = 0 \tag{77}$$

with an effective potential

$$V_{eff} = \frac{E^2}{\hbar^2c^2} - \left(1 - \frac{r_S}{r}\right)\frac{l\,(l+1)}{r^2}\ . \tag{78}$$

Introducing as in the last section instead of $\hbar^2 l\,(l+1)$ the square of the classical angular momentum this effective potential is equal to the potential found with the Schwarzschild solution and describes the trajectories of a massless particle i.e. light deflection.[18]

4.2.2 Shapiro effect

Using the coefficients given in equation (42) and keeping in mind that for massless particles $\kappa = k$ for large r the solution of equation (37) is given by

$$\psi = \left(\frac{1}{r} + \frac{z_2}{r^2} + \dots\right)e^{i\left(kct+kr+kr_S ln\left(\frac{r}{r_0}\right)\right)}. \tag{79}$$

r_0 is an arbitrary phase term introduced by dimensional reasons.

For light speed and phase velocity are the same. This allows deducting the speed of light in a gravitational field by examining the exponent of this equation.

Introducing wavelength λ and oscillation period Δ we find

$$kc\Delta = 2\pi \text{ and } k\left(\lambda + r_S\ln\left(1+\frac{\lambda}{r}\right)\right) \approx k\lambda\left(1+\frac{r_S}{r}\right) = 2\pi$$

resulting in

$$c_S = \frac{\lambda}{\Delta} \approx \frac{c}{1+\frac{r_S}{r}} \tag{80}$$

The so found reduction of the speed of light in a gravitational field is as Shapiro effect experimentally confirmed. [20, 21]

4.2.3 Redshift and time dilation

Gravitational redshift and time dilation are not effects of GR. They already follow from special relativity and the equivalence principle. Nevertheless they are often introduced with the experimental proofs of GR.

For UR it can be be deducted as follows: Equation (38) with $a = 0$ describes light moving in radial direction in the gravity field of a spherical body. It is formulated in a Riemannian metric.

Introducing a local time

$$cdt_l(r) = \sqrt{A}dT = \sqrt{1 - \frac{r_S}{r}}dT$$

its first term can be written as $\frac{\partial^2 \psi}{c^2 \partial t_l^2}$.

Introducing ρ as in equation (52) in the second term $\frac{\partial}{\partial r}\left(r^2 A \frac{\partial \psi}{\partial r}\right)$ and noting that with equation (54) $\Delta\rho \approx \Delta r$ shows that the effect of A is negligibly small.

With these measures equation (75) simplifies to an equation in an Euklidean space with the variable time t_l. This means that clocks located at fixed positions, but not too far apart, run at different speeds depending on the radius of their position.

The effect is known as gravitational time dilation.

Comparing to the run of time without gravity – what is equal to a position far away from the center – it holds $dt_r = dt_\infty \sqrt{1 - r_S/r}$. This means $dT = cdt_\infty$.

Introducing the frequency $\nu = 1/T$ gives $\nu_r = \nu_\infty / \sqrt{1 - r_S/r}$.

Relating the frequency of a signal in two distances r_1 and r_2 from the center with $r_2 > r_1$ we get $\frac{\nu_2}{\nu_1} = \sqrt{\frac{1-r_S/r_1}{1-r_S/r_2}}$. This variation of frequency is known as gravitational redshift, as the color of light with increasing distance to the center of a star is shifted to lower frequencies.

5 Particles interacting by gravity

There are two ways to express interaction by gravity in UR.

The exact way to describe for example a double star is looking for a symmetry in 6d which in 4d produces a metric that generates the effects on a planet particle expected by a double star. The found metric coefficients then allow drawing conclusions how the two stars interact.

As it is difficult to imagine such a 6d symmetry an inverse approach might be more advantageous. It means assuming promising symmetries of the 6d space and comparing their consequences in 4d with observation results.

An easier way to handle interaction is a generalization of the found interaction between central and planet particle with spherical symmetry.

Mass enters the 4d Lagrangian in two different ways. The metric coefficients define the mass of a gravitational center. It occurs only for special 6d symmetries. In contrast the mass of a planet particle is given by its Compton wavelength and stands for the two 6d momenta not accessible in 4d.

