


An alternative approach in General Relativity describing Gravitational RedShift, Black Holes without Singularities and Dark Matter.

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Abstract

Differently than in General Relativity, the Interaction between Gravity and Light [35] fundamentally has been based on the sum of the “Stress Energy Tensor” and the introduced “Gravitational Tensor”.

The theory describes “Gravitational-Electromagnetic Interaction” resulting in a mathematical Tensor presentation for BLACK HOLES. (Gravitational Electromagnetic Confinements) [1] The “Electromagnetic Energy Gradient” creates a second order effect “Lorentz Transformation” which results in the Gravitational Field of BLACK HOLES which determines the interaction force density between the confinement of Light (BLACK HOLE’s) and the Gravitational Field.

Einstein approached the interaction between gravity and light by the introduction of the “Einstein Gravitational Constant” in the 4-dimensional Energy-Stress Tensor $\kappa T_{\mu\nu}$ (1).

In this alternative approach related to General Relativity, the interaction between gravity and light has been presented by the sum of the Electromagnetic Tensor $T_{\mu\nu}$ and the Gravitational Tensor $J_{\mu\nu}$ (2) .

The new approach presents mathematical solutions for the BLACK HOLES (Gravitational Electromagnetic Interaction) introduced in 1955 by Jonh Archibald Wheeler in the publication in Physical Review Letters in 1955 [1]. The mathematical solutions for BLACK HOLES are fundamental solutions for the relativistic quantum mechanical Dirac equation (Quantum Physics) in Tensor presentation (35). Assuming a constant speed of light “c” and Planck’s constant \hbar within the BLACK HOLE, the radius “R” of the BLACK HOLE with the energy of a proton, is about 1% of the radius of the hydrogen atom (14).

The New Theory has been tested in an experiment with 2 Galileo Satellites and a Ground Station by measuring the Gravitational RedShift in an by the Ground Station emitted stable MASER frequency [2]. The difference between the calculation for

Gravitational RedShift, within the Gravitational Field of the Earth, in “General Relativity” and the “New Theory” is smaller than 10^{-16} (12) and (13).

In all “General Redshift Experiments” General Relativity and the New Theory predict a Gravitational RedShift with a difference smaller than 15 digits beyond the decimal point which is beyond the accuracy of modern “Gravitational Redshift” observations. Both values are always within the measured Gravitational RedShift in all observations being published since the first observation of the gravitational redshift in the spectral lines from the White Dwarf which was the measurement of the shift of the star Sirius B, the white dwarf companion to the star Sirius, by W.S. Adams in 1925 at Mt. Wilson Observatory.

Theories which unify Quantum Physics and General Relativity [32], like “String Theory”, predict the non-constancy of natural constants. Accurate observations of the NASA Messenger [11] observe in time a value for the gravitational constant “G” which constrains until $(|G|/G \text{ to be } < 4 \times 10^{-14} \text{ per year})$. One of the characteristics of the New Theory is the “Constant Value” in time for the Gravitational Constant “G” in unifying General Relativity and Quantum Physics.

Keywords

Classical Field Theory, Gravitation and Electromagnetism, General Relativity

1. An alternative approach in Gravity

Einstein approached the interaction between gravity and light by the introduction of the “Einstein Gravitational Constant” in the 4-dimensional Energy-Stress Tensor.

$$\mathbf{G}_{\mu\nu} + \Lambda \mathbf{g}_{\mu\nu} = \kappa \mathbf{T}_{\mu\nu} \quad (1)$$

In which $\mathbf{G}_{\mu\nu}$ equals the Einstein Tensor, $\mathbf{g}_{\mu\nu}$ equals the Metric Tensor, $\mathbf{T}_{\mu\nu}$ equals the Stress-Energy tensor, Λ equals the cosmological constant and κ equals the Einstein gravitational constant.

An alternative approach to Einstein’s expression with the tensor $\kappa \mathbf{T}_{\mu\nu}$, describing the curvature of the Space-Time continuum, is the sum of the Electromagnetic Tensor $\mathbf{T}_{\mu\nu}$ and the Gravitational Tensor $\mathbf{J}_{\mu\nu}$.

$$\kappa \mathbf{T}_{\mu\nu} \Leftrightarrow \mathbf{T}_{\mu\nu} + \mathbf{J}_{\mu\nu} \quad (2)$$

The 4-dimensional divergence of the sum of the Electromagnetic Stress-Energy tensor and the Gravitational Tensor expresses the 4-dimensional Force-Density vector (expressed in $[\text{N}/\text{m}^3]$ in the 3 spatial coordinates) as the result of Electro-Magnetic-Gravitational interaction.

$$f^\mu = \partial_\nu (\mathbf{T}^{\mu\nu} + \mathbf{J}^{\mu\nu}) \quad (3)$$

In vector notation the 4-dimensional Force-Density vector can be written as:

$$\vec{f}^4 = \begin{pmatrix} f_4 \\ f_3 \\ f_2 \\ f_1 \end{pmatrix} = \square \cdot (\vec{\mathbf{T}} + \vec{\mathbf{J}}) \quad (4)$$

The fundamental boundary condition for this alternative approach to gravity is the requirement that the Force 4 vector equals zero in the 4 dimensions, expressing a universal 4-dimensional equilibrium:

$$\vec{f}^4 = \begin{pmatrix} f_4 \\ f_3 \\ f_2 \\ f_1 \end{pmatrix} = \square \cdot (\vec{\mathbf{T}} + \vec{\mathbf{J}}) = \vec{0}^4 \quad (5)$$

The 3 spatial components of the Force-Density vector, as a result of Electro-Magnetic-Gravitational interaction can be written as:

$$\begin{aligned} \bar{f} = & -\frac{1}{c^2} \frac{\partial (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\partial t} + \epsilon_0 \bar{\mathbf{E}} (\nabla \cdot \bar{\mathbf{E}}) - \epsilon_0 \bar{\mathbf{E}} \times (\nabla \times \bar{\mathbf{E}}) + \\ & + \mu_0 \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{H}}) - \mu_0 \bar{\mathbf{H}} \times (\nabla \times \bar{\mathbf{H}}) + \gamma_0 \bar{\mathbf{g}} (\nabla \cdot \bar{\mathbf{g}}) - \gamma_0 \bar{\mathbf{g}} \times (\nabla \times \bar{\mathbf{g}}) = \bar{\mathbf{0}} \quad [\text{N/m}^3] \end{aligned}$$

in which: $\epsilon_0 (\nabla \cdot \bar{\mathbf{E}}) = \rho_E$ Electric Charge Density [C/ m³]

$\mu_0 (\nabla \cdot \bar{\mathbf{H}}) = \rho_M$ Magnetic Flux Density [Vs/ m³] or [Wb/ m³]

$\gamma_0 (\nabla \cdot \bar{\mathbf{g}}) = \rho_M$ Mass Density (Electromagnetic) [kg/ m³]

Electric Energy Density: $w_E = \frac{1}{2} \epsilon_0 E^2$

Magnetic Energy Density: $w_M = \frac{1}{2} \mu_0 H^2$

Gravitational Energy Density: $w_G = \frac{1}{2} \gamma_0 g^2$

In which E represents the electric field intensity expressed in [V/m], H represents the magnetic field intensity expressed in [A/m] and g represents the gravitational acceleration expressed in [m/s²]. The permittivity indicated as ϵ_0 , the permeability indicated as μ_0 and the gravitational permeability of vacuum as γ_0 .

For curl-free gravitational fields equation (6) can be written as:

$$\begin{aligned} \bar{f} = & -\frac{1}{c^2} \frac{\partial (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\partial t} + \epsilon_0 \bar{\mathbf{E}} (\nabla \cdot \bar{\mathbf{E}}) - \epsilon_0 \bar{\mathbf{E}} \times (\nabla \times \bar{\mathbf{E}}) + \\ & + \mu_0 \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{H}}) - \mu_0 \bar{\mathbf{H}} \times (\nabla \times \bar{\mathbf{H}}) + \bar{\mathbf{g}} \rho_M = \bar{\mathbf{0}} \quad [\text{N/m}^3] \end{aligned} \tag{7}$$

Substituting Einstein's $W = m c^2$ in (7) results in "Electro-Magnetic-Gravitational Equilibrium Field Equation" (8):

$$\bar{f} = -\frac{1}{c^2} \frac{\partial (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\partial t} + \varepsilon_0 \bar{\mathbf{E}} (\nabla \cdot \bar{\mathbf{E}}) - \varepsilon_0 \bar{\mathbf{E}} \times (\nabla \times \bar{\mathbf{E}}) + \quad (8)$$

$$+ \mu_0 \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{H}}) - \mu_0 \bar{\mathbf{H}} \times (\nabla \times \bar{\mathbf{H}}) + \frac{1}{2c^2} \bar{g} (\varepsilon E^2 + \mu H^2) = \bar{0} \quad [\text{N/m}^3]$$

The theory describes “Electromagnetic-Gravitational Interaction”, “Magnetic-Gravitational Interaction” and “Electric-Gravitational Interaction”. In this new theory particles do not interact with fields. The interaction between an electric charged particle and an electric field is not the interaction between a particle and a field but it is the interaction between the electric field of the particle interacting with the other electric field. Every interaction is an interaction between fields. Electric Fields interact with Electric Fields , Magnetic Fields interact with Magnetic Fields and Gravitational Fields interact with Gravitational Fields.

