Energy-momentum relation in curved space-time

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Abstract

In this paper we will obtain a variation of the energy-momentum relation:

$$E^2 = m^2c^4 + p^2c^2$$

With an element more that depends on the Ricci scalar curvature:

$$E^2 = m^2c^4 + p^2c^2 - R\hbar^2c^2$$

This leads to a variation in the definition the energy of a mass at rest:

$$E = mc^2\sqrt{1 - \frac{R\hbar^2}{m^2c^2}}$$

In small gravitational fields, the term $\frac{R\hbar^2}{m^2c^2}$ is so small that the equation is indistinguishable from the classical equation:

$$E = mc^2$$

But it can have its importance in high gravitational fields as a limit to the curvature is imposed:

$$R < \frac{m^2c^2}{\hbar^2} + \frac{p^2}{\hbar^2}$$

Also, we will get the variation Klein-Gordon equation in flat space-time:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2} - R\right)\psi(t, \boldsymbol{x}) = 0$$

And in curved space time:

$$\frac{-1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\psi\right) + \left(\frac{m^2c^2}{\hbar^2} - R\right)\psi = 0$$

The same way, we will obtain a variation to the Dirac equation, both in flat and curved space time:

$$i\gamma^\mu\partial_\mu\psi = \sqrt{\frac{m^2c^2}{\hbar^2} - R}\psi$$

$$i\gamma^{\mu}D_{\mu}\psi = \sqrt{\frac{m^2c^2}{\hbar^2} - R\psi}$$

Keywords

Geometric Algebra, Energy-momentum relation, Klein-Gordon Equation, Dirac equation, Ricci scalar curvature, non-Euclidean metric

1. Introduction

In this paper will use the previous paper [73] to explain the consequences of the equations obtained in that paper.

2. Energy-momentum relation

In the paper [73] the following equation was obtained using geometric algebra to merge the Klein-Gordon equation with the Einstein gravitational equation:

$$E_{particle} = mc^2 - \frac{\hbar^2}{m}R$$

Being R the Ricci scalar of the gravitational field where the particle lies in. The idea of this paper is to show all the possible consequences that this equation has.

In [73] we obtained the following factor in the Klein-Gordon equation of the part of the Energy of the particle (being the Ricci scalar curvature):

$$\frac{m}{\hbar^2}\left(mc^2 - \frac{\hbar^2}{m}R\right)$$

This is:

$$\frac{m^2c^2}{\hbar^2} - R$$

We see that the mass is squared, typical for equations for squared energy as in the squared Energy equation by Einstein [75] (and also in Klein-Gordon equation[67]). So, we keep this mass squared, but we can multiply by constants (changing the units but not the meaning of the elements). Multiplying now by $\hbar^2c^2$ to get units of energy squared in both elements.

$$m^2c^4 - R\hbar^2c^2$$

So, this is the element to be introduced in the squared equation of energy (energy-momentum relation) by Einstein [75]:

$$E^2 = m^2c^4 + p^2c^2$$

Leading to:

$$E^2 = m^2c^4 - R\hbar^2c^2 + p^2c^2$$

$$E^2 = m^2c^4 + p^2c^2 - R\hbar^2c^2$$

Being this squared energy equation including also the gravitational effects in the Energy (as one element is the Ricci scalar representing the effects of the gravitational field).

As we saw in [73] the element depending in the Ricci scalar is in general several orders of magnitude smaller than the one depending on the mass. Making it neglectable in general except in high gravitational fields as we will comment later.

If we consider a mass at rest $(p = 0)$ and in a gravitational field, we will have:

$$E^2 = m^2c^4 - R\hbar^2c^2$$

Taking the square root:

$$E = \sqrt{m^2c^4 - R\hbar^2c^2}$$

$$E = mc^2\sqrt{1 - \frac{R\hbar^2c^2}{m^2c^4}} = mc^2\sqrt{1 - \frac{R\hbar^2}{m^2c^2}}$$

Considering a small gravitational field, we can approximate with two elements of the Taylor series as:

$$E = mc^2\sqrt{1 - \frac{R\hbar^2}{m^2c^2}} \cong mc^2\left(1 - \frac{1}{2}\frac{R\hbar^2}{m^2c^2}\right) \cong mc^2 - \frac{1}{2}\frac{\hbar^2}{m}R$$

In (73) I did not consider this equation as quadratic and that is the reason, I calculated there wrongly this equation (without the square root or the ½ factor).

We can see that the energy of the particle is reduced by an element that depends on the Ricci scalar and inversely to the mass. Making the total energy of a particle smaller, as the gravitational field where it stands, is smaller.