But both types of mass have the same characteristics. To show this the center of symmetry is to be positioned in the center of the planet particle that is assumed to be spherical. As the mass of the planet particle in the Schrödinger equation does not influence the gravitational field of the central particle there is no feedback of the mass of the new planet particle – that in this situation is much larger than the central one – on the force it feels of the central particle. Therefore a calculation gives the same equation of motion with the two masses interchanged and describes correctly the trajectory of the new planet particle. It is the situation that the sun orbits the earth.

Equation (67) respective (68) show that the potential of the interacting force between the two particles is given by $\frac{m_0 c^2}{2}\frac{r_S}{r} = G\frac{m_0 M}{r}$ i.e. is proportional to the product of the two masses. The proportionality expresses Newton's third law $actio = reactio$.

This allows an extended interpretation of the interaction up to now understood as the influence of the metric characterized by a central attractor on a planet particle. The generation of entities with mass in 4d leads also to an interaction between these entities. This gravitation can be understood as a force between any two particles. If the two particles are spherical and far enough separated from each other the force is given by $K = -G\frac{m_0 M}{r^2}$ and acts from center to center. Regarding equation (53) an additional term proportional r_S^2/r^2 could make sense.

If there are several particles the force field of each of them is not affected by the masses of the other particles. Each particle sees these undisturbed fields of all other particles. So not only two but any number of particles can interact in this way.

This description of particles interacting by gravitation in some way is a justification of Newton's approach. But there are differences. In Newton's law the dependence of gravitation on distance is fixed whereas here if the particles are coming closer together more terms of the series describing gravitation can become active. So gravitational interaction between large masses and/or on short distances is different.

The approach neglects the impact of the metric on the angular momentum. It is therefore not exact but the neglects usually are small.

A galaxy can be understood as a collection of disjoint particles. The unique time of the last section and the description of interaction given here can serve as a relativistic justification of the often used Newtonian theory in calculating its behavior.

In section 8 will be described how starting with a wave function we can find the quasi-classical particles of Rydberg atoms and their interacting. We can reverse this concatenation from interacting classical particles to interacting quasi-particles to a wave function of interacting particles. The result is a Schrödinger equation with a structure known from interacting electrically charged particles (see e.g. [22])

$$i\hbar\frac{\partial X}{\partial t} = \sum_i -\frac{\hbar^2}{2m_i}\Delta X - G\sum_{i<j}\frac{m_i m_j}{r_{ij}}X \tag{81}$$

6 The UR counterpart to the energy-momentum tensor

The distinction in GR between internal and external solution with a defined boundary between the two areas of validity contradicts the behavior of a wave function and can hold only by approximation. In UR the difference between the two solutions must be justified by other arguments.

The symmetry of the 6d space in UR or of the energy-momentum tensor in GR define the structure of the fundamental equations (6d Lagrangian respective Einstein's equation). Solving Einstein's equation or adapting a 4d to the 6d Lagrangian defines the structure of the inherent metric, the metric coefficients and for UR also the wave function.

For the Schwarzschild solution e.g. the structure is expressed by two metric coefficients at their specific places. This structure defined by symmetry cannot be changed by choosing another energy-momentum tensor (with the same symmetry). The tensor can only modify the metric coefficients and the character of the gravitational center that is defined by the parameters of the metric coefficients.

The same holds for UR assuming spherical symmetry. It results the same structure with two metric coefficients. As for the Schwarzschild solution they are equal in the S-type solution but have to be different for a general solution.

To get more information about the gravitational center than only its mass in UR nothing like an energy-momentum tensor exists. Like mass also the other features must be generated by the transition from 6d to 4d. The possibility to do this is given by using eigenfunctions of higher order of the 6d equation of motion in generating the non-interpretable Lagrangian $\hat{\mathcal{L}}4$.

This means not to consider only the limit of large r but also the behavior for finite r with eigenfunctions depending also on the angular part of equation (13). As no matter how the angular term in 6d looks like the solutions of the angular part in 4d will be spherical harmonics to achieve adaption spherical harmonics of higher order must be used. (This is different to the procedure given in section 4. There external particles in the modified isotropic Klein-Gordon equation are considered, here the metric coefficients are modified.) The metric coefficients necessary to adapt an interpretable 4d Lagrangian then are more differentiated what gives more detailed information about the gravitational center.

