Background-free Relativity

Barbara Spangenberg

Abstract

Starting at the volume element of four-dimensional space-time and the determinant of the metric tensor as Einstein did in his derivation of his field equations in 1915, we want to try here to tread a new path to derive field equations for gravity. Here, in contrast to Einstein's work in 1915, the space *itself* is treated as the field of gravity, not its curvature as in general relativity. The newly derived field equations become astonishingly simple and *comprise* the well-known solutions within solar systems. However, they lead to an increased gravity for galactic systems.

Start at curved space, not at special relativity

In 1915, Albert Einstein derived his famous field equations for gravity.[1] He did that using only the laws of derivation of covariants and differential geometry. Grounded in his special relativity, he did not use any assumption on the coordinate system in which we have to calculate the field of gravity. Here, in contrast, we want to a certain amount *ignore* the insights of special relativity for the moment, especially the difficulties with the definition of simultaneousness. Instead, we want to start right at the precursor of general relativity, which is the invariance of the four-dimensional volume element and the determinant of the metric tensor. We believe that the insights of special relativity are possible to be included again later on in our Background-free relativity using coordinate transformations and energy considerations; however, that is not within the scope of this manuscript.

Since Einstein's theory of gravity, in contrast to the main precursor theory of gravity, namely Newton's, the **space-time is encountered to be curved**. It can be described as a four-dimensional space-time manifold. The curvature of space-time is described by the **symmetric metric tensor** at every point in space and time:

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix} \tag{1}$$

The metric tensor $g_{\mu\nu}$ characterizes the field of gravity. The gravitational field at every point in space and time is fully described by the metric tensor at this point. The gravitational field at a point is coordinate independent. If we decide for certain coordinates, the metric tensor gets components in dependence of these coordinates.

Now, we want to analyze this gravitational field and as a start **decide for a not-moving coordinate system**. This is a restriction, which is not done in general relativity. We again believe that this restriction may be eased later on using coordinate transformations. Using a not-moving coordinate system leads the mixed components of space and time, g_{0i} $(i \neq 0)$, to become zero:

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & g_{12} & g_{13} \\ 0 & g_{12} & g_{22} & g_{23} \\ 0 & g_{13} & g_{23} & g_{33} \end{pmatrix} \tag{2}$$

What have we gained here? (2) is a very simple description of curved space-time. **At every point**,

- this metric tensor may possess different values.
- the time flows with a certain "velocity", characterized by g_{00}.
- the spatial part g_{ij} with $i, j \in [1,3]$, may be arbitrarily (but differentiable) curved.

If we compare this description with the known solutions of general relativity, we ***may*** encounter singularities here. Those singularities may be possibly made to vanish with a choice of a moving coordinate system – however, at this stage we rather want to ignore some parts of the space-time than giving up the not-moving coordinate system. Therefore, if there is a singularity we will call the space-time "behind" the singularity simply ***not defined*** for the moment. There are metrics in general relativity, for example the Kerr-metric, which even do not allow for a transformation to (2) for expanded regions. These parts of the manifold within the event horizon of a black hole or the impossibility to be brought to the form (2) remain as ***not defined*** in background-free relativity (BFR) so far.

One important point of the model introduced with equation (2) is to assign a velocity of time g_{00} and a curvature of space g_{ij} to every point in space and time. If an observer is located at a certain point and is not moving with regard to the not-moving coordinate system, for them the time will run with the velocity characterized by g_{00} at this point. Allowing one marginalia regarding special relativity: If this observer moves with a certain velocity at this point, his flow of time will run slower, never faster, than characterized by g_{00} at this point.

This "not moving coordinate system" may look like being in a contradiction to what we know about relativity. It is at least another view of relativity than that of special relativity which strictly avoids the decision for any coordinate system. In background-free relativity (BFR), every defined point in space (and time) is characterized by that metric in (2). Importantly, **time runs differently fast at different points.** That is the relativistic principle in BFR.

