Stringy Motivic Spectra

Ryan J. Buchanan

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Dedicated to my mother on the occasion of her 50th birthday.

Abstract

We consider strings from the perspective of stable motivic, homotopical QFT. Some predictions for the behavior of gauginos in both a Minkowski light cone and 5-dimensional $\mathcal{A}d\mathcal{S}_5$-space are given. We show that there is a duality between working locking in a system of dendrites, and threshold edging at the periphery of a manifold.

This work extends the work of [4] and [7] by providing a more mathematical interpretation of the realization of quasi-quanta in open topological dynamical systems. This interpretation incidentally involves the category of pure motives over $\mathfrak{C}$, and projections of fiber spectra to the category of stable homotopies.

1 Prologue

In the weeks following up to this document, it had become increasingly necessary to me that I write a paper with the present title. A few facts have emerged recently which cemented this. Firstly, Haine's recent and beautiful paper [22], convinced me that the Betti realization was a manifestation of a perturbative kink in the noosphere, and secondly, the importance of Bettie realization in general has been reaffirmed by S. Mochizuki [23].

In this paper, I wanted to present an $\mathbb{A}^1$-local homotopical restriction of pre-brane in an unknowable vacuum space to ordinary cosmology. Along the way, the semantic realization that I had when working simultaneously with the words "brane" and "nerve" was by no means a coincidence. This quasi-biological metaphor is expected to be what Witten, Green, etc. must have at least partially felt as an intention. If no string theorist has exploited this coincidence yet, then I would be colored impressed.[1] It is only right to me that I can give at least a somewhat satisfactory case study of the technics at play by investigating what I have called here a "dendrite," but what is really a mathematical analogue of the nerve adjoining an $\mathbb{A}d\mathcal{S}_5$-brane to the category $\mathfrak{shad}$ of shadows.

[1] Depressed, moreso

The shadows I have considered so far have been specifically target groupoids of a representable functor out of an underlying category $\mathscr{A}$ of pre-real quasi-*intentities*. These int-entitaties take as their motivic pre-structure, meta-stable urelements in an encrypted (locked of some kind[2]) data structure, which encodes the homotopy types of images of Dennis traces. These are not fully mathematically rigorous as of yet. Physically, it is plausible that Dennis traces are the fermionic realization of a magnetic ghost field in quantum flux.[3] These are the non-nervous interactions at the stable range, which I approximate to be $\hbar \times .99823MJ$, the work needed to move a unit of thermodynamic energy of a quark.

The nervous model presented here is a measured response to [24]. Here, the Koszul duality is between a nerve of a selectively permeable brane, and a category of ineffective shadows which permute as intensifiers for nouns out of a language $\mathscr{L}\mathscr{A}$ of $\mathscr{A}$. This is, again, a non-mathematically precise linguistic description of the biological metaphor. Mathematically, however, we do have the option of thinking of $\mathscr{A}$ as a connective algebra, which simplifies many of the calculations involving simplicial sets. These calculations are combinatorial by nature, but no way do they have the *flavor* of a combinatorial problem.

Indeed, the present paper is mainly abstract (quasi-super)-algebraic. It consists in deriving functors

$$(spt_* \longrightarrow SH_{MU}(\mathfrak{X})) \simeq \mu_0^\dagger$$

which annihilate the motivic spectra of homotopy types of the free loop space over S^1. This does bear some similarity to Schreiber's "pre-quantum line bundle," and in fact his tadpole cancellation principle was a key guiding philosophy in my meditation on these topics.

By "annihilate," I mean specifically $\mathbb{P}$-annihilation of germs, or in other words, the adjoint of the creation map $Cr(\heartsuit) \rightsquigarrow \mathfrak{M}$. This is annihilation in the appropriate metaphysical sense. One bizarre and unexpected behavior of our regime is that the annihilation $q^\dagger$ of a quantum q does not result in the annihilation of every quasi-quantum $\hat{q}$ which lies within q. This anomaly can be accounted for by fixing a prescribed number of hypercharge directions, namely 32 in type II string theories.

1.1 Arithmetic invariants

Every number n can be endowed with any given property P in the appropriate context C. The Connes fusion of n with a topological invariant ρ gives us an entire family ρ_n of slice towers.

Developing the adequate context for a complete theory of ρ_n is an extremely demanding task. For starters, would need to compute trillions of homotopy types for arbitrary setoids equipped with binary operations. This would involve

[2]Perhaps work locked

[3]It is almost tempting to say pre-flux, but that would seem to imply that the flux is in a state of non-existence. Rather, it is in a state of meta-unstable (pseudo-non-degenerate) quasi-fluxes, cycling at a ratio of $\mathbb{K}/\mathbb{A}^1 : [\heartsuit]$.

performing isogeny computations for countless Lie group adjunctions, where by Lie groups we mean even the general flavor of a groupoid. As of the time of this writing, it is not even known whether quantum computations might simplify this problem at all.[4]