2 “Gravitational RedShift/ BlueShift in “Light (EMR)” due to “Electromagnetic Gravitational Interaction”

To test the New Theory, the Gravitational-Redshift experiment: “Test of the Gravitational Redshift with Galileo Satellites in an Eccentric Orbit” by S. Hermann et al, has been chosen [2]. In this experiment a stable “MASER” frequency from a ground station has been emitted to 2 Galileo Satellites, measuring the frequency difference between the Ground Station and the Satellites. The frequency shift has been caused by the gravitational field of the Earth and 2 satellites has been chosen to compensate for the eccentricity of the Galileo Orbit.

Assuming a gravitational field $g[z]$ depending on the radial direction in cartesian coordinates between the ground station and the satellites:

$$\bar{g}[z] = \left\{ 0, 0, \frac{G M_{Earth}}{4 \pi z^2} \right\} \quad (9)$$

In which G ($G = 6.67428 \cdot 10^{-11} \text{ Nm}^2 / \text{kg}^2$) equals the Gravitational constant, M_{Earth} the mass of the earth and r the radial distance from the centre of the earth. The mathematical solution [5] of equation (8) for plane electromagnetic waves (expressed in cartesian $\{x,y,z\}$ coordinates) related to the Electric Field Intensity equals:

$$\bar{\mathbf{E}} = \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \\ \mathbf{E}_z \end{pmatrix} = \begin{pmatrix} e^{-\frac{GM_{Earth} \epsilon_0 \mu_0}{8 \pi z}} h \left[\omega_0 e^{-\frac{GM_{Earth} \epsilon_0 \mu_0}{4 \pi z}} (t - \sqrt{\epsilon \mu} z) \right] \\ 0 \\ 0 \end{pmatrix} \quad (10)$$

And the mathematical solution of (8) for the Magnetic Field Intensity equals:

$$\bar{\mathbf{H}} = \begin{pmatrix} \mathbf{H}_x \\ \mathbf{H}_y \\ \mathbf{H}_z \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{\epsilon_0 \mu_0}} e^{-\frac{GM_{Earth} \epsilon_0 \mu_0}{8 \pi z}} h \left[\omega_0 e^{-\frac{GM_{Earth} \epsilon_0 \mu_0}{4 \pi z}} (t - \sqrt{\epsilon \mu} z) \right] \\ 0 \end{pmatrix} \quad (11)$$

In which ω_0 equals the original frequency of the MASER radiation propagating in the direction of the gravitational field $g[z]$ of the Earth in the z -direction. The exponential term demonstrates the Gravitational Redshift when the MASER radiation propagates in the direction of the Gravitational Field of the earth. The propagation speed of the Electromagnetic Radiation remains constant (the speed of light). But the amplitude of the field intensity and the frequency of the field intensity diminishes exponentially.

Calculations in Mathematica [5] demonstrate a difference between the calculation with General Relativity and the calculation with the New Theory. Choosing for the ground station a distance to the centre of the earth $z_1 = 6,378,000$ [m] (Radius of the Earth) and for the average distance of the ESA satellites in a Galileo orbit $z_2 = 23,222,000$ [m] (distance from the ESA satellite to the centre of the Earth), calculated with Mathematica, the Gravitational RedShift according General Relativity equals:

$$\Delta \omega_{GR} = 0.00000000004011815497097883 \text{ [s}^{-1}\text{]} \quad (12)$$

Calculated with Mathematica, the Gravitational RedShift according the New Theory, which is a solution of equation (8) equals:

$$\Delta \omega_{GR} = 0.00000000004011824206173742 \text{ [s}^{-1}\text{]} \quad (13)$$

Both calculated values are within the Range of the measured gravitational RedShift by the average values of both ESA satellites in the Galileo orbit

$$\Delta \omega_{\text{Measured}} = 0.000000000040118 \pm 2.2 \cdot 10^{-15} \text{ [s}^{-1}\text{]} \quad (14)$$

In [2] a factor α has been defined which presents the measured deviation α between the predicted Gravitational RedShift by General Relativity and the Measured Gravitational RedShift.

$$\alpha = \Delta \omega_{\text{MEASURED}} - \Delta \omega_{\text{GR}} = (2.2 \pm 1.6) \times 10^{-5} \quad (15)$$

A comparable factor α can be used to determine which theory (General Relativity or the New Theory) has the nearest approach to the experimentally measured data. Highly accurate measuring experiments are required with an accuracy higher than 16 digits beyond the decimal point.

3 Black Holes

3.1 Black Holes without Singularities with dimensions smaller than the diameter of the Hydrogen Atom

A second fundamental solution for equation (8) describes a **Gravitational Electromagnetic Confinement (BLACK HOLE)** [1] within a radial gravitational field with acceleration \vec{g} (in radial direction). This solution represents a Black Hole, the confinement of light due to its own gravitational field, and has no singularities. This solution for equation (8) describes Black Holes, dependent of time and radius, presenting discrete spherical energy levels, within a radial gravitational field with acceleration \vec{g} (in radial direction) [14] has been represented in (12) and (13).

$$\begin{pmatrix} E_r \\ E_\theta \\ E_\phi \end{pmatrix} = \begin{pmatrix} 0 \\ f(r) \sin(kr) \sin(\omega t) \\ -f(r) \cos(kr) \cos(\omega t) \end{pmatrix} \quad \begin{pmatrix} H_r \\ H_\theta \\ H_\phi \end{pmatrix} = \sqrt{\frac{\epsilon}{\mu}} \begin{pmatrix} 0 \\ -f(r) \sin(kr) \cos(\omega t) \\ -f(r) \cos(kr) \sin(\omega t) \end{pmatrix} \quad \vec{g} = \begin{pmatrix} \frac{G_1}{4\pi r^2} \\ 0 \\ 0 \end{pmatrix} \quad (12)$$

$$w_{em} = \left(\frac{\mu_0}{2} (\vec{m} \cdot \vec{m}) + \frac{\epsilon_0}{2} (\vec{e} \cdot \vec{e}) \right) =$$

$$f(r)^2 \left((\sin(kr) \sin(\omega t))^2 + (\cos(kr) \cos(\omega t))^2 + \frac{\epsilon}{\mu} (\sin(kr) \cos(\omega t))^2 + (\cos(kr) \sin(\omega t))^2 \right)$$

In which the radial function $f(r)$ equals:

$$f[r] = K e^{-\frac{G M_{BH} \epsilon_0 \mu_0}{8\pi r}} \quad (13)$$

G represents the Gravitational constant and M represents the total confined electromagnetic mass of the BLACK HOLE. Equation (12) presents a Standing (Confined) Electromagnetic Field Configuration with a phase shift of 90 degrees between the electric field and the magnetic field with the corresponding Nodes and AntiNodes. [13]. The solution has been calculated according [Newton's Shell Theorem](#).

Assuming a constant speed of light “ c ” and Planck’s constant \hbar within the BLACK HOLE, the radius “ R ” (with $n = 1,2,3,4,\dots$) of the BLACK HOLE with the energy of a proton, according $W = m_{\text{proton}} c^2$, would be: $1.5009211 \times 10^{-10}$ [J].

$$R_{\text{GEON}} = n \lambda = n \left(\frac{c}{f} \right) = n \left(\frac{c}{W} \right) \hbar = 7.1865 \cdot 10^{-26} \left(\frac{n}{W} \right) \quad (14)$$

$$R_{\text{GEON}} = n \cdot 3.82 \cdot 10^{-12} \text{ [m]}$$

Black Holes are varying from atomic dimensions with dimensions of 10^{-27} [kg], Page 39 [33] until Black Holes with dimensions of 10^{40} [kg], Page 67 [34]. At these dimensions Black Holes turn into Dark Matter. The fundamental boundary condition for the confinement of Electromagnetic radiation (BLACK HOLES) is that the energy flow (Poynting vector) $\vec{S} = \vec{E} \times \vec{H}$ equals zero at the surface of the confinement. This is possible at every “90 degrees Phase Shift Surface” (Sphere) between the Electric Field and the Magnetic Field.