Choosing eigenfunctions of higher order of the 6d equation of motion also does not change the symmetry of the spacetime considered and therefore the structure of the 4d metric.

The entire equation of motion given in equation (37) is equivalent to Einstein's equation with an energy-momentum tensor (at least in the case of spherical symmetry). This can be shown implementing the metric of the internal Schwarzschild solution (metric coefficients called g_{11} and g_{22}) in a Lagrangian of the type used in UR. The two originally different metric coefficients by multiplication with the factor $\sqrt{-g_{11}g_{22}}$ coming from the Jacobian will become equal as in the external Schwarzschild solution. The other parts will be multiplied with the factor. This is just equation (37).

This procedure could also be helpful in solving the double star problem.

7 Some conclusions

7.1 Absence of feedback

The Schrödinger equation shows that the mass of the (satellite) particle does not influence the gravitational field of the central particle. This behavior known also of two electrically charged particles where each particle sees only the uninfluenced field of the other is classically hard to explain. UR offers an explanation: The gravitational field and the particle together are the 4d realization of a 6d symmetry and cannot disturb the deducted symmetry. That not the combined field is acting is not caused by any smallness of the (satellite) particle ("test particle") but is based on the derivation of force and particle.

7.2 Principle of equivalence

Basic to GR is the equivalence principle i.e. the postulate of equality of inertial and gravitational mass. For UR this equality must not be postulated but is a result. It is based on the use of the mass term $a = \frac{m_0^2 c^2}{\hbar^2}$ of equation (23) to characterize the satellite particle. It is its only mass. As A and f do not depend on the mass of the (satellite) particle this unique mass enters the Schrödinger equations (64) or (65) in accord to the definition of inertial and gravitational mass in the terms of kinetic and potential energy.

7.3 Classical mechanics

Newton's second law of mechanics "force equals mass times acceleration" has the problem that the quantities "force" and "mass" are not defined independently but both together by this law. Introducing a gravitational force enabled him to calculate the orbits of the solar satellites. Inverting Newton's argumentation and using gravity as deducted from 6d symmetry to define an entity "force" solves the problem in defining the second law. The Hamilton function describing satellite orbits then can be used to deduct the whole of Newton's mechanics.

8 Macroscopic effects of quantum gravitation

In section 4.1.1 it is shown that the Schrödinger equation of gravitation describes the behavior of elementary particles in a gravitational field. But from the quantum character of gravity also macroscopic effects result. To find a starting point to examine how gravitation can describe these macroscopic effects it is necessary to analyze the meaning of "particle" in quantum mechanics.

There are two types of wave functions that come close to particles.

1. A vivid approach is given for a Schrödinger equation with a parabolic potential. The wave function is bell shaped. This is the only situation in which in Heisenberg's uncertainty principle the equal sign holds. The wave function is maximally concentrated so that it could be understood as something like a particle, a quasi-particle, oscillating around the minimum of the parable.
2. A more sophisticated approach are quasi-particles as they occur in Rydberg atoms. It is based on the correspondence principle formulated by Bohr at the very beginning of quantum physics: For effects with an action in the order of magnitude

of Planck's quantum of action quantum mechanics i.e. the wave function gives the right answer, for much larger action classical mechanics with its trajectories of particles.[23]

In the transition area a mixed form between wave function and particle should exist. Large action usually means high quantum numbers. So highly excited quantum systems should show elements of convergence to classical behavior.

And indeed in actual experiments it is demonstrated that this transient area between the two approaches exists. The behavior of highly excited quantum systems approaches that of classical systems and the behavior of its wave function approximates that of a particle.

This can be shown with Rydberg atoms, atoms in which one or more electrons are highly excited but still bound. The behavior of such an electron shows many aspects of a localized classical electrically charged particle orbiting the nucleus.[24] Position and momentum can be measured simultaneously with sufficient exactness.

"Particles" of this type ("Rydberg-particles") are another link between the particles of GR and the wave functions of UR. On the one hand the Schrödinger equation describes their behavior and on the other hand it can be attributed to a classical particle.

Both types are regarded in the following.

8.1 Planet orbits II

Planets are moving on ellipses. This can be shown with quasi-particles of the first type by examining the radial part of the Schrödinger equation nearby the minima of its effective potential, the positions where classical particles move.