Contents

2 Introduction

Let $\mathbb{L}^4$ denote the Minkoski lightcone. Recall that $\mathbb{L}^4$ admits a construction in the form of a modal diagram:

$$
\begin{tikzcd}
\diamond^2 & & \diamond^2 & & \diamond^2 \\
& \diamond \arrow[ul, dotted] \arrow[ur, dotted] & & \diamond \arrow[ul, dotted] \arrow[ur, dotted] & \\
& & \Box \arrow[ul, dashed] \arrow[ur, dashed] & & \\
& \diamond^{-1} \arrow[ur, squiggly] & & \Box^{-1} \arrow[ul] & \\
\diamond^{-2} \arrow[ur, "\shortmid" marking] & & \Box^{-2} \arrow[ul, "\shortmid" marking] \arrow[ur] & & \diamond^{-2} \arrow[ul, squiggly]
\end{tikzcd}
$$

[4]The canonical mantra is "they might."

such that the worldline $\mathfrak{W}_p$ of a particle $p \in \Box$ is a proper subset of the path $... \longrightarrow \Box^{-2} \longrightarrow \Box^{-1} \longrightarrow \Box$ consisting of solid arrows. Further, this diagram may be twisted and folded into a closed dynamical system $\hat{\mathcal{P}}$ represented by the pants diagram, which is effectively a brane of dimension $d > 3$. For more information on this construction, please skim [4] for the relevant details.

Let $\mathcal{B}^d$ be an arbitrary brane of dimension d, and let there be a Galois connection

$$Gal_\Gamma(d) \equiv \int_{\mathbb{L}^4} \mathfrak{I} \longmapsto \mathcal{B}^d$$

where $\mathfrak{I}$ is the Dirac index of a parton in $L^\sharp X \subset \mathbb{L}^4 \bigwedge \mathbb{G}_m$. Recall that $L^\sharp X$ is constructing by replacing the free loop space LX of a space X consisting of all fibers $S^1 \longrightarrow X$ by the set of maps

$$S^{p|q} \underset{q}{\overset{p}{\rightleftarrows}} LX$$

where $S^{p|q}$ is the supercircle [see 4], and is called the "free loop superspace."

For simplicity, let $\mathfrak{I} = \sum_{\omega=0}^{n} \frac{d\omega}{dt}$. This allows us to easily construct a group completion, $\mathfrak{I}^\circ : n \longrightarrow sup(\mathfrak{I})$, where $\mathfrak{I}$ is a proper subcategory of $\mathfrak{I}$, by the formula:

$$\mathfrak{I}^\circ = \underset{n \to sup(\mathfrak{I})}{colim} \sum_{\omega=0}^{n} \frac{d\omega}{dt}$$

where $\mathfrak{I}$ is locally identified with a field $\mathbb{K}$, such that $sup(\mathfrak{I})$ is the maximal ideal of $\mathbb{K}$.

Using a theorem of Stasheff [3], one can show that if M is a monoid, the canonical map $E \longrightarrow \Omega BM$ is an equivalence if, and only if, M is an $\mathbb{E}_1$-group. Recall that an $\mathbb{E}_1$ monoid is a space with a multiplication map which is associative up to coherent homotopy [1], and an $\mathbb{E}_1$ group is the underlying group of such a monoid. We can extend this theorem as such.

Theorem 2.1. *If E is a topological stack ($E \cong \mathfrak{X}$), then the map $E \longrightarrow \mathbb{L}^4$ is an equivalence if and only if:*

1. *E is singular, with singularity $\mathfrak{b}$.*
2. *$\mathbb{L}^4$ has the homotopy type of an Ω-spectrum.*

This is effectively the main conjecture of this paper. A further conjecture is the following:

Theorem 2.2. *Assume that [Theorem 1.1] holds. Then, given the spectrum $\Omega B\mathbb{L}^4$ of a classifying space $B\mathbb{L}^4$, there is an equivalence of homotopy types:*

$$H_n(\Omega B\mathbb{L}^4) \cong H_n(B\mathcal{B}^d)$$

for some brane $\mathcal{B}^d$, induced by the map

$$\mathfrak{Tw}_{\mathbb{L}^4} \bigwedge \mathfrak{Tw}_{\mathbb{L}^4} : \mathbb{L}^4 \longrightarrow \mathcal{B}^d$$

which we will call the "twist-and-fold" map.

Remark 2.1. *For short, we will write $\mathfrak{Tw}_{fld(X)}$ for the twist-and-fold map of an arbitrary object X. Our primary problem is finding a suitable twist (e.g., Dehn, etc.) for our model. In some sense, a twist may be thought of as a restriction on a sheaf; i.e., "twisting twice is the same as twisting all at once."*

Remark 2.2. *Recall that $\mathbb{L}^4$ has a nodal singularity $\mathfrak{b}$ which stratifies it into "past" and "future" components. We have effectively two options for working with this fact; firstly, we can treat (the inclusion of) $\mathfrak{b}$ as the group completion of some underlying index group, or we can treat $\mathfrak{b}$ itself as a virtual particle (e.g., instanton, parton, gaugino). We remain ambivalent as to our preferred choice of resolving the singularity.*

2.1 Realization

We should formalize our notion of a topological realization just a bit more here. As a proposal, let us examine one example of a realization functor.

