THE ARROW OF TIME
AND THE WHEELER-FEYNMAN TIME-SYMMETRIC THEORY

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Abstract: A method to constrain the Arrow of Time to the anterograde direction using the Wheeler-Feynman Time-Symmetric theory and four-vector potentials to model electrons as longitudinal electromagnetic scalar potential waves that only progress in the forward direction of Time.

§1 W.F.E.M.F.V.P.T.S.T.

In my previous paper *A Dynamical Theory of the Electromagnetic Potential* (O'Brien 2018), I suggested fermions can be modelled as longitudinal electromagnetic scalar potential waves. This required constructing the scalar potential across the Wheeler-Feynman time-symmetric theory with energy and momentum constraints leading to an electromagnetic wave that evolves longitudinally along the axis of time, this method is similar to Maxwell's transverse electromagnetic wave for light. It results in a particle model that has an exact value for the electron's reduced Compton wavelength that matches the observed wavelength of an electron in $\mathbb{R}^{1,3}$, a perfectly spherical charged electron in $\mathbb{R}^3$, and is intrinsically *on-mass shell.*

This was possible by rewriting the Wheeler-Feynman time-symmetric model from it's classic $\mathbf{E}$ notation with it's retarded (ret) and advanced (adv) fields,

$$\mathbf{E}_{total} + \mathbf{E}_{free} = \sum_n \tfrac{1}{2}\left(\mathbf{E}_n^{ret} + \mathbf{E}_n^{adv}\right) + \sum_n \tfrac{1}{2}\left(\mathbf{E}_n^{ret} - \mathbf{E}_n^{adv}\right) = \sum_n \mathbf{E}_n^{ret} \tag{1}$$

to four-vector potential form A_μ^{total} with ret and adv potentials instead of fields,

$$A_{\mu}^{total} = \sum_n \frac{1}{2}\left(A_{\mu,n}^{ret} + A_{\mu,n}^{adv}\right)_{total} + \sum_n \frac{1}{2}\left(A_{\mu,n}^{ret} - A_{\mu,n}^{adv}\right)_{free} \tag{2}$$

We can re-express this as a wavefunction with epoch angles ε, and where the ½ appears as a result of spitting the A_{μ} into two parts along the axis of Time into the anterograde direction and the retrograde direction,

$$\psi = \exp(-\tfrac{iq}{2\hbar}\int A_{\mu}dx^{\mu} + \varepsilon) \tag{3}$$

This turns the Wheeler-Feynman summation into a total wavefunction

$$\psi_{total} = \exp(-\tfrac{iq}{2\hbar}\int A_{\mu}^{\mathrm{ret}}\mathrm{dx}^{\mu} - \varepsilon_{\mathrm{ret}}) * \exp(-\tfrac{iq}{2\hbar}\int A_{\mu}^{\mathrm{adv}}\mathrm{dx}^{\mu} + \varepsilon_{\mathrm{adv}}) \tag{4}$$

To ensure constant phase the difference of the epoch angles ε is zero and is referred to as the interference,

$$\Delta\varepsilon = \varepsilon_{adv} - \varepsilon_{ret} = 0 \tag{5}$$

As a *mathematical trick* we can without loss of generality equate the interference to the sum of the advanced and retarded potentials of the Wheeler-Feynman free terms since they also sum to zero,

$$\varepsilon_{\mathrm{ret}} = -\tfrac{iq}{2\hbar}\int A_{\mu}^{\mathrm{adv}}\mathrm{dx}^{\mu} \quad \textit{and} \quad \varepsilon_{\mathrm{adv}} = -\tfrac{iq}{2\hbar}\int A_{\mu}^{\mathrm{ret}}\mathrm{dx}^{\mu} \tag{6}$$

The total wavefunction now becomes,

$$\psi_{total} = \exp(-\tfrac{iq}{2\hbar}\int A_{\mu}^{\mathrm{ret}} - A_{\mu}^{\mathrm{adv}}\mathrm{dx}^{\mu}) * \exp(-\tfrac{iq}{2\hbar}\int A_{\mu}^{\mathrm{adv}} + A_{\mu}^{\mathrm{ret}}\,\mathrm{dx}^{\mu}) \tag{7}$$

$$\psi_{total} = \exp(-\tfrac{iq}{2\hbar}\int A_{\mu}^{\mathrm{ret}} - A_{\mu}^{\mathrm{adv}} + A_{\mu}^{\mathrm{adv}} + A_{\mu}^{\mathrm{ret}}\,\mathrm{dx}^{\mu}) \tag{8}$$