This reduction is in general negligible [73], being several orders of magnitude below the normal energy. Anyhow, as the mass increases, the Ricci scalar increases also due to gravitational effects. As the Ricci scalar is being subtracted to the energy depending on the mass, the system will arrive to a balance before becoming a singularity.

We can see this, returning to the former squared energy equation:

$$E^2 = m^2c^4 + p^2c^2 - R\hbar^2c^2$$

$$E = \sqrt{m^2c^4 + p^2c^2 - R\hbar^2c^2}$$

If we consider that energy cannot be imaginary, then:

$$m^2c^4 + p^2c^2 - R\hbar^2c^2 > 0$$

$$m^2c^4 + p^2c^2 > R\hbar^2c^2$$

$$\frac{m^2c^2}{\hbar^2} + \frac{p^2}{\hbar^2} > R$$

$$R < \frac{m^2c^2}{\hbar^2} + \frac{p^2}{\hbar^2}$$

We see that the curvature created by the gravitational field has a limit that depends on the mass and momentum of the masses creating is. This is, the probabilities to arrive to a singularity clearly reduces. The higher the mass and momentum, the higher the curvature of the filed, is clear. But also, the higher the curvature the “power” of the energy and the momentum to increase the gravitational field starts reducing.

This means, in the end we arrive to a limit, that is the above equation. The curvature that can be created is limited by the mass and momentum that ironically have originally created it.

In fact, going even further talking about the nature of the elemental particles. If we consider the particle as an area of space that arrives to an equilibrium, we could consider that the masses are the discrete values (a kind of eigenvalues) where this equilibrium between the energy of the particle and the gravitational field created (represented by the Ricci scalar) arrives to a balance.

This could be one of the reasons that there are three generations or families of masses per type of particle [76]. Because, as we saw in [73] in this context, the three special dimensions are the ones to be taken into account, and therefore the tensors and matrices have indices going from 1 to 3 (three row or columns in the case of matrices with 3 eigenvalues). The reason of not considering time as a 4^{th} dimension, is because time corresponds to one of the 8 degrees of freedom created by these spatial dimensions (considered normally the trivector created by the three spatial dimensions or sometimes the scalars depending on the situation see Annex A1, [5][47] [63] [73].

3. Klein-Gordon equation

In our Energy-momentum relation equation:

$$E^2 = m^2c^4 + p^2c^2 - R\hbar^2c^2$$

Let's divide by $\hbar^2 c^2$:

$$\frac{E^2}{\hbar^2 c^2} = \frac{m^2c^2}{\hbar^2} + \frac{p^2}{\hbar^2} - R$$

The Klein-Gordon equation in Euclidean metric according to literature is [67]:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\psi(t, \boldsymbol{x}) = 0$$

The energy and the momentum are represented by the derivatives through tome and space respectively. And the mass element has its own element. We see that what is missing is the element regarding the Ricci Scalar. With the units that the equation is in now (L^{-2}) can include it directly in the equation (the units of R are also L^{-2}). See two equations above to check that.

So, the result is:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2} - R\right)\psi(t, \boldsymbol{x}) = 0$$

If you have seen my papers, you will know, that this equation is squared so it should be applied to the wavefunction collapsed (squared) not to the single function (but in standard algebra to do this is not possible). It is possible to do it in geometric algebra but this is another story that you can check in Annex A1.

According to literature the Klein Gordon equation [67] in curved spacetime is:

$$\frac{-1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\,\partial_\nu\psi\right) + \frac{m^2c^2}{\hbar^2}\psi = 0$$

Adding the new element:

$$\frac{-1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\psi\right)+\frac{m^2c^2}{\hbar^2}\psi-R\psi=0$$

$$\frac{-1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\psi\right)+\left(\frac{m^2c^2}{\hbar^2}-R\right)\psi=0$$

In geometric Algebra the one obtained is [73]:

$$e^\beta\nabla_\beta(\nabla_\alpha(\psi^\dagger\psi)e^\alpha)=\frac{m}{\hbar^2}\left(mc^2-\frac{\hbar^2}{m}R\right)\psi^\dagger\psi$$

$$e^\beta\nabla_\beta(\nabla_\alpha(\psi^\dagger\psi)e^\alpha)=\left(\frac{m^2c^2}{\hbar^2}-R\right)\psi^\dagger\psi$$