The assumption of a "not moving coordinate system" is in contradiction to the difficulties in describing simultaneous events in special relativity. With the assumption of the "not moving coordinate system", we assume that it is possible to "freeze" all objects simultaneously in the universe. Kinetic energy therein simply adds to the total energy of an object. In the universe of BFR, there are only lots of mass points with different mass-energy. In BFR, we assume that it is possible to "freeze" all those mass points at least mathematically at once for a moment to assign to every point in space a metric tensor of the form of equation (2). If g_{0i} of the metric tensor in equation (2) would *not* be zero, an observer would *have* to move at this point relative to the coordinate system. They could not be "frozen". We however need the observer to have the ability to "freeze" for the time of the measurement. It is similar to calculating the slope of a curve f(x): f(x) is ever changing, nevertheless we can calculate f'(x) at a "singular" point.

The postulate of a constant four-dimensional volume element

From (2), the determinant of the metric tensor may be calculated using the Laplace expansion:

$$|g_{\mu\nu}| = \left|\begin{pmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & g_{12} & g_{13} \\ 0 & g_{12} & g_{22} & g_{23} \\ 0 & g_{13} & g_{23} & g_{33} \end{pmatrix}\right| = g_{00} \cdot \left|\begin{pmatrix} g_{13} & g_{13} & g_{13} \\ g_{12} & g_{22} & g_{23} \\ g_{13} & g_{23} & g_{33} \end{pmatrix}\right| = g_{00} \cdot |g_{ij}| \tag{3}$$

The determinant of the metric tensor, $|g_{\mu\nu}|$, is a tensor density of weight 2. That means if we do a coordinate substitution $x \rightarrow x'$, the determinant of the metric tensor changes according to

$$|g_{x'}| = \left|det \frac{\partial x}{\partial x'}\right|^2 |g_x| \tag{4}$$

Now, we **postulate $|g_{\mu\nu}|$ to be constant** all over the defined manifold when $\left|det \frac{\partial x}{\partial x'}\right|^2$ is equal to one:

$$|g_{\mu\nu}| = g_{00} \cdot |g_{ij}| \stackrel{\text{def}}{=} const. \text{ for } \left|det \frac{\partial x}{\partial x'}\right|^2 = 1 \tag{5}$$

$$|g_{\mu'\nu'}| = g_{00} \cdot |g_{i'j'}| \stackrel{\text{def}}{=} \left|det \frac{\partial x}{\partial x'}\right|^2 const. \tag{6}$$

Leaving aside changes of coordinate density, which would be introduced with $\left|det \frac{\partial x}{\partial x'}\right|^2 \neq 0$, the determinant of the metric tensor shall be constant all over the manifold. That means if at a certain point P in space and time the time runs slower than at another point Q, the determinant of the spatial part of the metric tensor $|g_{ij}|$ has to become larger at Q than at P. At least qualitatively, this is what we know from general relativity: time runs slower near a massive object (time dilation) where more volume fits in (length contraction).

Minkowski space-time can be described for example using Cartesian coordinates or spherical coordinates. While in Cartesian coordinates, the "coordinate density" is the same everywhere, in spherical coordinates the coordinate density is highly dependent on the position. As a manifold, the four-dimensional space-time is locally Euclidean at every point. The Minkowski space-time in Cartesian coordinates therefore is an approximation at every point and ideal to calculate *which* constant value we obtain with the above postulate. The Minkowski metric is

$$x^2 + y^2 + z^2 - c^2t^2 \tag{7}$$

Here, the determinant of the metric is

$$|g_{\mu\nu}| = -c^2 \tag{8}$$

which we **postulate to hold for every metric** with a constant coordinate density (as x,y,z,t) and which leads to the equation

$$|g_{\mu'\nu'}| = -c^2 \left|det \frac{\partial x}{\partial x'}\right|^2 \tag{9}$$

If the coordinate density is not constant all over the manifold. Using (9) in (5) and (6), we gain:

$$g_{00} \cdot |g_{ij}| = -c^2 \tag{10}$$

For coordinates with constant coordinate density like (x,y,z,t), and

$$g_{00} \cdot |g_{i'j'}| \left|det \frac{\partial x}{\partial x'}\right|^{-2} = -c^2 \tag{11}$$

for coordinates without constant coordinate density like spherical or axial coordinates. Case (11) takes place for coordinates of which the Jacobian of the substitution from Cartesian is not constant. Equation (10), or, in general coordinates, equation (11), is the **first field equation of background-free relativity.**

Of note, g_{00} and $|g_{ij}|$ both are scalars. They are two scalars, which are connected through the first field equation of background-free relativity. Every point in space and time is characterized through one of those scalars. Gravity is described using one of those two scalars at every point.