3.1 Black Holes with a Singular point and Large dimensions

Fig 1 represents a Black Hole with a mass of 10^{35} [kg] and a radius of about 25 [km] controlled by a different mathematical solution for equation (8).

$$f[r] = K e^{\left(\frac{G M_{BH} \epsilon_0 \mu_0}{8 \pi r} - \log[r] \right)} \quad [\text{J} / \text{m}^3] \quad (15)$$

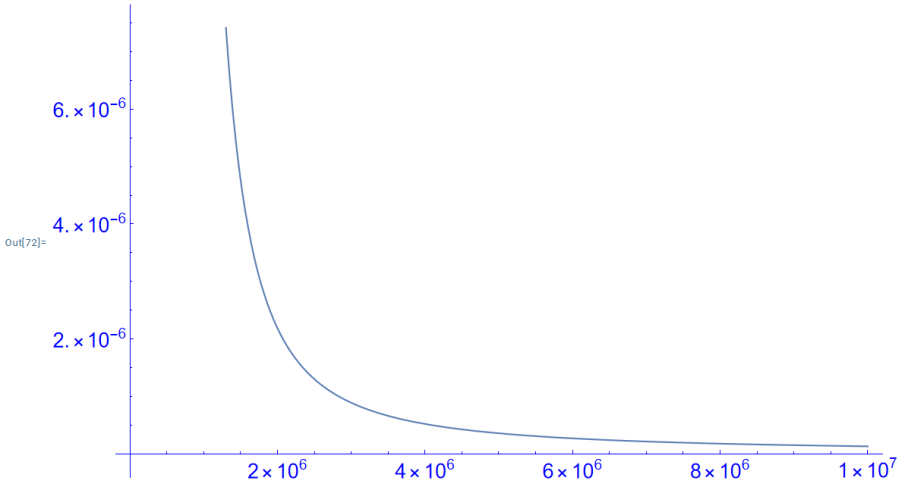


Fig. 1 The Energy Density [J/ m³] as a function of the Radius R = max 10⁷ [m] of the Black Hole.

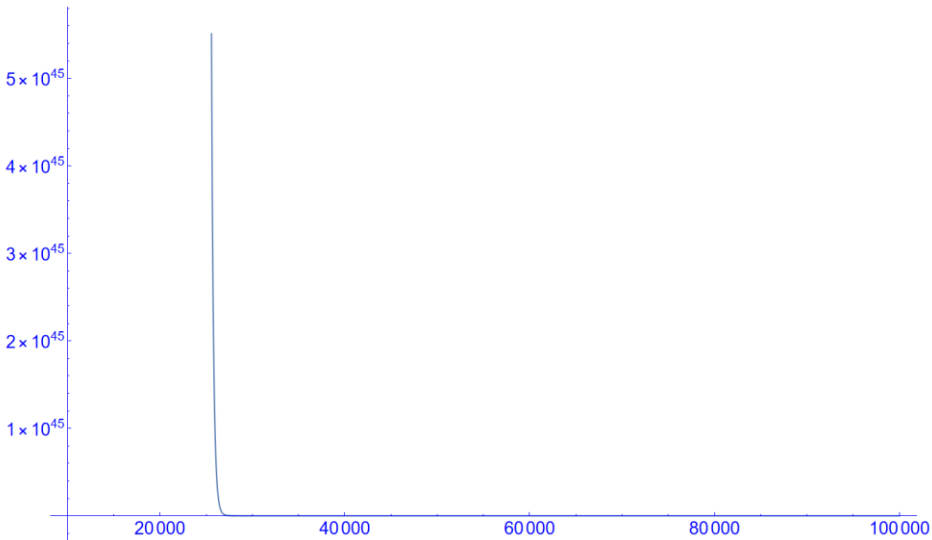


Fig. 2 The Energy Density [J/ m³] as a function of the Radius R = max 10⁵ [m]

Figure 1 and Figure 2 demonstrate the large effect of “Gravitational Intensity Shift” and “Gravitational RedShift” at the distance of 25 [km]. Over a distance of 10.000 [km] the intensity of the emitted light of the Black Hole with a mass of 10^{35} [kg] falls back with a factor of 10^{-51} . Also the frequency of the emitted light of the Black Hole falls back with a factor 10^{-51} . Emitted light in the visible spectrum of 10^{14} [Hz] falls back to a frequency of 10^{-37} [Hz]. These extreme low frequencies with extreme low intensities have never been measured which has result in the name “Black Hole” for the phenomenon of “Gravitational Intensity Shift” and “Gravitational RedShift” for a large mass. It follows from equation (8) and the solutions (10) and (11) that the speed of light does not change inside and around Black Hole. Only the direction of the propagation of light can change due to a gravitational field.

3.2 Dark Matter in the Universe

Fig 3 represents Dark Matter with a total mass of 10^{53} [kg] and a radius of about 10 times the size of the Milky Way Galaxy. The radius of the dark mass equals $5 \cdot 10^{21}$ [m] which has been controlled by a different mathematical solution (16) for equation (8).

$$f[r] = K e^{\left(\frac{G M_{\text{BH}} \varepsilon_0 \mu_0}{8 \pi r} - \log[r] \right)} \quad [\text{J} / \text{m}^3] \quad (16)$$

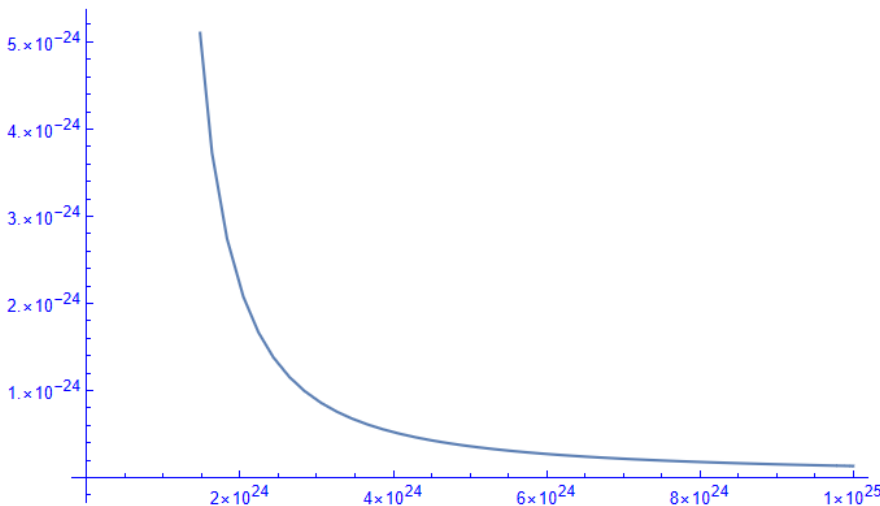


Fig. 3 The Energy Density [J/ m³] as a function of the Radius R = max 10²⁵ [m] of the Dark Matter.

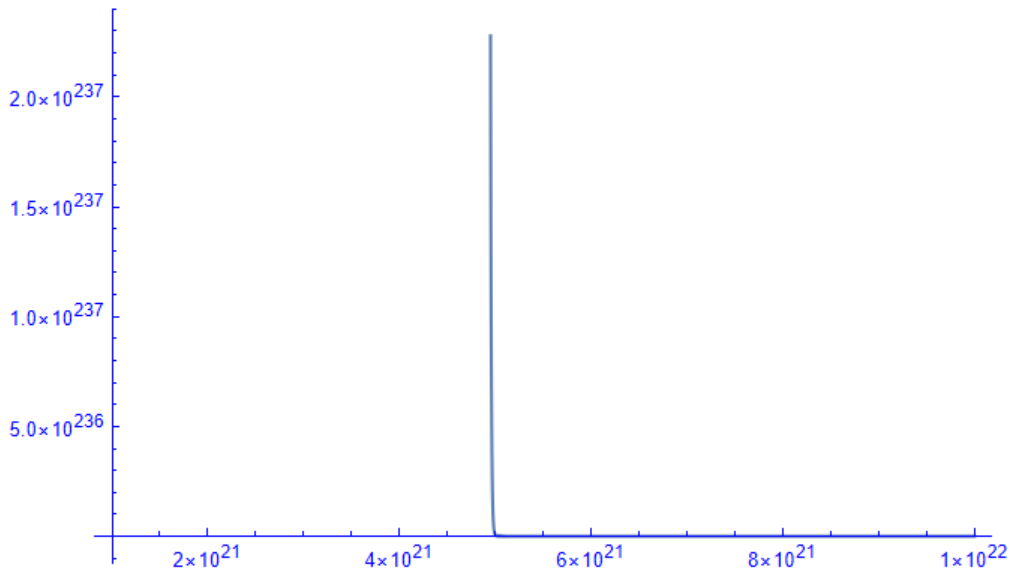


Fig. 4 The Energy Density [J/ m³] of the Dark Matter as a function of the Radius R = max 10²² [m]

Figure 3 and Figure 4 demonstrate the large effect of “Gravitational Intensity Shift” and “Gravitational RedShift” at the distance of $5 \cdot 10^{21}$ [m] which is 10 times the radius of the Milky Way Galaxy. Over the distance of $5 \cdot 10^{21}$ [m] the intensity of the emitted light of the Dark Matter with a mass of 10^{53} [kg] falls back with a factor of 10^{-261} . Also the frequency of the emitted light of the Black Hole falls back with a factor 10^{-261} . Emitted light in the visible spectrum of 10^{14} [Hz] falls back to a frequency of 10^{-247} [Hz]. These extreme low frequencies with extreme low intensities have never been measured which has result in the name “Dark Matter” for the phenomenon of “Gravitational Intensity Shift” and “Gravitational RedShift” for an extreme large mass. It follows from equation (8) and the solutions (10) and (11) that the speed of light does not change inside and around the Dark Mass. Only the direction of the propagation of light can change due to the gravitational field of the Dark Mass.