Where, as commented, the equation is applied to the wavefunction squared and is symmetric from left to right. Making it completely symmetric and taking the license of considering that ∇_α applies to the left (in [73] you can see a nomenclature for this operation, but here just consider that applies to the left):

$$e^\beta\nabla_\beta\left((\psi^\dagger\psi)\nabla_\alpha e^\alpha\right)=\sqrt{\frac{m^2c^2}{\hbar^2}-R}\psi^\dagger\psi\sqrt{\frac{m^2c^2}{\hbar^2}-R}$$

We can get the Dirac equation in geometric algebra:

$$e^\beta\nabla_\beta\psi^\dagger=\sqrt{\frac{m^2c^2}{\hbar^2}-R}\psi^\dagger$$

And:

$$(\psi\nabla_\alpha)e^\alpha=\psi\sqrt{\frac{m^2c^2}{\hbar^2}-R}$$

Or putting again ∇_α in the normal order and applying to the right again:

$$(\nabla_\alpha\psi)e^\alpha=\sqrt{\frac{m^2c^2}{\hbar^2}-R}\psi$$

We can see that they are the same equation but reversed and applying to a function that is reversed compared to the equation in ∇_β. You can see the meaning of reverse and geometric algebra in [73].

One thing we can see here is that the square root, this time does not include the momentum (the momentum is in the operators in the left-hand side). So probably the restriction to R would be even higher, and be:

$$\frac{m^2c^2}{\hbar^2}-R>0$$

$$R<\frac{m^2c^2}{\hbar^2}$$

Where all the considerations regarding the previous chapter (reducing the probabilities of singularities and that the masses of particles could be eigenvalues to an equilibrium limit between created gravitational field created and reduction of mass due to it) keeps being the same. Except that this equation is more restricting and does not include the momentum of the particle just the mass.

4. The Dirac equation

The Dirac equation in flat spacetime according to literature [13] is:

$$\left(i\hbar\gamma^{\mu}\partial_{\mu} - mc\right)\psi(x) = 0$$

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi - mc\psi = 0$$

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi = mc\psi$$

$$i\gamma^{\mu}\partial_{\mu}\psi = \frac{mc}{\hbar}\psi$$

As we know that the R has to be subtracted to the square of the element, we can do the following:

$$i\gamma^{\mu}\partial_{\mu}\psi = \sqrt{\frac{m^2c^2}{\hbar^2}}\psi$$

And now we can adapt it, just subtracting that element regarding R to the equation:

$$i\gamma^{\mu}\partial_{\mu}\psi = \sqrt{\frac{m^2c^2}{\hbar^2} - R\psi}$$

We can check in (74) that in curved space time, the equation in the literature is similar but applying covariant derivative instead of the derivative. So, we can follow, the same exact steps to obtain the equation:

$$i\hbar\gamma^{\mu}D_{\mu}\psi - mc\psi = 0$$

$$i\hbar\gamma^{\mu}D_{\mu}\psi = mc\psi$$

$$i\gamma^{\mu}D_{\mu}\psi = \frac{mc}{\hbar}\psi$$

$$i\gamma^{\mu}D_{\mu}\psi = \sqrt{\frac{m^2c^2}{\hbar^2}}\psi$$

Now we subtract the element R to the equation:

$$i\gamma^{\mu}D_{\mu}\psi = \sqrt{\frac{m^2c^2}{\hbar^2} - R\psi}$$

In the previous chapter we had already obtained the Dirac equation in geometric algebra as:

$$e^{\beta}\nabla_{\beta}\psi^{\dagger} = \sqrt{\frac{m^2c^2}{\hbar^2} - R\psi^{\dagger}}$$

$$(\nabla_{\alpha}\psi)e^{\alpha} = \sqrt{\frac{m^2c^2}{\hbar^2} - R\psi}$$

One applying to the reverse of the wavefunction [73] and the other to the wavefunction but giving both the same results.

5. Conclusions

In this paper we have obtained a variation of the energy-momentum relation:

$$E^2 = m^2c^4 + p^2c^2$$

With an element more that depends on the Ricci scalar curvature:

$$E^2 = m^2c^4 + p^2c^2 - R\hbar^2c^2$$

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Also, we have got the variation Klein-Gordon equation in flat space-time:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2} - R\right)\psi(t, \boldsymbol{x}) = 0$$

And in curved space time:

$$\frac{-1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\psi\right) + \left(\frac{m^2c^2}{\hbar^2} - R\right)\psi = 0$$

The same way, we have obtained a variation to the Dirac equation, both in flat and curved space time:

$$i\gamma^\mu\partial_\mu\psi = \sqrt{\frac{m^2c^2}{\hbar^2} - R}\psi$$

$$i\gamma^\mu D_\mu\psi = \sqrt{\frac{m^2c^2}{\hbar^2} - R}\psi$$

Bilbao, 29th December 2023 (viXra-v1).