Example 2.1. *The Betti realization functor*

$$SH^{\mathbb{A}^1}(\mathfrak{C}) \longrightarrow SH$$

where $SH^{\mathbb{A}^1}(\mathfrak{C})$ denotes the triangulated category of $\mathbb{A}^1$-stable homotopies over the complex number field, and SH is the classic stable motivic homotopy category, is a topological realization in case its image is homeomorphic to the image of a projective functor

$$\mathfrak{X} \twoheadrightarrow B\Omega\mathfrak{X}$$

with $B\Omega\mathfrak{X}$ denoting the classifying loop space generated by the idempotents of a topological stack.

For every $\mathbb{E}_1$-group, there is an implied derived functor

$$\mathbb{E}_1 \times \mathfrak{X} \in \mathfrak{Stk} \longmapsto Map_{\mathbb{K}}(\mathfrak{X}, \mathfrak{X} \cdot \mathbb{E}_1) \in \mathfrak{St}^{\mathfrak{et}}$$

to the category of etale stacks.

Remark 2.3. *The realization we are working with here is of a categorically different nature than the one we worked with in [7], where we dealt with quasi-quanta. Notice that in that case, the appropriate choice of category for the submersion is technically a brane (perhaps in, say, $\square$), and is a map out of the canonical frame $\mathscr{A}$. This is somewhat akin to our Galois connection $Gal_{\Gamma}(d)$ onto a brane. Our realization is instead a refinement of the the category of pure motives over $\mathfrak{C}$.*

Proposition 2.1. *There is an equivalence*

$$\mathbb{E}_n \times \mathfrak{X} \cong MotPur_{\mathfrak{C}}$$

between the abelian multiplication of an $\mathbb{E}_n$ group ($n < p$ for some small prime p) with a topological stack, and the category of pure motives over $\mathfrak{C}$.

Proof. Since there is an equivalence of bundles:

$$Bun_{\mathbb{E}_{n<p} \times \mathfrak{X}} \cong BunMotPur_{\mathfrak{C}}{}^{ad}$$

such that the right-hand-side is the adjoint bundle of the left, we have

$$(\mathbb{E}_n \times \mathfrak{X})/Bun_{\sim} = 1_{\mathbb{E}_n \times \mathfrak{X}, MotPur_{\mathfrak{C}}}$$

which means the two categories are equivalent modulo an adjunction. □

In [8], an equivalence between the homotopy category $M\mathbb{Z}{-}mod$ and Voevodsky's big category of motives $\mathbf{DM}_k$ for k a field of characteristic zero, was proven. Furthermore, it was shown that this equivalence preserves the monoidal and triangulated structures. Thus, if

$$\mathfrak{x}_0 \longrightarrow Ext_i^{\mathbb{A}^1}\mathfrak{x}_0 \longrightarrow \mathfrak{x}_0{}^{ad} \longrightarrow ...$$

is a triangle in $M\mathbb{Z} - mod$, then

$$\mathfrak{x}_0{}' \longrightarrow Ext_i^{\mathbb{A}^1} \longrightarrow \mathfrak{x}_0{}'^{ad} \longrightarrow ...$$

is in DM_k. Thus, $\mathfrak{x}' \in (\mathfrak{x}/\cong)$, where $\cong$ means the fiber spectrum is preserved across the transition maps $\mathfrak{x} \leftrightarrows \mathfrak{x}'$.

2.1.1 Nervous Realization

We define once and for all the nervous realization

$$||*|| : * \longrightarrow \mathcal{N}_*^m + n$$

once and for all. Let $\mathscr{G}$ be the isotropy group of the point $*$. Then the above map may be written (substituting $*$ for a Lie groupoid):

$$||*|| : \mathscr{G}_s \times_t \mathscr{G} \longrightarrow SH_{MU}(X_{et})$$

This is a form of etale realization which sends $n \leq 1$-cells to the stable homotopy category of Thom spectra.

Axiom 2.1. *If*

$$\begin{array}{ccc} X & \longrightarrow & X' \\ \updownarrow & & \updownarrow \\ Y & \longrightarrow & Y' \end{array}$$

is a commutative square in $\mathscr{G}$, then there is a corresponding square

$$\begin{array}{ccc} X & \xrightarrow{\mathcal{N}^0_{Kosz}} & X_{et} \\ \updownarrow{\scriptstyle Cr} & & \updownarrow{\scriptstyle Cr} \\ Y & \xrightarrow{\mathcal{N}^1_{Kosz}} & Y_{et} \end{array}$$

in $SH_{MU}(X_{et})$, where $Cr = Cr_{fib}(X_{et}, Y_{et}) \vee Cr_{fib}(X'_{et}, Y'_{et})$.

Remark 2.4. *This applies the creation map to a block matrix consisting of permutations of $\mathfrak{M} = \begin{bmatrix} X & X_{et} \\ Y & Y_{et} \end{bmatrix}$. The function $diag(\mathfrak{M})$ gives an inverse of the spectral specialization map $SH \rightsquigarrow \Omega X$.*

2.1.2 Etale realization

We describe the minimal case for the etale realization functor:

$$||\tilde{x}||_{et} : \tilde{x} \longrightarrow sSets$$

which is by no means canonical.