4 The relationship between Black Holes and Quantum Physics

Introducing the Quantum Vector Function $\bar{\phi}$,

$$\bar{\phi} = \sqrt{\frac{\mu}{2}} \left(\bar{H} + i \frac{\bar{E}}{c} \right) \quad (17)$$

Substituting (17) in (12) results in the quantum presentation for the BLACK HOLE:

$$\overline{\Phi(r, \theta, \varphi)} = \sqrt{\frac{\mu}{2}} \left(\bar{H} + i \frac{\bar{E}}{c} \right) f(r) \begin{pmatrix} \Phi_r \\ \Phi_\theta \\ \Phi_\varphi \end{pmatrix} \quad (18)$$

$$\overline{\Phi(r, \theta, \varphi)} = K \sqrt{\frac{\epsilon}{\mu}} e^{-\frac{G I \epsilon_0 \mu_0}{8 \pi r}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\sin(k r) & \sin(k r) \\ 0 & -i \cos(k r) & i \cos(k r) \end{pmatrix} \begin{Bmatrix} 0 \\ \cos(\omega t) \\ i \sin(\omega t) \end{Bmatrix}$$

With “K” a constant value dependend of the mass of the BLACK HOLE. The Dot product between the unit vector and the Quantum Vector Function $\bar{\phi}$ represents the quantum mechanical probability function $\Psi[r, t]$ which is a fundamental solution of the Schrödinger Wave Equation.

$$\overline{\Phi(r, \theta, \varphi)} = K \sqrt{\frac{\epsilon}{\mu}} e^{-\frac{G1 \epsilon_0 \mu_0}{8 \pi r}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\text{Sin}(k r) & \text{Sin}(k r) \\ 0 & -i \text{Cos}(k r) & i \text{Cos}(k r) \end{pmatrix} \begin{Bmatrix} 0 \\ \text{Cos}(\omega t) \\ i \text{Sin}(\omega t) \end{Bmatrix} \quad (19)$$

$$\Psi(r, t) = \begin{Bmatrix} 1 & 1 & 1 \end{Bmatrix} \begin{Bmatrix} 0 \\ \text{Cos}(\omega t) \\ i \text{Sin}(\omega t) \end{Bmatrix} K \sqrt{\frac{\epsilon}{\mu}} e^{-\frac{G1 \epsilon_0 \mu_0}{8 \pi r}} = K \sqrt{\frac{\epsilon}{\mu}} e^{-\frac{G1 \epsilon_0 \mu_0}{8 \pi r}} e^{i \omega t}$$

The Scalar function $\Psi[r, t]$ represents a fundamental solution of the Quantum Mechanical Schrödinger wave equation.

4.1 Black Holes with Discrete Spherical Energy Levels

An essential requirement for the confinement of Electromagnetic Energy is that the Poynting vector equals zero at the (spherical) surface of the confinement. For the confinement within a sphere, a standing electromagnetic wave pattern has been required which exists of concentric spheres, at every sphere an antinodal plane for E (or B) with a radius distance between each sphere of half the wavelength of the confinement. The constant $k = n \pi / \lambda$, “n” a natural number (1,2,3,4.....) and λ the wavelength.

4.1.1 Time and Radius dependent BLACK HOLES with discrete Energy Levels. Confinements of Electromagnetic Radiation within spherical Regions.

Every concentric sphere represents an anti-nodal surface for the Electric Field (E) or the Magnetic Field (H). The Poynting Vector: $\vec{S} = \vec{E} \times \vec{H}$ at this spherical surface equals zero at any time and at any location at this sphere. The Electromagnetic Energy remains always within this sphere and the next concentric sphere. The concentric spheres have a difference in radius of one half wavelength of the electromagnetic radiation within the confinement and a different discrete energy level. Every concentric sphere represents an anti-nodal surface of the electric field or the magnetic field.

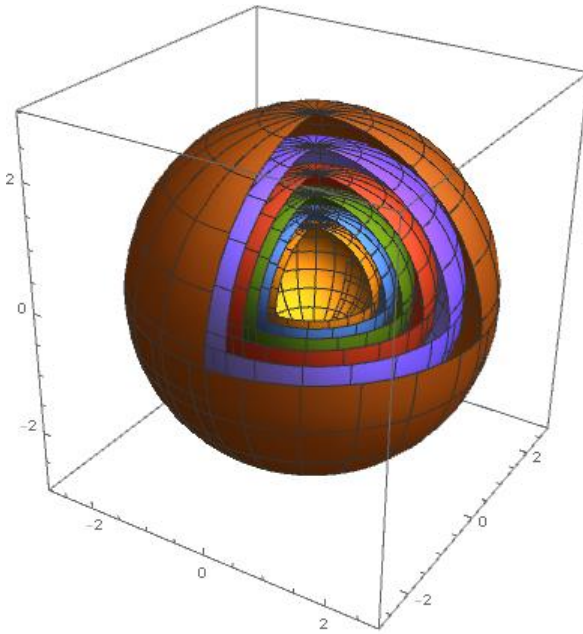


Fig. 5 Nodal and Antinodal Spheres for Standing (Confined) Spherical Electromagnetic waves with a 90 degrees phase shift between the Electric field and the Magnetic field. Equation (9)

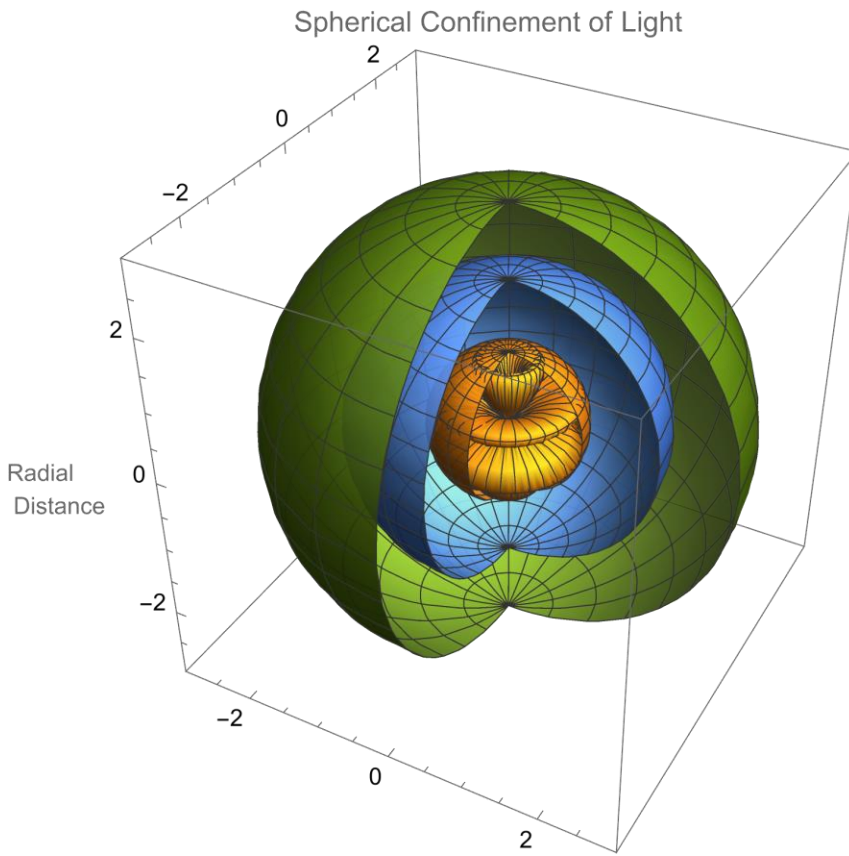


Fig. 6 Nodal- and Antinodal Spheres (k = 3) for Standing (Confined) Spherical Electromagnetic waves with a 90 degrees phase shift between the Electric field and the Magnetic field. Equation (9)

Equation (20) describes a Time and Radius dependent BLACK HOLE.

