Method of listing finite and infinite decimals simultaneously: The simplest proof of countability of real numbers

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Abstract: The finite and infinite decimals of binary are listed one by one at the same time, thus completely proving that the real numbers are countable. This paper will completely rewrite the history of mathematics. Because there are lots of errors in Cantor's theory, and the uncountability of real numbers is only one of them, it is necessary to launch a campaign to crack down on false and correct errors, lest these errors continue to destroy the normal capacity of human thinking and continue to mislead people.

Key words: real number; Power sets; Measure theory; Diagonal Argument; Transfinite number; Continuum hypothesis

The uncountability of real numbers is the basis of modern mathematics, has important applications in measure theory, and is the basis of the so-called theory of transfinite number [1]; and the continuum hypothesis.

Is this a sound foundation? It needs to be examined.

In my previous work [2], Appendix, it was pointed out that proofs such as Diagonal Argument [3] introduce unproven assumptions and therefore cannot be held. This article will show a more powerful way to prove that real numbers are countable: list both finite and infinite decimals.

As is well known, for the set of natural numbers N= {1,2,3...}, the elements of its power set, i.e., P(N) are:

{} ,{1},{2},{1,2},{3},{1,3},{2,3},{1,2,3},{4},{1,4},{2,4},{3,4},{1,2,4}, {1,3,4}, {2,3,4}, {1,2,3,4}， ……， {2,3,4,5……}， {1,2,3,4……} （1）

In the above arrangement, there are 2 subsets when the value of natural number in the sets <=1, 4 subsets when <=2, and 8 subsets when <=3, ...

If the natural number of (1) is specified as the position of "1" after the decimal point, for example, {} means 0.000…, {1} means 0.1, {2} means 0.01, {1, 2, 3} means 0.111, {1,3} means 0.101, N means 0.111...and so on, then the binary decimal numbers corresponding to (1) are:

0,0.1,0.01,0.11,0.001,0.101,0.011,0.111,……, 0.0111…,0.111… （2）

Equation (2) lists not only the finite decimals of binary, but also the infinite decimals of binary (see the last few decimals).

Since the definition of a power set is all subsets of N, i.e., there cannot be a subset of N that is not in P(N), that is, there cannot be a decimal that is not in (2). Thus, (2) has actually listed any finite and infinite decimals.

However, in eq. (2), because there is an ellipsis in the middle, any infinite decimal cannot be numbered by natural number, so it cannot be convincingly proved that the decimals must correspond to the natural numbers one by one. Therefore, we can get the elements of the set $P^*(N)$ by subtracting the subsets of (1) from N one by one: N- {} = {1,2,3...}, N- {1} = {2,3......}, ..., N - {2, 3, 4...} = {1}, N-N= {}, thus, we obtain its elements:

{1,2,3…}, {2,3,4…}, {1,3,4…}, {3,4,5…}, {1,2,4, 5…}, {2,4,5,6….} …….{1}，{} （3）

It is not difficult to find that the first term of eq. (3) is exactly the last term of eq. (1), and the second term of eq. (3) is exactly the penultimate term of eq. (1) …… Thus, the elements of $P^*(N)$ simply reverse the order of the elements of P(N).

In fact, since every element of (3) is also a subset of N, and (3) can correspond to (1) one by one, it is easy to show that $P^*(N)$ and P(N) are just the same set written differently, that is, $P^*(N)$= P(N), and therefore their union

$P(N)UP^*(N) = P(N) = P^*(N)$， （4）

But the form changes: the elements of $P(N)UP^*(N)$ can be arranged as:

{},{1,2,3……},{ 1},{2,3,4……},{2},{1,3,4……},{1,2},{3,4,5……},{3},{1,2,4,5,….}……. （5）

The decimals that correspond one to one with (5) are

0, 0.111…, 0.1, 0.0111…, 0.01, 0.10111…, 0.11, 0.00111…, 0.001, 0.110111…, …… （6）

The last elements of (1) and (3) do not appear in (5) and (6) because the duplicate elements have been removed.

In this way, we have listed both finite and infinite decimal numbers in binary one by one, and we still cannot find any binary decimal that is not in (6) according to eq. (4): If there is a decimal that is not listed, then the corresponding subset of that decimal will not exist, contradicting the definition of a power set.

Since the decimals can be listed one by one, of course, the natural numbers can be used to number the decimals one by one, that is, it proves that the decimals can correspond one by one to the natural numbers.

Since it is impossible to find any binary decimal that is not listed in eq. (6), any "proof" (such as Diagonal Argument) that attempts to find a decimal that is not listed is obviously inconsistent with the facts, and can only serve as an example of poor thinking.

In other words, the so-called uncountability of real numbers has become a historical joke caused by poor thinking.

Anyone who is familiar with the history of mathematics will know what this short passage means to the history of mathematics. In fact, the uncountability of real numbers and its proof method, the Diagonal Argument, not only have a great impact on the history of mathematics, but also on the history of philosophy and even logic. Therefore, once the uncountability of real numbers is overturned, it should be a big earthquake to the academic circle.

Clearly, this paper will completely rewrite the history of mathematics.

Because there are a lot of errors in Cantor's theory [2], and the uncountability of real numbers is only one of them, it is necessary to launch a campaign to crack down on false and correct errors, lest these errors continue to destroy the normal capacity of human thinking and continue to mislead people.

Literature:

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Appendix: Errors in the Diagonal Argument

Before a strict proof that the real numbers are uncountable, there is no reason why the real numbers cannot be listed one by one, so the real numbers may be listed one by one:

$a_1=0.a_{11}a_{12}a_{13}....$

$a_2=0.a_{21}a_{22}a_{23}....$ (A1)

$a_3=0.a_{31}a_{32}a_{33}....$

....

The subscripts on the right side of the equals sign form an infinite matrix.

In the Diagonal Argument, Cantor let.

$b=0.b_1b_2b_3...$， (A2)

Were,

$b_k \neq a_{kk}$, （k=1,2,3,...) (A3)

The value of *b* seems to be different from any of the decimals listed in (A1), and thus seems to contradict the fact that (A1) has already listed the decimals one by one, and Cantor thus argued that it is not possible to list the real numbers one by one, thus establishing the so-called uncountable theory which has been so influential in the history of mathematics.

However, it is not difficult to see that since the same *k* is used to represent the both row and column notations of the matrix in (A3), this fact shows that the Diagonal Argument is made under the assumption that the number of rows is strictly equal to that of decimal columns, which is hereinafter referred to as the equality assumption for ease of narration.

However, no one has proved the equality assumption. This fact makes Diagonal Argument without any general significance, and it is not surprising that the resulting contradiction, which has no necessary relationship with the countability or uncountability.

It is not difficult to see that the number of rows in (A1) represents the number of decimals listed, and the number of columns represents the number of places behind the decimal point, so an extra row (*b*) simply means that there are more the decimals than the places. Is there any reason why the number of the places and the number of decimals must be exactly equal?

In fact, it can be seen from eq. (1), that if N is used to represent the number of places, the decimals can correspond to P(N) one by one, that is, the number of decimals is much more than the number of places. The equality assumption is not true at all. What's so weird about an extra *b*? Thus, the Diagonal Argument proves nothing.

Is it any wonder that in any derivation, the introduction of fundamentally incorrect assumptions leads to contradictions? Can these contradictions prove anything? It's just a big joke

It is regrettable that such an obvious logical error should have gone undetected for a long time and that the so-called Diagonal Argument should be regarded as a classic.

The history of mathematics has thus taken a large and unnecessary detour, which shows how serious the consequences of careless thinking can be.