Let us consider an exit path

$$Pur \twoheadrightarrow SH$$

which takes quasi-simplicial objects in Pur and maps them directly and faithfully onto the stable homotopy category.

Morally,

$$\Sigma^\infty(SH) = min(sSets)$$

where sSets is taken to be a graph of a quasi-category. Let us construct a nerve

$$\mathcal{N}^1_{et} \cong ||\tilde{x}||_{et}$$

which descends to a degenerate open.

We can see from [21] that if

$$\begin{array}{ccccc} \tilde{x}_0 & \longrightarrow & \tilde{x} & \longrightarrow & \tilde{x} \\ & & \downarrow & & \downarrow \\ & & x & \longrightarrow & x \end{array}$$

is a "frying pan," then $\tilde{x}_0$ is a non-degenerate closed subscheme of the chain complex

$$\begin{array}{ccccccccc} ... & \longrightarrow & \tilde{x}_0 & \longrightarrow & \tilde{x}_0 & \longrightarrow & \tilde{x}_0 & \longrightarrow & ... \\ & & \downarrow & & \downarrow & & \downarrow & & \\ ... & \longrightarrow & \tilde{x} & \longrightarrow & \tilde{x} & \longrightarrow & \tilde{x} & \longrightarrow & .. \end{array}$$

2.2 Jet Bundles

We will prove the following facts now for later convenience:

Proposition 2.2. *Let $\mathfrak{I}$ admit a smooth embedding $\mathfrak{I} \underset{\longrightarrow}{\subset} \mathcal{J}^\infty$ of infinite jet bundles. Then, there is a smooth embedding $\mathfrak{I} \hookrightarrow \mathcal{J}^\infty$, which is a submersion.*

Proof. Assume that $\mathfrak{I}$ is presentable as a category. Using Ehresmann's lemma, and Halmos' axiom of extension, it is almost tautological to say that if $\mathfrak{J} \subset \mathfrak{I}$ is proper.

Let $\mathcal{J}^\infty$ be representable as a topological space. Then, the topological realization

$$||\mathcal{C}|| : \mathcal{C} \longrightarrow Top$$

is an immersion, and since the map $|\mathfrak{J}| \hookrightarrow \mathcal{J}^\infty$ is a monorphism (i.e., is injective), we have that the realization functor is a submersion for all $\alpha \in \mathfrak{I}$. We extend this to all proper subcategories (specifically, all proper subsets) of $\mathfrak{I}$, such that $\mathfrak{I} \supset \mathfrak{J} \ni \alpha$. $\square$

Proposition 2.3. *If $\mathcal{C}$ is an ∞-category, then topological realization functor*

$$||\mathcal{C}|| : \mathcal{C} \longrightarrow Top$$

is a totally lossless projection.

Proof. Let $\mathcal{C}$ be an ∞-category. Then, $||\mathcal{C}|| \cong C^\infty_\bullet$ is a homomorphism. Assuming $\mathcal{C}$ is a perfect category gives us a perfect inverse:

$$Top^{op}_{\mathcal{C}} \xrightarrow{Perf} \mathcal{C}$$

$\square$

On an abstract level, there is an intimate connection between the infinite jet bundle, and an O_X-module. Indeed, in the etale realization, $\mathcal{O}_X$-modules of a perfect space are presheaves of the etale infinite jet bundle; viz., for $O_X \in sm/\mathbb{K}$[5]:

$$|Pshv_{O_X}| = \mathcal{J}^\infty_{et}$$

The physical interpretation of this fact may be that there is a ghost superstring:

$$\underset{x_i \longrightarrow \aleph_0}{colim}\, x_i \in O_X$$

arising from a blow-up around a crepant resolution (modeled as the appropriate vacuum) of the O_X-module.

[5] $sm/\mathbb{K}$ being the category of smooth, separated schemes over $\mathbb{K}$

Remark 2.5. *Notice that there is also a rich link between $\mathcal{J}^k$ and C^k spaces; we obtain this isomorphism by localizing the adjunction $\mathcal{J}^\infty \dashv C^\infty_\bullet$ at a finite integer $k \in \mathbb{K}$. We shall call this k-localization. This is effectively a map:*

$$Mot_{\mathfrak{C}} \xmapsto{k} Mot_{\mathfrak{D}}$$

where the elements of $\mathfrak{D}$ form a proper subset of the elements of $\mathfrak{C}$.

Remark 2.6. *Note that the functor k, as above, is essentially a specialization of the homotopy category of MU to a homotopy type $H_n(MU)$ of MU.*

A proof that $Mot_{\mathfrak{D}} \in Mot_{\mathfrak{C}} \backslash (\mathfrak{C} \cap Pur)$ is possible, but tedious, and therefore outside the scope of this paper.[6]

2.3 Gauginos

This part is purely speculative, and so may be skipped at a first reading, and read at a later, more convenient time.