$$\vec{E} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \begin{pmatrix} 0 \\ \text{Sin}[k r] \text{Sin}[\omega t] \\ -\text{Cos}[k r] \text{Cos}[\omega t] \end{pmatrix} \quad (20)$$

$$\vec{H} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \sqrt{\frac{\epsilon_0}{\mu_0}} \begin{pmatrix} 0 \\ \text{Sin}[k r] \text{Cos}[\omega t] \\ -\text{Cos}[k r] \text{Sin}[\omega t] \end{pmatrix}$$

Equation (20) represents by the function $\text{Sin}[k r]$ ($k = 1,2,3,4,\dots$) the confinement of electromagnetic radiation between two concentric spheres. K represents the amplitude of the Electric/ Magnetic Field Intensity. [14]

4.1.2 Time and Polar Angle dependent Black Holes

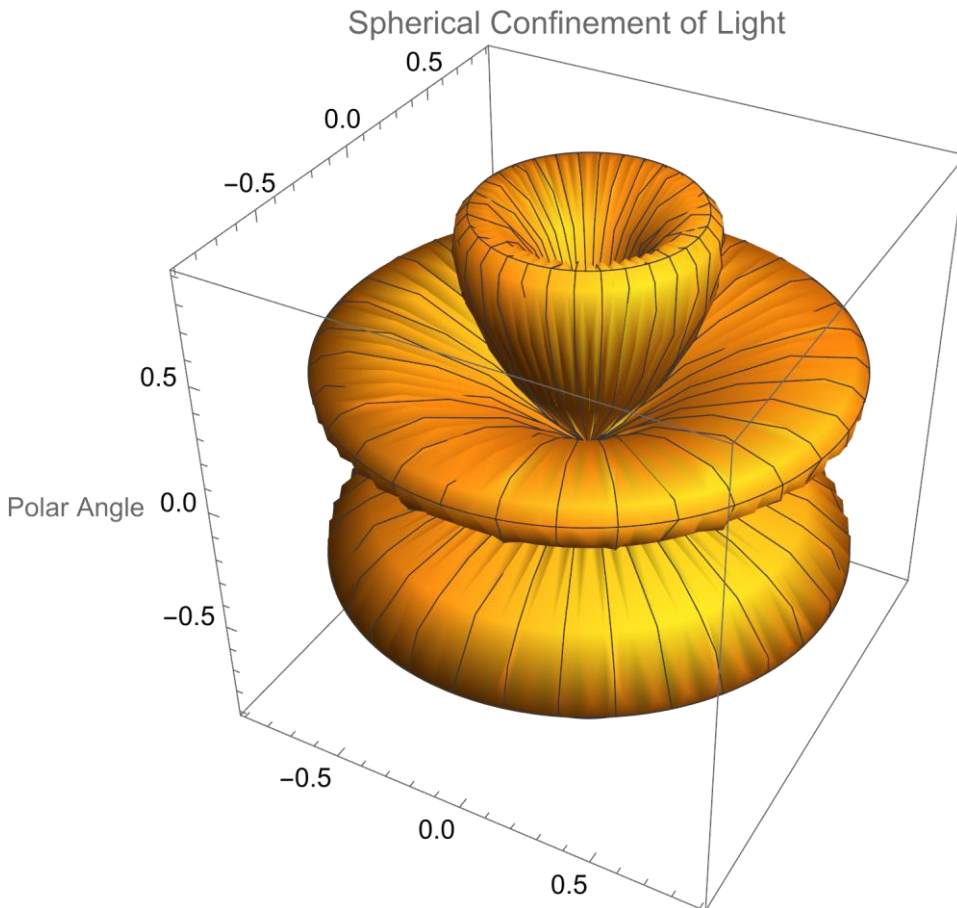


Fig. 7 Nodal- and Antinodal Polar Angle Regions ($m = 3$) for Standing (Confined) Spherical Electromagnetic waves with a 90 degrees phase shift between the Electric field and the Magnetic field. Equation (15)

Equation (21) describes a Time and “Polar Angle” dependent BLACK HOLE

$$\bar{E} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \begin{pmatrix} 0 \\ \text{Sin}[m \theta] \text{Sin}[\omega t] \\ \text{Sin}[m \theta] \text{Cos}[\omega t] \end{pmatrix} \quad (21)$$

$$\bar{H} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \sqrt{\frac{\epsilon_0}{\mu_0}} \begin{pmatrix} 0 \\ \text{Sin}[m \theta] \text{Cos}[\omega t] \\ -\text{Sin}[m \theta] \text{Sin}[\omega t] \end{pmatrix}$$

Equation (19) represents by the function $\text{Sin}[m \theta]$ ($m = 1,2,3,4,\dots$) the confinement of electromagnetic radiation between two Polar Angular Regions [15].

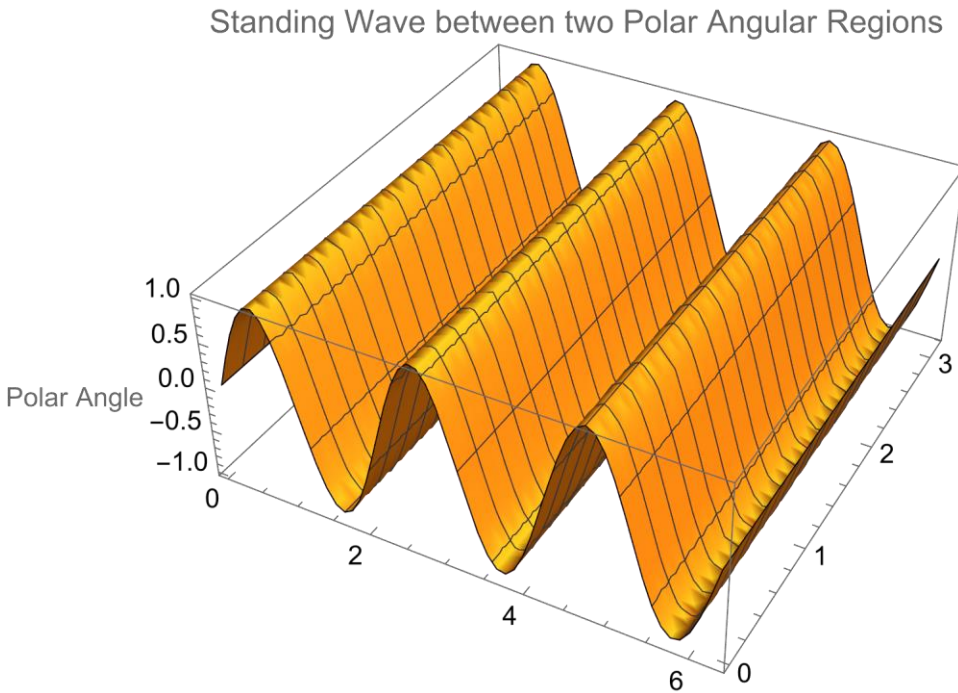


Fig. 8 Nodal- and Antinodal Polar Angle Regions ($m = 3$) for Standing (Confined) Electromagnetic waves with a 90 degrees phase shift between the Electric field and the Magnetic field. Equation (15)