Constraining the metric determinant to a constant is known as unimodular gravity. It's been explored extensively in the literature, see for example Álvarez et al.[3]

The second field equation of background-free relativity

From equation (10) or (11), we want to go further to derive a ***set*** of new field equations for gravity. The field of gravity around a spherical symmetric mass (point mass) needs to be spherically symmetric as well. A field of something is supposed to vanish in infinity. As the product of the velocity of time g_{00} and the spatial determinant $|g_{ij}|$ is constant, the only possible change is increase of one factor while the other decreases. As we know from General Relativity, $|g_{ij}|$ near the masses increases, while $|g_{00}|$ decreases. In contrast to "mass *curves* space-time" of General Relativity we want to try out a new path: Namely that of "mass is the *source* of the space itself". This means we regard space itself as a field, which is the surrounding field of every mass. To describe a classical scalar field with its sources, in general the Poisson equation is applicable.

$$\text{div grad}\,|g_{ij}| = -4\pi G\rho(\boldsymbol{r}) \tag{12}$$

If coordinates without constant density are used, the addendum as from equation (10) to (11) has to be done. Using equation (4) in (12), the result is:

$$\text{div grad}\,|g_{i'j'}|\left|det\frac{\partial x}{\partial x'}\right|^{-2} = -4\pi G\rho(\boldsymbol{r}) \tag{13}$$

Therein, ρ is the mass density and G is the gravitational constant. Equation (12) looks very similar to the classical Poisson equation of the gravitational field $\boldsymbol{g}$: $\text{div grad}\,\boldsymbol{g} = -4\pi G\rho(\boldsymbol{r})$. However, there is a huge difference: $|g_{ij}|$ is not the gravitational field $\boldsymbol{g}$, but the volume of space itself. $|g_{ij}|$ characterizes the density of space at every point. Using the gravitational field $\boldsymbol{g}$, space and time are the stable background ***on which*** the gravitational field takes effect – using instead the volume of space $|g_{ij}|$, the ***space itself*** is curved and bound to vanish in infinity.

Equation (12) for coordinate systems with unique density and equation (13) otherwise is the **second field equation of background-free relativity.**

The solution of equation (12) is well-known. For a known mass distribution it's

$$|g_{ij}| = -G\int\rho(\boldsymbol{r})\frac{r-r'}{|r-r'|^3}dx'dy'dz' \tag{14}$$

And for a single mass point of mass M in spherical coordinates we use equation (13) and get

$$|g_{i'j'}|\left|det\frac{\partial x}{\partial x'}\right|^{-2} = -\frac{GM}{r} \tag{15}$$

To derive the metric tensor, which belongs to the solution of equation (15), we need to take into account that we're using spherical coordinates. The Jacobi determinant to change from Cartesian to spherical coordinates is $r^2\sin\theta$. We need to use equation (11) to derive g_{00}.

$$g_{\mu\nu} = \begin{pmatrix} -c^2\frac{r}{GM} & 0 & 0 & 0 \\ 0 & \frac{GM}{r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta \end{pmatrix} \tag{16}$$

We hypothesize without the attempt to show here that this metric tensor leads to gravitational forces that are similar to those of Mordehai Milgrom's modified Newtonian dynamics (MOND).

In the framework of background-free relativity, the metric (16) is the solution of the curved space of a mass point without any background field. If only one mass point were in the whole universe, this would be its gravitational field. Galactic systems do not encounter much influence of even larger, bigger, more compact objects. Therefore, we hypothesize that galactic systems can be regarded as such an isolated object in first approximation.

The summation rule for densities

In background-free relativity, the density of space $|g_{ij}|$ is a scalar field. As a density, it follows a simple summation rule. Densities at every point sum up. The velocity of time then follows an inverse summation rule, as equation (10) or (11) has to be followed.

Solar systems as our own reside within the influence of a much larger and more compact object: the galactic center with black holes and the mass of several millions of sun masses. The gravitational field of the galactic center is enormous in comparison to the extension of our solar system. Its slope is negligible on the extension of our solar system and that leads to a flat background for our solar system. The flat background $|g_{ij}| = 1$ has to be added to the solution of a point mass. The result is

$$g_{\mu\nu} = \begin{pmatrix} -c^2 \frac{1}{1+\frac{GM}{r}} & 0 & 0 & 0 \\ 0 & \left(1+\frac{GM}{r}\right) & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta \end{pmatrix} \tag{17}$$

where g_{00} has been calculated using the inverse summation rule and equation (11).

