Fine Structure Constant
and Proton / Electron Mass Ratio

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December 8, 2023

Keywords: fine structure constant, proton radius, proton electron mass ratio, charge shell.

Abstract: In this note a model is put forward whereby the proton has mass and charge shell radii in the ratio 1:1.68. The fine structure constant is proportional to the thickness of the shell. Two new formulae for calculating α are introduced. Eq 5 and Eq 6 are the centrepiece of this note. These make use of the usual set of fundamental constants, including the proton / electron mass ratio. Eq 6 gives the same value for α as standard formulae. However, it is suggested that in an optimal physics, this method with reasonable confidence, gives a slightly lower value for α reliable to 12 decimal places, i.e., 0.007 297 352 566. Whether this is the actual fine structure constant depends on the veracity of the model and the accuracy of the proton /electron mass ratio.

Fundamental Constants: The NIST (2018) recommended values for the relevant constants are as follows.

$e\ = 1.602\,176\,634 \times 10^{-19}$ (exact) C.

$\mathrm{c}\ = 299\,792\,458$ (exact) m s^{-1}.

$h\ = 6.626\,070\,15 \times 10^{-34}$ (exact) J Hz^{-1}.

$\hbar\ = h/2\pi$ J Hz^{-1}.

$\varepsilon_0\ = 8.854\,187\,8128(13) \times 10^{-12}$ F m^{-1}.

$$
\begin{aligned}
\tfrac{m_p}{m_e} &= 1836.152\,673\,43(11) \\
\alpha &= 0.007\,297\,352\,5693(11).
\end{aligned}
$$

As three of the constants are now exact this means from the point of view of the standard formula for α, the uncertainty is due to the electric constant ε_0. I.E.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}. \tag{1}$$

If we plug the above values into Eq 1, the resulting output value is closer to $0.007\,297\,352\,569\,278$ (see Eq 6).

Model: The proton's mass is treated as a 3-sphere, such that r_p is the mass radius. The number $r_p - 2$ is either equal or very close to $(m_n - m_p)/m_e$. On this view, echoing Heisenberg, the neutron and the proton are different states of the same particle. The neutron's mass is equal or very close to the volume delimited by the proton's outer charge radius L_2. $L_2 - L_1$ is interpreted as the thickness of the charge shell. The proton therefore has three radii, with the shell's L radii easily mistaken for a single radius. This gives possible theoretical support for a recently reported charge radius larger than the mass radius (Duran et al., 2023). The r_p/L_2 ratio of 1:1.68 is a fit for Duran et al.

Method: To derive α from our model we need to know the values of three radii based on the proton / electron mass ratio. I.E.

$$r_p = \sqrt[3]{\frac{m_p}{2\pi^2 m_e}} \tag{2}$$

$$L_1 = \sqrt[3]{\frac{3}{4\pi}\frac{m_p}{m_e}} \tag{3}$$

$$L_2 = \sqrt[3]{\frac{3}{4\pi}\left(\frac{m_p}{m_e} + r_p - 2\right)} \tag{4}$$

The radii and the fundamental constants combine to form the following equivalence.

$$\frac{3e^2}{4\pi\varepsilon_0 hcL_2 - 4\pi\varepsilon_0 hcL_1} + \frac{5e^2}{18\pi\varepsilon_0 hc} - 1 = \Delta. \tag{5}$$

Plugging NIST (2018) values into Eq 5, $\Delta \approx 4.031\,435 \times 10^{-10}$. Missing out a few trivial steps, from Eq 5 we get Eq 6.

$$\alpha = \frac{2\pi}{\left(\frac{3}{L_2 - L_1}\right) + \frac{10}{9}} \times (1 + \Delta). \tag{6}$$

Due to Eq 5, Eq 1 = Eq 6. However, an obvious speculation (preferred here), is to think that in an optimal physics Δ ought to be zero. E.G.

$$\begin{aligned} \frac{m_p}{m_e} &= 1836.152\,673\,43 \\ \Delta &= 0, \\ \alpha &= 0.007\,297\,352\,566\,336. \end{aligned} \tag{7}$$

This last value for α agrees with the NIST (2014) recommendation. As long as future adjustments to m_p/m_e are within five standard deviations of the present NIST (2018) recommended value, and $\Delta = 0$, then the number derived at (7) is accurate to 12 decimal places, with only the final three decimals in doubt. Whether this is the actual fine structure constant to 12 decimals rests on the veracity of the three radii model, and the accuracy of the present proton / electron mass ratio.

References

Duran, B., Meziani, Z.-E., Joosten, S., Jones, M., Prasad, S., Peng, C., ... others (2023). Determining the gluonic gravitational form factors of the proton. *Nature*, *615*(7954), 813–816.