8.1.1 Eccentricity and perihelion rotation

Considering at first the simplified effective potential given in equation (72) we find minima for

$$r_{min_l} = r_B l\,(l+1)\,. \tag{82}$$

A series expansion around a minimum V_{min_l} gives in second order

$$\bar{V}_{eff_l}\,(r) = \frac{1}{2} m_0 \omega_{C_l}^2 \bar{r}_l^2 \tag{83}$$

with

$$\bar{V}_{eff_l}\,(r) = V_{eff_l}\,(r) - V_{min_l},\ \bar{r}_l = (r - r_{min_l}) \text{ and } \omega_{C_l} = \frac{\hbar}{r_B^2 m_0}\sqrt{\frac{1}{l^3\,(l+1)^3}}\,.$$

It is known that the solution of a Schrödinger equation with this parabolic potential can be understood as describing the oscillation of a quasi-particle with frequency ω_{C_l} . Introducing the oscillation period by $T_{C_l} = \frac{2\pi}{\omega_{C_l}}$ we find

$$\frac{T_{C_l}^2}{r_{min_l}^3} = \frac{4\pi^2}{GM}\,. \tag{84}$$

This period is identical with the known orbital period of a planet with distance $r_{min\,l}$ from the sun given by Kepler's third law. As oscillation period and orbital period are equal closed orbits result.

Combining the circular orbit and the radial oscillation in first order of the oscillation amplitude it results according to Kepler's first law an ellipse with the sun being placed in one of the focal points and the eccentricity being defined by the oscillation amplitude.

As the effective potential is not affected by the oscillation eccentricity depends only on the mass of the planet and not on the mass of the central star. Because the effect of distortions is greater on a small object this could explain why eccentricity in cosmic dimensions is a fast changing parameter.

Oscillation period and orbital period are identical only for a Newtonian style gravitational force. If there occur additional terms in the effective potential not only the minima will shift but also the shape of the minima and hence the width of the parable and the frequency of oscillation. By this the accord between orbital and oscillation period gets lost, the ellipses no longer close and rosettes are produced. As the aberration in our solar system proceeds very slowly it can be interpreted as a rotation of the apsides of the planet orbits.

Starting with the effective potential of the full Schrödinger equation given in equation (74) and calculating like above the oscillation period ω_{R_l} (the index R means "relativistic") we get

$$\frac{\omega_{R_l}^2}{\omega_{C_l}^2} = 1 + \frac{3r_S}{r_B l\,(l+1)}\ . \tag{85}$$

By this nonsynchronous oscillation the perihelion angle advances during each orbit of the planet by an angle

$$\Delta\phi = \frac{3r_S\pi}{r_B l\,(l+1)} = \frac{3r_S\pi}{r_{min_l}}\ . \tag{86}$$

A result that coincides with that of GR.

8.1.2 Disk structure

A similar approach as in the last section when the effective potential was used to calculate the radial behavior can be employed on the polar angle.

The part depending on the polar angle in the Schrödinger equation given in equation (64) can be written as

$$\frac{\partial}{\partial\mu}\left(\left(1-\mu^2\right)\frac{\partial P}{\partial\mu}\right) + \left(l\,(l+1) - \frac{m^2}{(1-\mu^2)}\right)P = 0\ . \tag{87}$$

The second term can be understood as an effective potential. Its minimum is $\mu = \cos(\theta) = 0$ i.e. $\theta = \frac{\pi}{2}$ and $m = 0$. This means that for all l the planets orbit in a plane going through the center of the central body.

8.1.3 Structure of a solar system

Using Rydberg-particles means mixing features of particle- and gravity-physics. This approach is new, so new results can be expected.

In section 4.1 it was found that Laguerre functions are a good approximation of the solution of the Schrödinger equation for bound particles. Examining their behavior in highly exited states i.e. for large main quantum numbers n should give this information.