6. Acknowledgements

To my family and friends. To Paco Menéndez and Juan Delcán. To Cine3d [32], To the current gods of Geometric Algebra: David Hestenes, Chris Doran, Anthony Lasenby, Garret Sobczyk, Joy Christian and many others. To A. Garret Lisi and J.O Weatherall.

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If you consider this helpful, do not hesitate to drop your BTC here:

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A1. Annex A1. Comments regarding single and squared wavefunctions

In chapter 3 we have taken this equation from literature:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\psi(t,\boldsymbol{x}) = 0$$

You can see that is a squared equation applying to a single wavefunction. In standard algebra you cannot apply it to the wavefunction squared as the wavefunction squared is normally a probability (a number) or maximum a vector related to probability current. In geometric algebra you can do it like that because the product of a multivector of 8 components (this is the wavefunction by its reverse will give another multivector). So, you can work with the original wavefunction of with its squared with no issues and being completely coherent.

This is the reason in geometric algebra, the Klein-Gordon equation can be applied to the square of the wavefunction with no issues (∇_α applies to the left in this case).

$$e^\beta \nabla_\beta \left((\psi^\dagger\psi)\nabla_\alpha e^\alpha\right) = \sqrt{\frac{m^2c^2}{\hbar^2} - R}\,\psi^\dagger\psi\,\sqrt{\frac{m^2c^2}{\hbar^2} - R}$$

And as commented, it is almost immediate to obtain Dirac equation:

$$e^\beta \nabla_\beta \psi^\dagger = \sqrt{\frac{m^2c^2}{\hbar^2} - R}\,\psi^\dagger$$

$$(\nabla_\alpha \psi)e^\alpha = \sqrt{\frac{m^2c^2}{\hbar^2} - R}\,\psi$$

It is like there are two different worlds regarding the reality. One is the square of the wavefunction (the collapse of the wavefunction), that represents the events that the observers agree that have happened. What we normally call reality or macroscopic reality.

And the other one is the square root of the reality (or the single wavefunction not squared) where what we normally call "Quantum mechanics effects" apply. And it is a world where everything can happen, but it cannot be observed until, this world (or wavefunction) is squared creating what we call "real events" that can be confirmed by observers to have occurred in the "squared world" or reality.

Probably the issue that all elements, instruments… we use (including ourselves) belong to the squared world makes us call it reality, because is the world where we can interact, observe and modify. The square root of the reality (the original wavefunction not squared) is something that we consider have "strange" effects just because of the fact that it is in a world that we cannot control.

And for me it is important that anytime we apply an equation to the wavefunction, we know if it has to be applied to the squared wavefunction (as it should be in the Klein-Gordon equation) or to the non-squared wavefunction (as in the Dirac equation).

Coming back to this, in most of my papers I have considered the time to be the trivector in geometric algebra. Whereas our APS [43] friends consider it to be the scalars. I guess both are right. In the non-squared world probably the trivector and (also the scalars?) could represent what we call the dimension of time. In the squared world (the reality), the trivector has been squared to a negative scalar (and probably from that moment on the time could be represented by negative scalars as APS [43] friends say). But if due to asymmetries when you square a wavefunction, you still have a coefficient for the trivector different from zero, it could be still representing the time.

A2. Annex A2. The electromagnetic trivector

I do not want to miss this opportunity to talk about the electromagnetic trivector. This trivector affects the particles but not changing its “average” speed or direction (as the electric and magnetic moments do).

It has two effects:

- One is changing the orientation of the particle (affecting the bivector planes that represent its orientation).
- It also affects the direction and speed but in an “oscillatory” way. This means it can create momentary changes in the direction and speed in that go in the opposite direction moments later. So it does not affect the “average” trajectory or speed.

This makes the trivector almost invisible, as the effects it has, or either they cannot be measured directly (as orientation of the particle) or are considered noise or uncertainties (for the oscillatory movement). In fact, ipse David Hestenes [77] considers that the zitterbewegung (oscillatory movement) of the electron is related to geometric algebra considerations (being the trivector one of the mostly iconic elements of geometric algebra by definition).

But the issue is that the presence of an electromagnetic trivector could explain effects as the quantum entanglement or certain uncertainties in quantum mechanics.

Regarding the electromagnetic trivector you can find more info in [6][26][47].