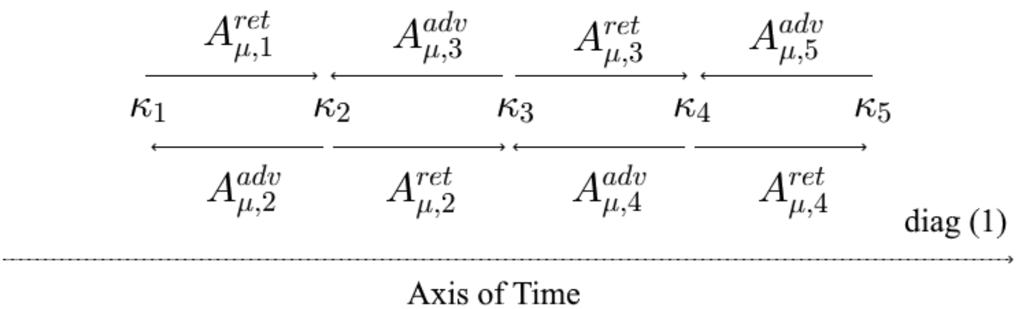
After simplifying the ½ vanishes giving a unitary total wavefunction Ψ_{total}

$$\Psi_{total} = \exp(-\tfrac{iq}{\hbar}\int A_{\mu}^{\mathrm{ret}}\,\mathrm{dx}^{\mu}) \tag{9}$$

§2 Arrow of Time

In this present paper it will be shown there can be no resultant A_μ from the Wheeler-Feynman summation *even for antimatter*, and the worldlines of all on-mass fermions must necessarily be in the forward direction of Time.

Consider diagram (1) where the sources of the four-vector potentials A_μ are a series of virtual charges κ_i separated by a distances Δx,



diag (1)

It can be seen each of the κ_i has incoming and outgoing potentials which match the premise in the Wheeler-Feynman model,

$$A_\mu^{total} = \sum_n \frac{1}{2}\left(A_{\mu,n}^{ret} + A_{\mu,n}^{adv}\right)_{total} + \sum_n \frac{1}{2}\left(A_{\mu,n}^{ret} - A_{\mu,n}^{adv}\right)_{free} \tag{10}$$

On taking the limit of the infinitesimal of $\Delta x \to 0$, where Δx is the distance between the particles, the charges approach simultaneous points in spacetime and the sum becomes a single entity, at which point the terms become indistinguishable and on dropping the summation over many potentials,

$$A_\mu^{total} = \lim_{\Delta x \to 0} \frac{1}{2}\left(A_\mu^{ret} + A_\mu^{adv} + A_\mu^{ret} - A_\mu^{adv}\right) \tag{11}$$

Then once more to preserve phase we require,

$$\Delta\varepsilon = \varepsilon_{adv} - \varepsilon_{ret} = 0 \tag{12}$$

Now we see why the retarded and advanced epoch angles are allowed to take the values of the advanced and retarded potentials as at that point the epoch angles of the advanced and retarded potentials are indistinguishable.

Since the sign of the epoch angles is important then equally we can write,

$$\Delta\varepsilon = \varepsilon_{adv} + \varepsilon_{ret} = 0 \tag{13}$$

$$\Delta\varepsilon = -\varepsilon_{adv} - \varepsilon_{ret} = 0 \tag{14}$$

However, in these two cases, (13) and (14), the interference of the phases are identically zero, thus these are excluded as that means the free potentials are also identically zero which entails a non-physical system.

We can also reverse of the order of the epochs in eqn (12),

$$\Delta\varepsilon = \varepsilon_{ret} - \varepsilon_{adv} = 0 \tag{15}$$

In this case (15) it requires *the states of the future determine the states of the past* and that possibility is excluded as being non-causal, *implying but not necessarily proving* that we cannot write,

$$\Psi_{total} = \exp(-\tfrac{iq}{\hbar} \int A_{\mu}^{\text{adv}}\, \text{dx}^{\mu}) \tag{16}$$

Since under eqn (12) the advanced total wavefunction vanishes, and since advanced wavefunctions are acasual – this suggests Causality as a *lesser condition* of the Wheeler-Feynman summation.

Finally, a *greater condition* can be given by considering the affect of Time Reversal on the components of the four-vector potential total A_{μ}^{total},

$$A_{\mu}^{total} = \lim_{\Delta x \to 0} \tfrac{1}{2}\left(A_{\mu}^{\text{ret}} - A_{\mu}^{\text{adv}} + A_{\mu}^{\text{adv}} + A_{\mu}^{\text{ret}}\right) = A_{\mu}^{\text{ret}} \tag{17}$$

The crucial step appears by noting that as the direction of Time reverses then the TCP theorem also inverts the components of the four-potentials, and again the Wheeler-Feynman summation returns a retarded potential instead of an advanced potential,

$$A_{\mu}^{total} = \lim_{\Delta x \to 0} \tfrac{1}{2}\left(-A_{\mu}^{\text{ret}} + A_{\mu}^{\text{adv}} - A_{\mu}^{\text{adv}} - A_{\mu}^{\text{ret}}\right) = -A_{\mu}^{\text{ret}} \tag{18}$$