A completely satisfactory theory of gauginos has not been fully worked out. Ideally, one would want to understand the cosmic Galois connection as a string-to-blackhole transition of sorts.

[6]For clarity, $\mathfrak{C} \cap Pur = Mot_{\mathfrak{C}} \cap MotPur_{\mathfrak{C}}$; in other words, we take the intersection *universe-wise*, where, for a Grothendieck universe $\mathcal{V}$, we restrict ourselves to $\mathcal{V}$-small refinements. Geometrically these may be thought of as convex sets, as they lie in the interior of the universe and are therefore δ_i-small with respect to $\widehat{\Box} = (\Box \ni \mathcal{V}) \cup Pur$.

Let $G^\sharp$ be a supergroup. Let $\mathbb{L}^4 = \mathbb{L}^{1,3}$, $\mathcal{B}^d$, and $\widehat{\mathcal{P}}$ be as before. There is a diamond:

$$\begin{tikzcd}
 & G^\sharp \arrow[dl, "O_X"'] \arrow[dr, "Map"] & \\
\mathbb{L}^4 \arrow[rr, leftrightarrow, "\sim"] \arrow[dr, leftrightarrow, "Fun(QCoh(Ho(\mathbb{L}^4)))"'] & & \mathcal{B}^d \arrow[dl, leftrightarrow, "Fun(QCoh(Ho(\mathcal{B}^d)))"] \\
 & \widehat{\mathcal{P}} &
\end{tikzcd}$$

We have

$$H_n(\mathbb{L}^4_k) \cong H_n(\mathcal{B}^d)$$
$$\cong H_n(\mathfrak{Tw}\widehat{P}_k)$$

and

and

$$\prod_{k=0}^{\infty} Coho(\mathbb{L}^4_k)$$
$$= \mathbb{L}^\infty - \hbar^q$$

where

$$q_\alpha \leq i \leq (k \sim \aleph_0) \leq \infty \leq q_\beta$$
$$\hbar = 1_i = Id_i \qquad \forall i \in \mathbb{H}$$

We define a q-loop by

$$q_\ell = [q, \infty) \times (\infty, q]$$

$$q_\ell^2 = (\mathbb{C} \otimes [q, \infty)) \times ((q, \infty] \times \mathbb{C})$$
$$q_\ell^3 = (\mathbb{H} \wedge [q, \infty)) \otimes^{\wedge^2} [[q, \infty]] \otimes^{\wedge^2} ((q, \infty] \wedge \mathbb{H})$$

Complexification is then defined by the functor $q_\ell \longmapsto q_\ell^2 \simeq S^n \longmapsto S^{n+q}$

$$= S^n \longmapsto LS^n$$

$$= S^n \longmapsto \Omega_{Seg} S^n$$

where Ω_{Seg} is Segal's classical loop classifying space

$$(\cong H_n(BLS^n)) = Map(H_n(BLS^n), S^n)/\cong$$

, where, in this context, $\cong$ is all the cofibrations and weak equivalences. The following question arises: "how does a gaugino with a non-vanishing worldline behave under complexification?"

The answer to this question is actually quite subtle, and rather involved. One obtains a superalgebra

$$\mathbb{L}^{4,\pm} = \ell \mapsto End(\ell^2)$$

which gives us a Yukawa coupling at the singularity $\hat{b}$, which now becomes a bordism. The associated crepant resolution can then be used to model a string-blackhole transition. This duality trivializes the ghost fields over the brane containing the topological realization of $\aleph_0$, which is thus adjoint to the $U(1)$-bundle[7] containing $inf(\mathfrak{X}) = q$.

A very interesting potential future direction is to explore gauge connections, and higher holonomy on the brane associated with the pre-quantum line bundle, which is isomorphic to the restriction of the principle geodesic containing q to a closed planar disc. I can imagine that if one brings equivariant homotopy and torsion theories into the mix, with a dash of Virasoro algebra, one could model each pair of branes of opposing charge or chirality as a dual set of categories with one trivial isomorphism (bijection on fibers), and a non-trivial symplectic connection of some kind.

In the $\mathbb{Z}_2$-graded case, there is a rough correspondence between p-torsion theories on the bulk and Reidemeister moves on the boundary. That is to say,

$$Tor^{\mathbb{K}}_{k,q} \cong \mathfrak{tors}_{k,q}\, \mathbb{K}^{\circ}$$

The hypercharge directions of the gaugino are parameterized by 32 possible states, which are graded by a symmetric bi-monoidal category $\mathscr{C}_2$. This is an $\vec{e}_n \cdot \vec{e}_{n+1}$ theory; see, e.g. [13]. One obtains, in the passage to a localization over the algebra of observables, a map

$$\begin{array}{c} \mathbb{K}^{\circ} \\ \downarrow \\ \mathfrak{sim}_k \end{array}$$

for sheaves simplicially enriched over k.