4.1.3 Time and Azimuthal Angular dependent BLACK HOLES

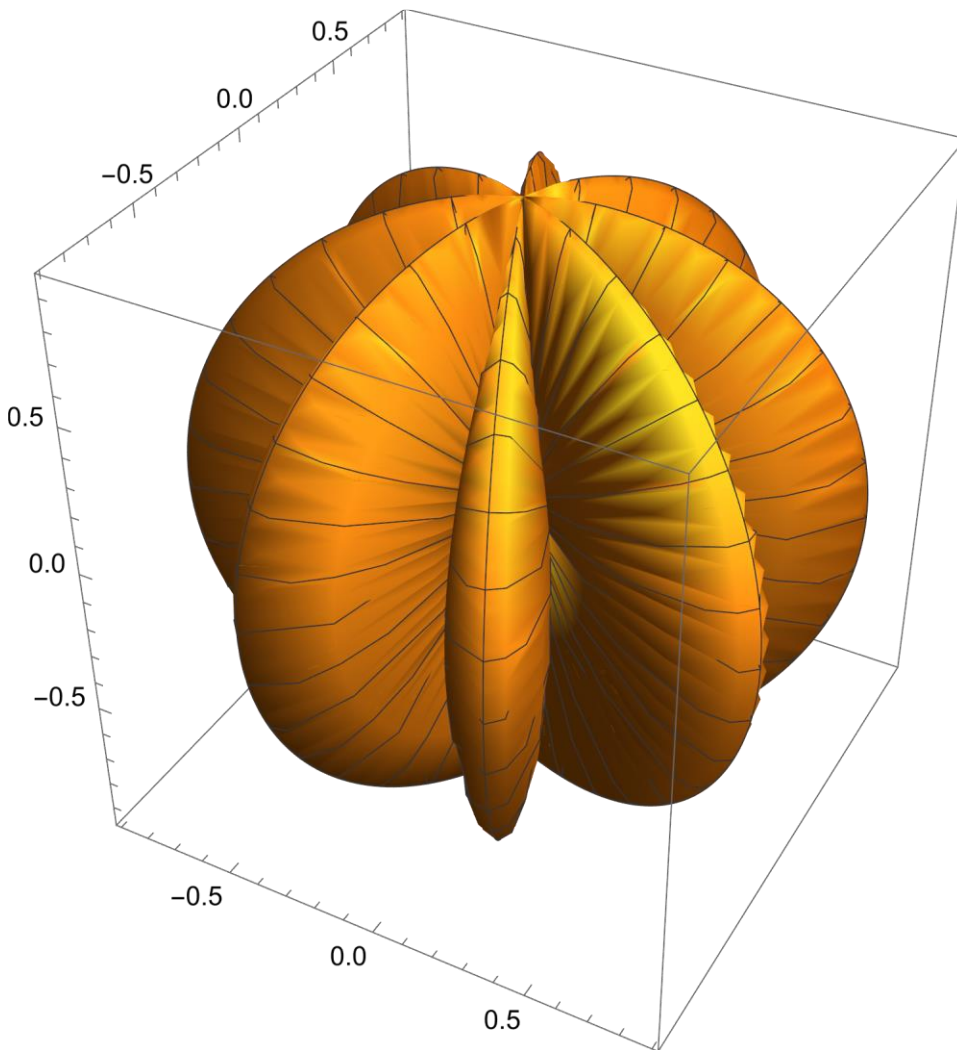


Fig. 9 Nodal- and Antinodal Azimuthal Angular Regions ($n = 3$) for Standing (Confined) Electromagnetic waves with a 90 degrees phase shift between the Electric field and the Magnetic field. Equation (16)

Equation (22) describes a Time and “Polar Angle” dependent BLACK HOLE

$$\bar{\mathbf{E}} = \mathbf{K} e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \begin{pmatrix} 0 \\ \text{Cos}[n \varphi] \text{Sin}[\omega t] \\ \text{Cos}[n \varphi] \text{Cos}[\omega t] \end{pmatrix} \quad (22)$$

$$\bar{\mathbf{H}} = \mathbf{K} e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \sqrt{\frac{\epsilon_0}{\mu_0}} \begin{pmatrix} 0 \\ \text{Cos}[n \varphi] \text{Cos}[\omega t] \\ -\text{Cos}[n \varphi] \text{Sin}[\omega t] \end{pmatrix}$$

Equation (22) represents by the function $\text{Sin}[n \varphi]$ ($n = 1,2,3,4,\dots$) the confinement of electromagnetic radiation between two Azimuthal Angular Regions [16].

4.1.4 Time, Polar Angular and Azimuthal Angular dependent BLACK HOLES

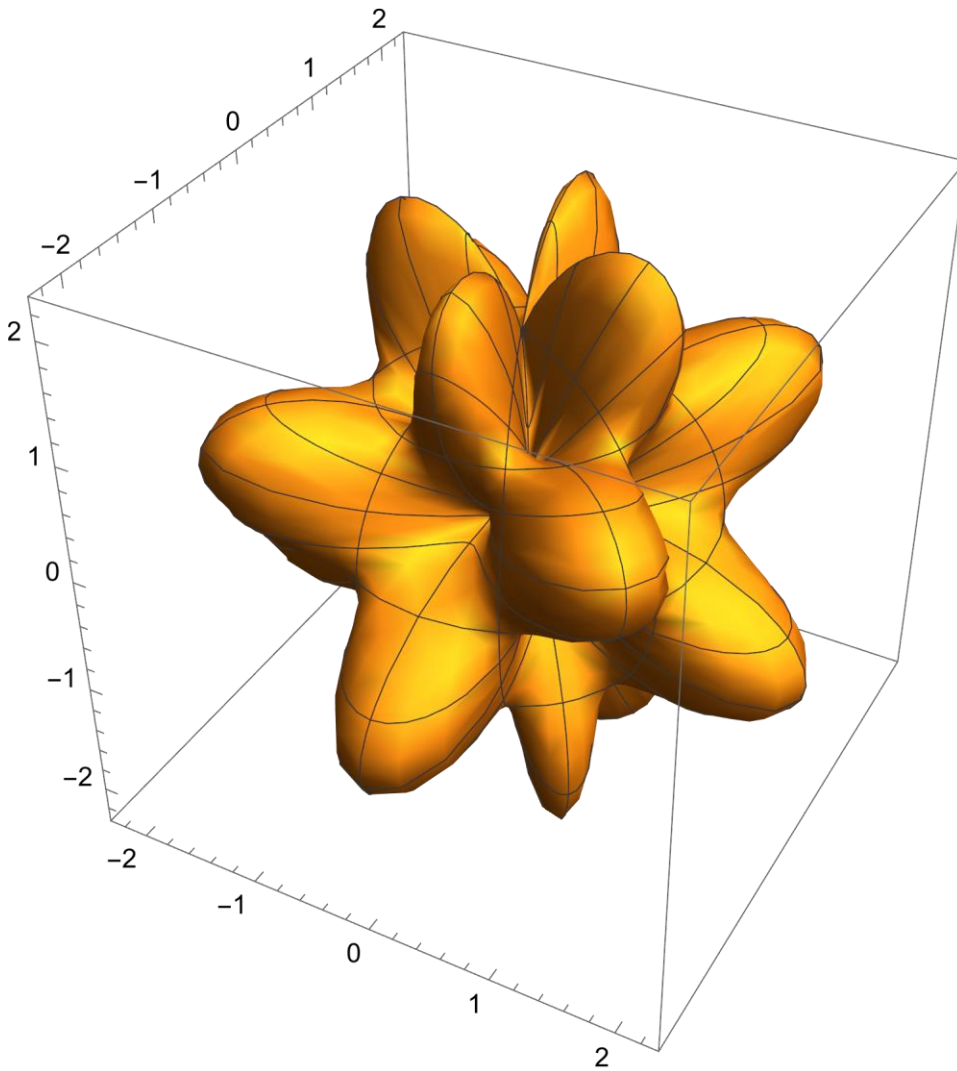


Fig. 10 Nodal- and Antinodal Polar Angular and Azimuthal Angular Regions ($n = 4$ and $m = 4$) for Standing (Confined) Electromagnetic waves with a 90 degrees phase shift between the Electric field and the Magnetic field. Equation (17)

Equation (23) describes a Time “Azimuthal Angle” and “Polar Angle” dependent
BLACK HOLE

$$\vec{E} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \begin{pmatrix} 0 \\ \text{Cos}[n \varphi] \text{Sin}[m \theta] \text{Sin}[\omega t] \\ \text{Cos}[n \varphi] \text{Sin}[m \theta] \text{Cos}[\omega t] \end{pmatrix} \quad (23)$$

$$\vec{H} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} \sqrt{\frac{\epsilon_0}{\mu_0}} \begin{pmatrix} 0 \\ -\text{Cos}[n \varphi] \text{Sin}[m \theta] \text{Cos}[\omega t] \\ \text{Cos}[n \varphi] \text{Sin}[m \theta] \text{Sin}[\omega t] \end{pmatrix}$$

Equation (21) represents by the function $\text{Cos}[n \varphi]$ ($n = 1,2,3,4,\dots$) and $\text{Sin}[m \theta]$ ($m = 1,2,3,4,\dots$) the confinement of electromagnetic radiation between two Azimuthal Angular Regions and two Polar Angulars Regions [17].