With increasing n the Laguerre polynomials contain increasing powers of r. The exponential damping factor nevertheless allows the Laguerre functions to be normalized. The damping effect however occurs for larger and larger r. The limit is reached for the asymptotic approximation $R_{n,l_{as}}(r)$

$$R_{n,l_{as}}(r) = (-1)^l\,\frac{n^{l+1}}{\sqrt{\pi}}\left(\frac{r_B}{2r}\right)^{\frac{3}{4}}\cos\left(2\sqrt{\frac{2r}{r_B}} - \frac{3\pi}{4}\right) \tag{88}$$

when the polynoms are able to neutralize the damping factor.[25]

The probability $dW(r)$ to find a particle in a spherical shell with radius r then is given by

$$\begin{aligned} dW(r) &= 4\pi|R_{n,l_{as}}(r)|^2 r^2 dr \\ &= r_B^2 n^{2(l+1)}\sqrt{\frac{2r}{r_B}}\cos^2\left(2\sqrt{\frac{2r}{r_B}} - \frac{3\pi}{4}\right)dr. \end{aligned} \tag{89}$$

This probability is divergent what means that the asymptotic wave function describes no quantum physical state.

As the asymptotic wave function is the limit of convergent wave functions it can be assumed that for large but limited n wave functions exist that in the area of interest are

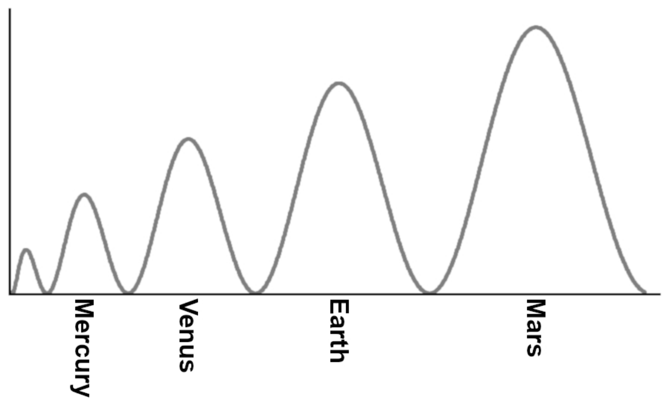

Figure 1: Matter density in the surrounding of a central star depending on distance from center

described by the asymptotic wave function only marginally influenced by the damping factor. Outside of this area the damping factor acts stronger and stronger and ensures convergence.

The factor $r_B^2 n^{2(l+1)}$ then can be implemented in the normalization constant.

This modified asymptotic approximation shows that single quantum numbers in the transition area are no longer appropriate to characterize a state. The wave function is the same for all n and l. The emerging particles are to be understood as the result of a superposition of many wave functions.

Table 1: Correlation between planet radii and maxima of radial mass density

Planets	Maxima of matter density
Mercury	2^{nd} maximum
Venus	3^{rd} maximum
Earth	4^{th} maximum
Mars	5^{th} maximum
Jupiter	10^{th} maximum
Saturn	13^{th} maximum
Uranus	19^{th} maximum
Neptune	19^{th} maximum

The graph of $dW(r)/dr$ of equation (89) as shown in figure 1 demonstrates in the surrounding of a gravitational center for the probability density a sequence of areas with slowly increasing maxima separated by areas with vanishing probability. Introducing a running index $\nu = 1, 2 \ldots$ counting the radii of maximal density r_{max_ν} starting at the center the distance between two maxima increases roughly with the square of ν.

Summing over many particles probability density can be understood as matter density. So the graph means that in the surrounding of a central star by quantum physical reasons there are areas where matter cannot stay permanently and areas where matter accumulates. The maxima therefore are the areas where planets could evolve. The minima can be seen in the gaps of ring systems as e.g. that of Saturn.
As mentioned in section 2 introducing fictive particles the mass of these entities is undefined and must be defined by experiment. This allows using the mass dependent parameter r_B here as an adaption parameter.

Correlating the (numerically calculated) positions of the maxima of the matter density with the positions of the planets Mercury till Neptune for $r_B = 5.83 * 10^9$ m as shown in table 1 for the inner planets Mercury till Mars generates perfect coincidence in sequence and radius. Also the outer planets can be aligned but there are many unoccupied maxima so that it is not quite clear, why a planet is just in a specific maximum.

The forbidden areas can be used to argue why the peripheral parts of the cloud our solar system once arose out of did not vanish in space but could built up a structure.

The exponential damping factor allowing the wave function to be normalized can help to argue why the trans-Jupiter planets become smaller.

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