3 Dendrites

I will attempt to define here an original construction of dendrites, consisting of the pre-established notion of the Koszul nerve[8].-

Let $\mathcal{N}^n_{Kosz}$ denote an nth degree Koszul nerve. Then, the map

$$\mathcal{N}^n_{Kosz} \hookrightarrow \mathcal{N}^{n+1}_{Kosz}$$

restricts to an epimorphism in the category of spectra. Write $\mu_0 = MU|_0$ for the vacuum of a dendrite[9]. Then, we have $\mu_0 \twoheadrightarrow (\mu_1)^{-1}$, where $(\mu_1)^{-1}$ is a

[7]Here isomorphic to a simple copy of the closed disc $S^1 \times \mathbb{A}^1 = \bar{S}^1$.

[8]A Koszul nerve is an nth-order restriction on the set of sections that admits a stratification into singleton sets, which represent arrows in their respective diagram.

[9]I.e., the finest calcite of a skeleton such that $cal_{\hbar} \in sk_{\mathbb{H}}$ holds

section of the adjoint bundle of a space which is enriched with the structure of a superalgebra fibered in groupoids.

Let $\mathfrak{D}^m$ denote the maximal dendrite; i.e., the dendrite corresponding to $sup(n+1) \propto sup(\mathcal{N}^{n+q}_{Kosz})$ for $q \geq 1$. This dendrite is a graph (not necessarily commutative), in which the range of all interactions are spanned by $\Psi(q^{\heartsuit})$.[10]

Lemma 3.1. *Promotion of any given creation map $Cr_q : ||-|| \longrightarrow q$ to a genuine creation operator results in an unwellfounded set theory in the image of the projection from $\mathscr{A}$.*

That is to say, the class of fiber spectra becomes unpresentable as a set, and the map

$$\tilde{q} \in Pur \longrightarrow \tilde{q}_{Set}$$

ceases to exist. This may be a form of spontaneous symmetry breaking. The analogous case in the gaugino picture we have painted here, is that a q-loops splits under the map $q^m_\ell \mapsto q^{m+k}_\ell$. This gives us the span (in the sense of Segal)

$$q^{m+k}_\ell \longleftarrow q^m_\ell \longrightarrow q^{mn+k}_\ell$$

where n and k are Pauli matrices.

3.1 Work Locking

Motivic invariants may be projected as cardinals. A recent example of this is the phenomenon of "work locking:"[11] a finite member of the superalgebra generating the (quantum) field of a ghost mode may be measured as a set of internal directional energy (work) elements. It is thought that particles typically live in "topologically protected" states, where the transition from vacuum phase expectation to a relative cohomological invariant is "shielded" (e.g. Faraday) from external cofibrant objects. Presumably, these states involve some knot invariants like the Khovonov invariant, or the Jones polynomial.

Defects of work-locking mechanisms may manifest themselves in circuits, by the induction of a twisted bi-category over a Cartesian prism.

$$\begin{tikzcd}
\mathfrak{D} \arrow[dr, rightsquigarrow] & & \\
& p_x \arrow[r] \arrow[d] \arrow[dr, rightsquigarrow] & x \arrow[d] \\
& x \arrow[r] & \bar{x}
\end{tikzcd}$$

In the above diagram, the ray emanating from $\mathfrak{D}$ induces a stable equivalence of fiber spectra in a sufficiently small neighborhood $\mathcal{U}(p_x) \in \mathbb{E}^{\mathcal{N}}$. This creates a

[10]Here, $\Psi(q^{\heartsuit})$ consists of all of the suitable motives for the class of all isotopy groups of a singular Serre fibration; i.e., the set of all diffeomorphisms of internal homs for an arbitrarily chosen Hopf bimonad. This amounts to all of the creation maps in [4].

[11]see 15

short trip (specialization) $p_x \rightsquigarrow \bar{x}$, in which the residue field of $\mathcal{N}$ is represented by the ambient topology of a generic torus. This representation is given by the moment map[12]

$$p_x \longmapsto (p_x)'$$

which is realized as the Dennis trace map of Dendrites:

$$\mathfrak{D} \overset{Den}{\longmapsto} \mathfrak{D}^{\mathbb{H}}$$

sending each pair of H_n-constructible faces to a single H_{n+2} constructable hypermanifold, $\mathcal{M}^{hyp}_{const}$, which consists of a collection of pencils emanating from a source S. Viz.:

$$S \hookrightarrow \mathcal{M}^{hyp}_{const} \cong \int_{\mathbb{L}^{p,q}} \mathbb{P}_\xi$$

3.1.1 Work Locking in Maximal Dendrites

Let $\mathfrak{D}^m$ denote the maximal dendrite, whose definition we gave earlier. Let $\underline{\widetilde{Cob_m}}$ be the multicategory constructed in [16, lemma 3.5].

We define the "work locking functors

$$\mathfrak{Walls} : \mathfrak{D}^m \longrightarrow int(\mathfrak{D}^m)$$

and

$$\mathfrak{D}^m \times \underline{\widetilde{Cob_m}} : m \longrightarrow n$$

for a gaugino inhabiting a complex of dendrites in the neighborhood $\mathcal{U}(\mathfrak{D}^m) = U(m) \circledcirc \int_{\mathbb{E}^m+n} \mathcal{N}^{m-n}_{Kosz}$. Notice that $\mathfrak{Walls}$ is just the classical "wall crossing functor" acting on the sum of internal fiber spectra $\sum_{i=0}^{m+n} X_{b_i} \in int(\mathfrak{D}^m)$, where $X_{b_i} = 1_{b_i}$ for all $(b, i) \in \mathfrak{X}$.