4.1.5 Spherical Confinement of Light between two Concentric Spheres within BLACK HOLES

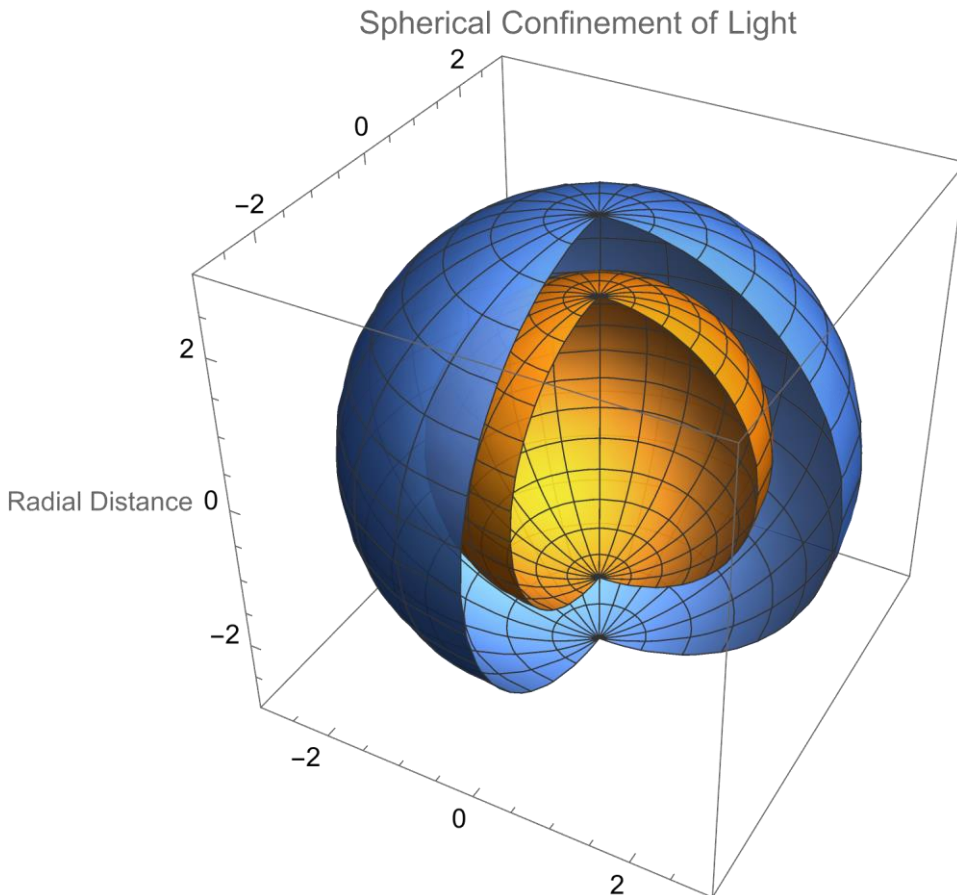


Fig.11 Nodal- and Antinodal Regions for Standing (Confined) Electromagnetic waves with a 90 degrees phase shift between the Electric field and the Magnetic field. Equation (14)

Equation (18) represents the reflection of the Confined Electromagnetic Energy within the BLACK HOLE between two concentric spheres while the speed of light, depending on the variable “ r ”, changes in direction with the frequency of the confined light (Electromagnetic Radiation).

A BLACK HOLE can split into two new BLACK HOLES with different radii. The original BLACK HOLE falls back into a lower energy level while the new BLACK HOLE represents the difference in Energy Levels comparable with an atom falling back into a lower energy level.

$$\bar{E} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} f \left[t - \frac{\sqrt{\epsilon_0 \mu_0} \text{Cos}[2 k r]}{2 k} \right] \begin{pmatrix} 0 \\ \text{Sin}[k r] \text{Sin}[\omega t] \\ -\text{Cos}[k r] \text{Cos}[\omega t] \end{pmatrix} \quad (24)$$

$$\bar{H} = K e^{-\frac{G1\epsilon_0\mu_0}{8\pi r}} f \left[t - \frac{\sqrt{\epsilon_0 \mu_0} \text{Cos}[2 k r]}{2 k} \right] \sqrt{\frac{\epsilon_0}{\mu_0}} \begin{pmatrix} 0 \\ -\text{Sin}[k r] \text{Cos}[\omega t] \\ -\text{Cos}[k r] \text{Sin}[\omega t] \end{pmatrix}$$

Spherical Confinement of Light between two Concentric Spheres

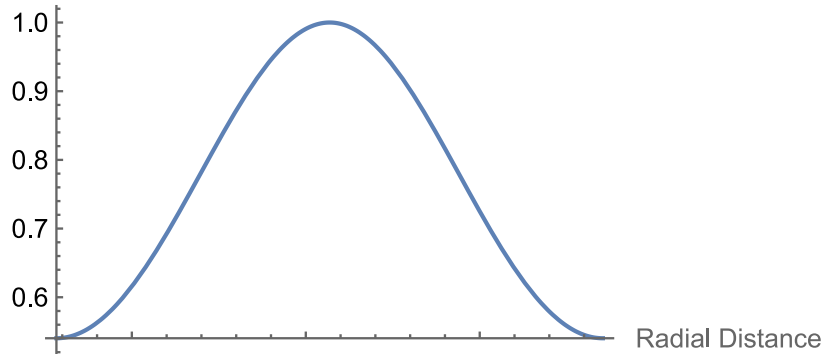


Fig. 12 Nodal- and Antinodal Regions for Standing (Confined) Electromagnetic within two concentric spheres. Equation (18)

5 Universal Equilibrium in the “Concept of Quantum Mechanical Probability” in “The New Theory”.

The 4-dimensional notation for the divergence of the Stress-Energy Tensor (25) expresses in the 4th dimension (time dimension) the law of [Conservation of Energy](#). For an Electromagnetic Field the law for conservation of Energy has been expressed as:

$$\bar{f}^4 = \begin{pmatrix} f_4 \\ f_3 \\ f_2 \\ f_1 \end{pmatrix} = \square \cdot \bar{\bar{T}} = \begin{pmatrix} \nabla \cdot \bar{S} + \frac{\partial w}{\partial t} \\ f_3 \\ f_2 \\ f_1 \end{pmatrix} = \bar{0}^4 \quad (25)$$

From the equation for the “Conservation of Electromagnetic Energy” (19) the “Fundamental Equation for Confined Electromagnetic Interaction” in “The New Theory” will be derived, which equals the Relativistic Quantum Mechanical “Dirac” equation and the Schrödinger wave equation at velocities relative low compared to the speed of light.

The “Fundamental Equation for Confined Electromagnetic Interaction” in “The Proposed Theory” can be considered to be the relativistic version of the Quantum Mechanical Schrödinger wave equation, which equals the Quantum Mechanical Dirac Equation.

5.1 Confined Electromagnetic Energy within a 4-dimensional Equilibrium

The physical concept of quantum mechanical probability waves has been created during the famous [1927 5th Solvay Conference](#). During that period there were several circumstances which came just together and made it possible to create a unique idea of “**Material Waves**” (Solutions of Schödinger’s wave equation) being complex (partly real and partly imaginary) and describing the probability of the appearance of a physical object (elementary particle) generally indicated as “**Quantum Mechanical Probability Waves**”.

The idea of complex (probability) waves is directly related to the concept of confined ([standing](#)) waves. Characteristic for any [standing](#) acoustical wave is the fact that the Velocity and the Pressure (Electric Field and Magnetic Field in QLT) are always shifted over 90 degrees. The same principle does exist for the [standing \(confined\) electromagnetic waves](#),

For that reason every confined (standing) Electromagnetic wave can be described by a complex sum vector $\bar{\phi}$ of the Electric Field Vector \bar{E} and the Magnetic Field Vector \bar{B} (\bar{E} has 90 degrees phase shift compared to \bar{B}).

The vector functions $\bar{\phi}$ and the complex conjugated vector function $\bar{\phi}^*$ will be written as:

$$\bar{\phi} = \frac{1}{\sqrt{2\mu}} \left(\bar{B} + i \frac{\bar{E}}{c} \right) \quad (26)$$

\bar{B} equals the magnetic induction, \bar{E} the electric field intensity (\bar{E} has + 90 degrees phase shift compared to \bar{B}) and c the speed of light.

The complex conjugated vector function $\bar{\phi}^*$ equals:

$$\bar{\phi}^* = \frac{1}{\sqrt{2\mu}} \left(\bar{B} - i \frac{\bar{E}}{c} \right) \quad (27)$$

The dot product equals the electromagnetic energy density w :

$$\bar{\phi} \cdot \bar{\phi}^* = \frac{1}{2\mu} \left(\bar{\mathbf{B}} + i \frac{\bar{\mathbf{E}}}{c} \right) \cdot \left(\bar{\mathbf{B}} - i \frac{\bar{\mathbf{E}}}{c} \right) = \frac{1}{2} \mu \mathbf{H}^2 + \frac{1}{2} \varepsilon \mathbf{E}^2 = w \quad (28)$$

Using Einstein's equation $W = m c^2$, the dot product equals the electromagnetic mass density w :

$$\bar{\phi} \cdot \bar{\phi}^* \frac{1}{c^2} = \frac{\varepsilon}{2} \left(\bar{\mathbf{B}} + i \frac{\bar{\mathbf{E}}}{c} \right) \cdot \left(\bar{\mathbf{B}} - i \frac{\bar{\mathbf{E}}}{c} \right) = \frac{1}{2} \varepsilon \mu^2 \mathbf{H}^2 + \frac{1}{2} \varepsilon^2 \mathbf{E}^2 = \rho \text{ [kg/m}^3\text{]} \quad (29)$$

The cross product is proportional to the Poynting vector [\(Ref. 3, page 202, equation 15\)](#).

$$\bar{\phi} \times \bar{\phi}^* = \frac{1}{2\mu} \left(\bar{\mathbf{B}} + i \frac{\bar{\mathbf{E}}}{c} \right) \times \left(\bar{\mathbf{B}} - i \frac{\bar{\mathbf{E}}}{c} \right) = i \sqrt{\varepsilon \mu} \bar{\mathbf{E}} \times \bar{\mathbf{H}} = i \sqrt{\varepsilon \mu} \bar{\mathbf{S}} \quad (30)$$

This article presents a new “Gravitational-Electromagnetic Equation” describing Electromagnetic Field Configurations which are simultaneously the Mathematical Solutions for the Scalar Quantum Mechanical “Schrodinger Wave Equation” and more exactly the Mathematical Solutions for the Tensor representation of the “Relativistic Quantum Mechanical Dirac Equation” (37).

