Complex Dynamics and the Age of the Universe

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Abstract

There are growing indications today that *complex dynamics of far-from-equilibrium systems* lies at the root of primordial cosmology and the ultraviolet (UV) sector of particle physics. We recently pointed out that dimensional fluctuations of the UV sector can reproduce the morphology of the cosmic web. Expanding on the same line of inquiry, this provisional report explores the link between the *long-range temporal correlations* of critical phenomena and primordial cosmology. Excluding systematic measurement errors, our report sheds new light on the tension in the age of the Universe sparked off by the latest observations of the James Webb Space Telescope (JWST).

Key words: Far-from-equilibrium phenomena, Complex dynamics, Critical slowing down, Long-range correlations, Early Universe cosmology.

1. Introduction

Many contemporary theoretical and experimental studies point out that the ultraviolet (UV) sector of particle physics and the dynamics of the primordial Universe unfold as *complex evolution outside thermodynamic equilibrium* [see e.g., 1- 4, 12 - 16]. For example, it was recently argued that a viable description of the primordial Universe must be built from concepts relevant to complex dynamics such as Self-Organized Criticality and Multifractal Geometry [1]. This approach is likely to bring fresh insights into the early genesis of Dark Matter and into some open challenges of standard cosmology.

The primary driver of complex dynamics in high-energy particle physics and primordial cosmology is the scale-dependent *continuous dimensional deviation*,

$$\varepsilon(\mu) = 4 - d(\mu) = O[m^2(\mu)/\Lambda_{UV}^2] \tag{1}$$

where μ is the running observation scale, d space dimensionality, m the representative mass scale of the theory and $\Lambda_{UV} >> m$ the UV cutoff. Building off these premises, the goal of this report is to briefly survey the link between the *long-range temporal correlations* of critical phenomena and primordial cosmology.

The report is organized as follows: the first section contains a short pedagogical review of critical behavior; second section delves into the topic of temporal correlations and their relevance to primordial cosmology. The Appendix section touches upon the remarkable analogy between critical phenomena and classical chaos theory.

2. Brief Synopsis of Critical Phenomena

Phase transitions represent sudden changes in the behavior of collective phenomena and are divided into *discontinuous* (first order) and *continuous* (second order). Given a representative thermodynamic property of a system called *control parameter* (λ), the distinction between the two types boils down to whether the transition stems from a discontinuous or continuous variation of λ. Continuous phase transitions are typically associated with the onset of *critical phenomena*. Near the transition point (λ_c), several observables of interest [$O(\lambda)$] diverge following the generic scaling law,

$$O(\lambda) \propto (\lambda - \lambda_c)^{-\sigma_0}; \;\; \sigma_0 > 0 \tag{2}$$

A key feature of continuous phase transitions is that the scale of correlations becomes unbounded at the critical point $(\lambda \to \lambda_c)$. Different thermodynamic phases are characterized by a locally defined *order parameter* $s(x)$ whose fluctuations are specified by their *correlation function*. Away from criticality, the correlation function of $s(x)$ decays as [5-10],

$$C(x) = \langle s(x)s(0) \rangle - \langle s \rangle^2 \propto \exp(-x/\xi); \;\; |x| \to \infty \tag{3}$$

where the *correlation length* exhibits the asymptotic behavior

$$\xi(\lambda) \propto |\lambda - \lambda_c|^{-\nu}; \;\; \nu > 0 \tag{4}$$

As $\lambda \to \lambda_c$, the correlation length diverges, which means that critical fluctuations in the order parameter do not have any characteristic length scale. This property is largely known as *scale invariance* and is also a representative attribute of *fractals* and *multifractals*. In contrast to (3), critical correlation functions in d dimensions assume the form,

$$C(x) \propto |x|^{-(d-2+\eta)} \tag{5}$$

in which η stands for the *Fisher exponent*.