The difference between $g_{11} = \left(1+\frac{GM}{r}\right)$ here (BFR) and $g_{11} = \left(\frac{1}{1-\frac{GM}{r}}\right)$ of the Schwarzschildmetric (SSM) is negligible. The first are the first two terms of the infinite geometric series for which the latter is the sum. The difference of {the difference of g_{11} at the orbit (36000 km) around earth (radius 6371 km) and the surface of the earth (rs = 9 mm)} between BFR and SSM is only at the 12th position after decimal point, which is presumably not detectable by now.

Remarks

The best way to falsify or verify background-free relativity will be to model galactic rotation curves and the movement of galaxies in galaxy clusters. There, the deviation of background-free relativity from general relativity is the most prominent (see equation (16)).

Birkhoff's theorem is often cited that demonstrates that every spherical symmetric solution of Einstein's field equations is identical to the exterior SSM. It needs to be emphasized here that the premise of Birkhoff's theorem is the validity of Einstein's field equations. However, in BFR, Einstein's field equations of general relativity are only valid in a special case: If there is a constant flat background space (that can be "curved" by masses). Otherwise not. We start at the assumption of curved space *without* Einstein's field equations. Then, we postulate the constancy of the four-dimensional volume element, which is the first field equation of BFR. After that, we transform the empirically derived hypothesis "mass is the source of space" into mathematics as it is done for other fields and their sources in physics using the Poisson equation, which leads to the second field equation of BFR. Neither Einstein's field equations nor Birkhoff's theorem are valid here which allows us for the derivation of a metric tensor that is different to the Schwarzschild solution (see equation (16)) and will lead to gravitational forces that are stronger than Newton's for galaxies.

Special relativity needs to be included again in the framework of BFR. We suggest that this may be done using the kinetic energy of a moving object. The kinetic energy may possibly be regarded as part of the total mass-energy of the system and therefore may lead to a transformation of the spatial field at the place of the moving object. Further research is needed to fully elaborate that.

With background-free relativity and $|g_{ij}|$ approaching zero between the galaxies, the empty space between the galaxies would be thin space with fast-running time. Light would undergo a gravitational redshift when it moves *into* these regions and a gravitational blueshift while it moves *out* of these regions. The postulation of $|g_{ij}| \to 0$, and, equivalently, the postulation that the "space surrounds each massive object like its own field" leads to the hypothesis that the amount of space is conserved. While the universe is expanding and most galaxies in it are moving apart, the space between the galaxies would become thinner. Therefore, the light would experience a net gravitational blueshift while moving through the expanding universe with regions of thin space. This net gravitational blueshift would be the greater the longer the distance the light traveled before reaching the observer. Dark energy has been postulated because distant galaxies appear to be less redshifted than near ones. The net gravitational blueshift gained from $|g_{ij}| \to 0$ could possibly become an alternative explanation for the decreased redshift of the distant stars.

In 2020, Migkas et al.[2] presented a study in which the isotropy of dark energy has been questioned. That leads to the following possibility of a validation (or falsification) of the hypothesis of background-free relativity. With the postulation of $|g_{ij}| \to 0$ and dark energy being only a secondary effect of light traveling through the expanding universe, the value of the dark energy would be strongly dependent on the amount of empty space or, other the way round, strongly dependent on the amount of mass between the light source and the observer. Within the next decade, it should be possible to test whether the differences of redshift derivations, which are presently ascribed to dark energy, correlate with the amount of empty space or mass between the source and the observer. If light from two otherwise equivalent objects at the same distance experiences different redshifts, the light, which travels though more empty space would experience less redshift if the assumptions of background-free relativity hold.

Conclusion

Background-free relativity (BFR) has been introduced. For coordinates with constant density as (x,y,z,t), the two equations of BFR are $g_{00} \cdot |g_{ij}| = -c^2$ and $\mathrm{div\ grad}\ |g_{ij}| = -4\pi G\rho(\boldsymbol{r})$. Two dependent scalar fields characterize the gravitational field. BFR leads for solar systems to similar gravitational laws as known from general relativity. However, for galactic systems, the gravitation is stronger than that from general relativity. This difference possibly may account for the dark matter effect.

References

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