The 4-dimensional divergence of the sum of the Electromagnetic Stress-Energy tensor expresses the 4-dimensional Force-Density vector (expressed in $[\text{N/m}^3]$ in the 3 spatial coordinates) as the result of Electro-Magnetic-Gravitational interaction.

$$f^\mu = \partial_\nu \mathbf{T}^{\mu\nu} = 0 \quad (31)$$

In vector notation the 4-dimensional Force-Density vector can be written as:

$$\bar{f}^4 = \begin{pmatrix} f_4 \\ f_3 \\ f_2 \\ f_1 \end{pmatrix} = \square \cdot \bar{\mathbf{T}} = 0 \quad (32)$$

The fundamental boundary condition for this alternative approach to gravity is the requirement that the Force 4 vector equals zero in the 4 dimensions, expressing a universal 4-dimensional equilibrium:

The 3 spatial components of the Force-Density vector, as a result of Electro-Magnetic-Gravitational interaction can be written as:

Substituting the electromagnetic values for the electric field intensity “E” and the magnetic field intensity “H” in (33) results in the 4-dimensional representation of the Electro-Magnetic-Gravitational Fields Equation (35):

$$\begin{aligned}
 & \text{Energy-Time Domain} \\
 (f_4) \Leftrightarrow \nabla \cdot (\bar{\mathbf{E}} \times \bar{\mathbf{H}}) + \frac{1}{2} \frac{\partial (\epsilon_0 (\bar{\mathbf{E}} \cdot \bar{\mathbf{E}}) + \mu_0 (\bar{\mathbf{H}} \cdot \bar{\mathbf{H}}))}{\partial t} = 0 \\
 & \text{3-Dimensional Space Domain} \tag{33} \\
 \begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} \Leftrightarrow -\frac{1}{c^2} \frac{\partial (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\partial t} + \epsilon_0 \bar{\mathbf{E}} (\nabla \cdot \bar{\mathbf{E}}) - \epsilon_0 \bar{\mathbf{E}} \times (\nabla \times \bar{\mathbf{E}}) \\
 + \mu_0 \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{H}}) - \mu_0 \bar{\mathbf{H}} \times (\nabla \times \bar{\mathbf{H}}) = \bar{\mathbf{0}}
 \end{aligned}$$

In which f_1, f_2, f_3 , represent the force densities in the 3 spatial dimensions and f_4 represent the force density (energy flow) in the time dimension (4th dimension). Equation (31) can be written as:

$$\begin{aligned}
 & \text{Energy-Time Domain} \\
 & \text{Conservation of Energy} \\
 & \text{B-7} \\
 (f_4) \quad \nabla \cdot \bar{\mathbf{S}} + \frac{\partial w}{\partial t} = 0 \tag{34.1} \\
 & \text{3-Dimensional Space Domain} \\
 & \text{B-1} \quad \text{B-2} \quad \text{B-3} \\
 \begin{pmatrix} f_3 \\ f_2 \\ f_1 \end{pmatrix} \quad -\frac{1}{c^2} \frac{\partial (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\partial t} + \epsilon_0 \bar{\mathbf{E}} (\nabla \cdot \bar{\mathbf{E}}) - \epsilon_0 \bar{\mathbf{E}} \times (\nabla \times \bar{\mathbf{E}}) + \\
 \quad \text{B-4} \quad \text{B-5} \\
 + \mu_0 \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{H}}) - \mu_0 \bar{\mathbf{H}} \times (\nabla \times \bar{\mathbf{H}}) = \bar{\mathbf{0}} \tag{34.2}
 \end{aligned}$$

The 4th term in equation (45.1) can be written in the terms of the Poynting vector “S” and the energy density “w” representing the electromagnetic law for the conservation of energy (Newton’s second law of motion).

5.3 The 4-dimensional Dirac Equation

Substituting (27) and (28) in Equation (32.1) results in The 4-Dimensional Tensor presentation for the relativistic quantum mechanical Dirac Equation (33):

$$(x_4) \quad \nabla \cdot (\bar{\phi} \times \bar{\phi}^*) + \frac{i}{c} \frac{\partial \bar{\phi} \cdot \bar{\phi}^*}{\partial t} = 0 \quad (35)$$

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} \frac{i}{c} \frac{\partial (\bar{\phi} \times \bar{\phi}^*)}{\partial t} - (\bar{\phi} \times (\nabla \times \bar{\phi}^*) + \bar{\phi}^* \times (\nabla \times \bar{\phi})) + (\bar{\phi} (\nabla \cdot \bar{\phi}^*) + \bar{\phi}^* (\nabla \cdot \bar{\phi})) = 0$$

To transform the electromagnetic vector wave function $\bar{\phi}$ into a scalar (spinor or one-dimensional matrix representation), the Pauli spin matrices σ and the following matrices (Ref. 3 page 213, equation 99) are introduced:

$$\bar{\alpha} = \begin{bmatrix} 0 & \sigma \\ \sigma & 0 \end{bmatrix} \quad \text{and} \quad \bar{\beta} = \begin{bmatrix} \delta_{ab} & 0 \\ 0 & -\delta_{ab} \end{bmatrix} \quad (36)$$

The Equations (6), (32) and (34) can be written in tensor presentation as the 4-Dimensional Relativistic Quantum Mechanical Dirac Equation: [3] (Equation 102, page 213)

$$(x_4) \quad \left(\frac{i m c}{h} \bar{\beta} + \bar{\alpha} \cdot \nabla \right) \psi = -\frac{1}{c} \frac{\partial \psi}{\partial t} \quad (37.1)$$

$$\begin{pmatrix} x_3 \\ x_2 \\ x_1 \end{pmatrix} -\frac{1}{c^2} \frac{\partial (\bar{\mathbf{E}} \times \bar{\mathbf{H}})}{\partial t} + \epsilon_0 \bar{\mathbf{E}} (\nabla \cdot \bar{\mathbf{E}}) - \epsilon_0 \bar{\mathbf{E}} \times (\nabla \times \bar{\mathbf{E}}) + \quad (37)$$

$$+ \mu_0 \bar{\mathbf{H}} (\nabla \cdot \bar{\mathbf{H}}) - \mu_0 \bar{\mathbf{H}} \times (\nabla \times \bar{\mathbf{H}}) + \gamma_0 \bar{\mathbf{g}} (\nabla \cdot \bar{\mathbf{g}}) - \gamma_0 \bar{\mathbf{g}} \times (\nabla \times \bar{\mathbf{g}}) = \bar{0} \quad (37.2)$$

6. Conclusions

Based on the assumption of the zero rest mass of photons, General Relativity describes the interaction between Gravity and Light within a 4-dimensional curvature in Space and Time due to a gravitational field. Light follows a path defined by this curved 4-dimensional Space and Time geometry.

The proposed theory presented here, describes a bi-directional separation between mass and inertia for light (photons). Inertia only in the direction of propagation of the beam of light (photons) which determines the speed of light. Mass of the beam of light (photons) only in the plane perpendicular to the direction of propagation, which determines the deflection of a beam of light (photons) by a gravitational field in the plane perpendicular to the direction of propagation.

BLACK HOLES (Gravitational-Electromagnetic Confinements) are fundamental solutions of the relativistic quantum mechanical Dirac equation. Black Holes represent the large impact of “Gravitational Intensity Shift” and “Gravitational RedShift” due to a gravitational field. Both phenomena can maybe observed in the future with extremely sensitive observatories at extreme low frequency levels.

Within a 4-Dimensional Equilibrium and taking into account the inertia- and the gravitational- force densities within the electromagnetic field configurations, Gravitational Electromagnetic Confinements (BLACK HOLES at sub-atomic dimensions) are a physical reality and are solutions of the Relativistic Quantum Mechanical Dirac Equation (35, 37) and present spherical confinements with discrete separate energy levels. .

To test the proposed theory with General Relativity, an experiment [2] has been required which measures the interaction between gravity and light within a well-defined gravitational field like the gravitational field of the earth. The difference between the calculation for Gravitational RedShift, within the Gravitational Field of the Earth, in “General Relativity” and “The Proposed Theory” is smaller than 10^{-16} and cannot be determined with present observation equipment (maximum accuracy of 10^{-15} for GRS). Validation of both theories requires higher accuracies.

5.1 Data Availability

All Data and Calculations have been published at:

<https://quantumlight.science/>

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https://community.wolfram.com/groups/-/m/t/2900869?p_p_auth=yxR9nZu6
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https://community.wolfram.com/groups/-/m/t/2901457?p_p_auth=H4jjDHmQ
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