Let $n \leq m$. Then, the map $m \longrightarrow n$ as above becomes a specialization

$$\tilde{m} \rightsquigarrow \tilde{n}$$

. We have

$$\underset{n \leftarrow m}{colim} = \tilde{n}$$

where $\tilde{n}$ is the nervous realization of n.[13] In the $\mathbb{Z}_2$-graded case (i.e., the superalgebraic case), whence $\tilde{m}$ and $\tilde{n}$ are of the same grade (polarity), we obtain the annihilation map. In this special case,

Proposition 3.1. *the annihilation map is the moment for one of the following sequences of fiber spectra:*

$$(X_{b_i} \longrightarrow Y_{b_i} \longrightarrow Y^{et}_{b_i}) \vee (X_{b_i} \longrightarrow X_{b_i} \longrightarrow Y^{et}_{b_i})$$

.

[12] The construction is unique to this document, and is in no sense canonical

[13] See section 1.1.1

or

$$(Y_{b_i} \longrightarrow X_{b_i} \longrightarrow X^{et}_{b_i}) \vee (Y_{b_i} \longrightarrow Y^{et}_{b_i} \longrightarrow X^{et}_{b_i})$$

Proof. This follows from the diagram of fibers in [section 1.1.1]. These are the four possible paths in the preimage of a nervous realization. We shall call our choice of sequence $Path(fib(b,i))$. □

Remark 3.1. *It is a good question as to which category these sequences shall live in. I am fond of SH_{Fin_*}, where Fin_* denotes the category of finite pointed sets.*

3.2 Perverse sheaves over $\mathfrak{D}^m$

Proposition 3.2. *Let $\Sigma_{g,n}$ be a manifold with genus g and crossing number n. Let T be a torsion theory. Then, $T \circ \Sigma_{g,n} \simeq Path(fib(b,i))$.*

Alternatively, we can give a description in terms of the underlying motive by which the manifold is generated:

Proposition 3.3. *For a motive $Mot_{g,n}$ consisting of the ring $\mathbb{G}_m \sqcup \mathbb{N}$, there is a unique left action sending the germs of $\mathbb{G}_m$ to a perverse sheaf over $\mathbb{N}$.*

A proof of the above proposition would involve showing that for all perverse sheaves over $\mathbb{N}$ (denote by $Perv_{\mathbb{N}}$), the homeomorphism

$$Perv_{\mathbb{N}} \mapsto Perv'_{\mathbb{N}}$$

is the only one in $Aut(\mathbb{N})$, and is trivial. A sketch of this proof would likely involve some telescopic descent condition imposed over the original perverse sheaf which reduces $Perv_{\mathbb{N}}/\cong$ to $spt(Perv_{\mathbb{N}})$.

Proposition 3.4. *Let $\Sigma_{g,n}$ be as above, and hyperkaehler. Then, there is a unique pyknotic object lying in $Perv_{\mathbb{N}}$ whose image under any map is the identity on $\Sigma_{g,n}$*

Proof. Since $Sigma_{g,n}$, this means that the dimension of the space becomes $4n$. Allowing each dimension to serve as a basis vector for a complex copy $\mathbb{P} \times \mathbb{A}^1$, we obtain a representative $\wp \in \mathbb{P}$ such that $\wp \cdot \pi_n(\mathbb{A}^1)$ is reduced to a rational point at ∞.

This point is then the crepant resolution for our manifold, which is unique. □

The above proof rests upon a hitherto uninvestigated correspondence between the pyknoticity of [17], the exodromy of [18], and the theory of Hyperkaehler manifolds. Essentially, for any Hyperkaehler manifold $\mathcal{H}$, one can assign a Serre fibration:

$$\mathcal{H}^d \longrightarrow \mathcal{H}^{d-2}$$

which is represented by a contraction of the mapping class group to a point:

$$MCG(\mathcal{H}) \rightsquigarrow *$$

3.2.1 Physical interpretation

Conceptually, the above map may be conjectured as the equivariant (with respect to the fiber spectrum group $\mathfrak{G}_{spt}$) instantiation of a hypercharged parton (say, a gaugino) on a real $\mathcal{A}d\mathcal{S}_5$-brane.

The brane in this specific case is:

$$\mathcal{B}^{d,d-2} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times SU(2)$$

$$= sep_* \ltimes \mathbb{L}^4$$

Where sep_* denotes the category of separated, pointed spaces. However, we may substitute $SU(2)$ for an finite, simple Lie group and obtain the same effects, up to a wrapping of the world-brane of the parton around a gravitationally significant neighborhood of appreciable weight. This gives us the canonical Sati-Schreiber tadpole cancellation, $\not{T}$:[14]

$$\pi_n(Hom(*, \mathscr{A}) \longleftarrow \Omega\pi_n(\mathcal{H}) \longrightarrow \pi_n(*)$$

The set of all blow-ups of $*$ is given by the frequency spectrum of a real Hermitian quantum embedded flatly in ordinary Euclidean space. This is represented in our actual world as a coherency class of quasi-quanta being transformed into unlocked work; i.e., outwards projective, directed currents of energy. In steady-state terms, the Liousville current of a contracted body is “effective enough” for the body to either expand or radiate thermal energy.