Critical behavior may be divided into three groups, namely,

1. ***Static phenomena*** occur in thermodynamic equilibrium. They are described using an effective time-independent Hamiltonian $H[s]$, which is a functional of $s(x)$ at a given control parameter λ. The canonical example is thermal critical behavior whereby $\lambda = T$ and the probability of finding the configuration $s(x)$ at T follows the Boltzmann-Gibbs distribution,

$$P_{eq}[s] \propto \exp(-H[s]/k_B T) \tag{6}$$

2. ***Dynamic phenomena*** develop outside thermodynamic equilibrium. Their description requires a *stochastic framework* typically formulated in terms of nonlinear Langevin equations or equations with fractional derivatives and integrals [7-9, 19]. In this case, dimensional deviation (1) from the upper critical dimension d_c (written as $\varepsilon = d - d_c$) takes on a leading role. A lesser-known embodiment of dynamic phenomena is *critical behavior in continuous dimension,* where (1) acts as control parameter (that is, $\lambda = \varepsilon$) and plays a key role in the emergence of *fractal spacetime* above the Fermi scale and in *dimensional reduction* conjectured to take place near the Planck scale.

At criticality, the relaxation time of fluctuations in the order parameter diverges as

$$t_{r,c}(\lambda) \propto \xi(\lambda)^z \propto \left|\lambda - \lambda_c\right|^{-z\nu} \tag{7}$$

where $z > 0$ is the dynamic critical exponent. In a nutshell,

$$\boxed{t_{r,c}(\lambda_c) \to \infty \text{ and } \xi(\lambda_c) \to \infty} \tag{8}$$

Relation (8) describes the process of *critical slowing down*, where fluctuations near criticality relax infinitely slow to equilibrium. It can be also shown that an identical singular behavior occurs for the *time correlation function* whose scaling law is given by [9]

$$t_{corr}(\lambda) \propto (\lambda - \lambda_c)^{-\Delta} \tag{9}$$

as in

$$\boxed{t_{corr}(\lambda_c) \to \infty} \tag{11}$$

3. ***<u>Self-organized criticality</u>*** (SOC) is a subset of dynamic phenomena based on *self-sustained* critical behavior of large-scale systems evolving outside equilibrium. The trademark signature of SOC is two-fold:

a) it occurs in complex ensembles of interacting components,

b) it is characterized by a power-law distribution of "avalanche" sizes. Nowadays, SOC is considered a generic paradigm for a large variety of scale-invariant phenomena ranging from spin glasses, magnetic domains and turbulent flows to traffic jams, cardiac and neuronal activity, economic processes, earthquakes, percolation clusters, forest fires, cellular growth, weather patterns, social behavior patterns and so on [1].

3. Temporal Correlations in Primordial Cosmology

According to [2-3], primordial cosmology can be modeled as *critical behavior in continuous dimension*. This viewpoint leads to the expectation that critical slowing down and the onset of long-range temporal correlations necessarily develop according to (8) – (11). It is conceivable that long-range time correlations may account for the latest JWST data which report a surprising similarity between the properties of *far-field* and *near-field cosmic structures* (galaxies and Black Holes), in terms of size, shape, dynamics, distribution, and internal composition.

We close by noting that the same findings may likely provide clues to the *horizon problem* of cosmology, which is still outstanding today (see e.g. [17-18]). The horizon problem may be concisely stated as follows: distant regions of space in opposite directions of the sky are widely separated. Assuming standard Big Bang expansion, they can never have been in *causal contact* with each other. This is because the light travel time between them exceeds the age of the universe. Yet the uniformity of the cosmic microwave background (CMB) temperature tells us that these regions must have been in contact with each other in the past.

APPENDIX

Kolmogorov Entropy and Relaxation Time in Nonlinear Dynamics

A remarkable analogy may be drawn between critical phenomena, on the one hand, and the approach to chaos in nonlinear dynamics, on the other. To unveil this analogy, refer first to [4], which bridges the gap between Kolmogorov entropy (S_K), topological entropy (S_0) and dimensional deviation (1),

$$S_K \Leftrightarrow S_0 \propto \log(\varepsilon^{-D_0}) \tag{A1}$$

Consider next and the link between the relaxation time to equilibrium in nonlinear dynamics and the rate of Kolmogorov entropy in phase-space given by [11],

$$\tau_r \propto (dS_K/d\rho)^{-1} \tag{A2}$$

in which $d\rho$ stands for the differential volume in phase-space. (A1) shows that, as dimensional deviation approaches its upper bound, ($\varepsilon \to \varepsilon_{MAX}$) near the Planck scale and spacetime dimension collapses to zero, Kolmogorov entropy (S_K) assumes a constant value. By (A2), the entropy rate in phase-space drops to zero and the relaxation time to equilibrium diverges to infinity, in a manner entirely consistent with (8) – (11).

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