This result is consistent with the positron model as being *the negative energy mode of an electron moving forward in Time,* where note the absence of a negative sign in the exponential of (19) determines this particle as antimatter,

$$\Psi_{total}(antimatter) = \exp(\tfrac{iq}{\hbar}\int A_{\mu}^{\mathrm{ret}}\,\mathrm{dx}^{\mu}) \tag{19}$$

The assumption that $-\left|A_{\mu}^{adv}\right| = \left|A_{\mu}^{ret}\right|$ is justified by seeing that reversing the sense of the advance potential is equal to the sense of the retarded potential, and the conservation of energy requires the individual elements of A_{μ}^{adv} must necessarily equal the elements of A_{μ}^{ret}, all of which is necessary for the conservation of phase between the ψ_{ret} and the ψ_{adv} wavefunctions.

We can simplify the above process by reiterating the identities from the previous paper [O'Brien 2018],

$$\begin{aligned}
|\psi_{\mathrm{ret}}| &= |\psi_{\mathrm{adv}}| \\
\psi_{\mathrm{ret}} &= \psi_{\mathrm{adv}}^{*} \\
\psi_{\mathrm{adv}} &= \psi_{\mathrm{ret}}^{*} \\
\psi_{\mathrm{ret}} \cdot \psi_{\mathrm{ret}}^{*} &= I \\
\psi_{\mathrm{total}} &= \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{adv}} \\
\psi_{\mathrm{interference}} &= \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{adv}}^{*} = I
\end{aligned} \tag{20}$$

Giving the W.F.E.M.F.V.P.T.S.T. in its most elegant form, this is only possible if and only if $\psi_{ret} = \psi_{adv}^{*}$, and if and only if the magnitudes $|\psi_{\mathrm{ret}}| = |\psi_{\mathrm{adv}}|$ are identical, then and only then does this formulation work for wavefunctions.

$$\begin{aligned}
\Psi_{\mathrm{total}} &= \psi_{\mathrm{total}} \cdot \psi_{\mathrm{interference}} \\
&= \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{adv}} \cdot \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{adv}}^{*} \\
&= \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{ret}}^{*} \cdot \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{ret}}^{**} \\
&= \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{ret}}^{*} \cdot \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{ret}} \\
&= \psi_{\mathrm{ret}} \cdot \psi_{\mathrm{ret}} \\
&= \Psi_{\mathrm{ret}}
\end{aligned} \tag{21}$$

The interference product $\psi_{\text{ret}} \cdot \psi^*_{\text{adv}} = I$ in (21) is the key step, if $\psi_{\text{ret}} \cdot \psi^*_{\text{adv}} \neq I$ then not only do the equations not balance but the resultant particles are virtual and intrinsically off-mass shell. The Ψ_{adv} explicitly vanishes even for antimatter, as reversing the direction of Time also inverts both the charge and the parity, so regardless of the species of particle the model must conform to the Wheeler-Feynman summation of excluding the advanced potentials. *It is the interference of the advanced and retarded potentials that imposes the Arrow of Time on the wavefunctions regardless of the species of the particles.*

It becomes apparent that it is impossible to construct the wavefunction for a fermion using the Wheeler-Feynman summation which is solely based on the A_μ^{adv} potential, or,

$$\Psi_{tot} \neq \exp\left(\tfrac{iq}{\hbar} \int A_\mu^{\text{adv}}\, \mathrm{d}x^\mu\right) \tag{22}$$

Therefore the resultant Wheeler-Feynman summation of the A_μ is always the A_μ^{ret} potential and the A_μ^{adv} potential always vanishes, and necessarily the only direction in Time that matter can travel is always in the forward direction of Time which is in sense of direction of the A_μ^{ret} potential, thereby giving an explanation for both the Problem of the Arrow of Time and the Principle of Causality.

The above method yields a covariant anterograde wavefunction for fermions that satisfies conservation of energy and is inherently *on-mass shell.* It was also shown in the previous paper [O'Brien 2018] the electron's reduced Compton wavelength matches the observed wavelength of an electron in $\mathbb{R}^{1,3}$, and results in a perfectly spherical charged massive electron in $\mathbb{R}^3$ which matches recent experimental results on the sphericity of electrons [Hudson et al 2011]. This result is highly significant in the context of the Big Bang as it implies that in a sea of virtual particles the Universe must evolve forward away from the Singularity, and therefore the Arrow of Time for the history of the Universe must be in the anterograde direction — consequently the Universe must move forward in Time.

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