Remark 3.2. *The cancellation of work locking may be written*

$$\diamond\Box \longrightarrow (E\Box - \frac{1}{n}Ek(*))$$

where $Ek(*)$ is the energy required to lower a representative point on the body by a single degree Kelvin:

$$Ek(*) = \frac{\partial^n Lv}{\partial^{n-m}t}$$

where Lv is the Liousville current of the atom containing the point.

The map $Ek(*) \xrightarrow{+2d} Ek(\mathcal{U}(*))$ is given for the transition of the electromagnetic field in $+2$ dimensions. This represents the inflation of a pyknotic $\delta_\emptyset$-small object in the category $\mathscr{A}$ of absolute frames to a 2-dimensional conformal slice of an $\mathcal{A}d\mathcal{S}_5 \times \pi_1(U(1))$-brane.

Axiom 3.1. *The map out of a $\delta_\emptyset$-small object is the creation map of a fiber spectrum:*

$$\delta_\emptyset \mapsto \delta_i = Cr_i(X_{b_i})$$

which is the Betti realization of some $\delta_0 \not\approx \delta_\emptyset$.

[14] See [19] for the full scoop. Also [20], in a paper describing the tadpole cancellation as a nonabelian anomaly on a probe, the wrapping gets discussed.

Remark 3.3. *There is something to be said for including $\delta_\emptyset$-small objects in Pur. Firstly, note that a projection from Pur does not necessarily preserve the homotopy type of the subclass of Pur from which it is derived. In fact, this subclass itself may not even have a homotopy type. In that case, the homotopy type is instantaneously generated by the transformation of $\delta_\emptyset$ along a motive. This describes the case of unfaithful maps.*

3.3 Shadows of dendrites

For any dendrite $\mathfrak{D}$, construct a map

$$\mathfrak{D} \mapsto \mathfrak{shad}$$

the symmetric monoidal category of shadows of [22], consisting of $\mathcal{N}^1$-homotopies of perverse sheaves. Explicitly:

$$\pi_1^{ad}(\Pi_\infty(\mathcal{D}) \supset \mathfrak{shad}) = \mathfrak{shad}$$
$$= \mathcal{N}^1(\mathfrak{D}^{eff})$$

gives the *shadow nerve* of a brane. This is to be thought of as a Koszul dual for the pre-triangulated category Υ of pure motives over unramified groups.

A direct computation of the homotopy type of Υ would be impossible, but one can estimate its homology class by observing log Fano varieties in the projective quiver which are crepant resolutions of $\mathscr{O}_X$.

$$\begin{tikzcd} \infty \arrow[r] & ... \arrow[r] & F/\mathbb{R} \arrow[r] \arrow[rr, bend left] & F/\mathbb{Q} \arrow[r] & F/\mathbb{Z} \arrow[r] & ... \arrow[r] & 0 \end{tikzcd}$$

In the above diagram, each set contains at least one correlation between a crepant resolution and the spectrum of primes. We obtain one map

$$Map(F/\mathfrak{K}^\circ, \mathfrak{K}_+) \longmapsto \mathfrak{shad}$$

for every ring considered,[15] which is associative up to all higher homotopies.

Proposition 3.5. $\mathfrak{shad}^+ \sqcup \mathfrak{shad}^-$ *is an H-space.*

Proof. Since the map from the inverse fiber spectrum of every object in $\mathfrak{shad}$ is associative up to all higher homotopies, all we need now to prove that this is an H-space is to show that commutativity holds. Let s_t^1 be the Tate circle. Let there be a map out of the fiber spectrum of every point in s_t^1 onto an equivariant system of modules

$$\mathfrak{D}^1_{mod} \otimes \mathfrak{D}^2_{mod} \times ... \times \mathfrak{D}^m_{mod}$$

[15] This is effectively an abelizanization.

Then, let $\Pi_\infty(\mathfrak{D}^n_{mod})$ be a topological space for all $n < m$. Then, there is a diagram

$$\begin{tikzcd}
\mathfrak{D}^1_{mod} \arrow[r] \arrow[d] & ... \arrow[r] & \mathfrak{D}^n_{mod} \\
\phi_0 \arrow[r] & ... \arrow[r] & \phi_n \arrow[u]
\end{tikzcd}$$

which is essentially Cartesian. The above diagram commutes, so we have shown that the superalgebra of $\mathfrak{shad}$ generates only representative associative and commutative spaces up to all higher homotopies.

Thus,

$$H_n(\mathfrak{shad}) = H_p(\mathfrak{shad}^+) + H_q(\mathfrak{shad}^-)$$

$\square$

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