Transverse relative time and length shift explained

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Abstract: A source S moving with a constant velocity v emits a signal s moving with the velocity of light c relative to source S position with time t. The velocity v of signal s is the sum of v and c. The time of flight t for the signal s when emitted from S and observed at O is calculated using purely Galilean transformation of velocities in Euclidean Space Geometry. O must reside in the s light cone to observe s and avoid the artificially introduced infinities that plague classical relativity models. The geometrical interpretation of the physics is valid for velocities greater than c.

Introduction

Assume a source S moves with constant velocity on the x axis in Euclidean Space Geometry. When S crosses the origin it emits a spherical signal s_1 which moves with velocity c. The center of s_1 remains coincident with S. An observer O at coordinate x, y, receives s_1 at t_1. S at time τ_0 emits s_2. O receives signal s_2 at time t_2. O measures the time τ', t_2 - t_1, a transverse relative time shift. The derivation of equations to calculate τ' with S velocity at 2 times c, 3×10^8 (m/s), are presented. The scale used in the figures is 1 unit of time in seconds (s) that light travels with velocity c in meters (m) when source S has velocity 0 (m/s).

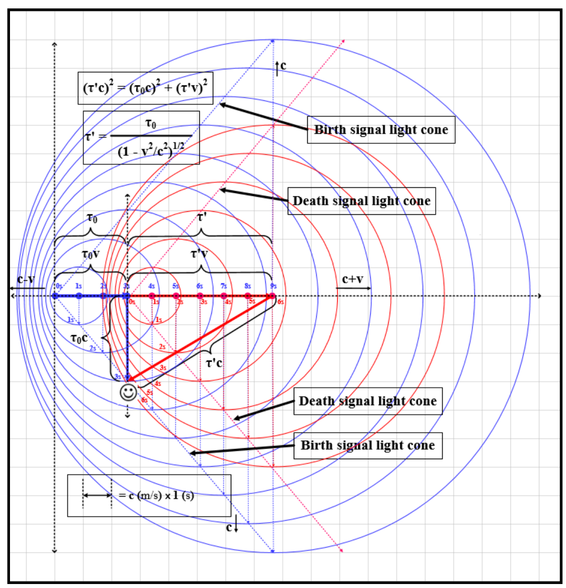

Figure 1: Transverse Relative Time Shift.

Geometry v near c

The extinction shift principle[1] shows when a stationary Observer O is perpendicular to Source S moving with velocity v when S emits signal s_2 O will measure τ' (1) a transverse relative time shift, **not** a time dilation. Refer to Figure 1 for geometric relationships.

$$\tau' = \frac{\tau_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (1)$$

From geometry we get (2).

$$(\tau' c)^2 = (\tau_0 c)^2 + (\tau' v)^2 \qquad (2)$$

Now the Observer is limited on capability to receive s_2 as v approaches c (3×10^8 m/s). This limitation is removed when O is positioned inside s_2 light cone. The next section will detail the geometric layout for O to receive s_1 and s_2 and get τ'.

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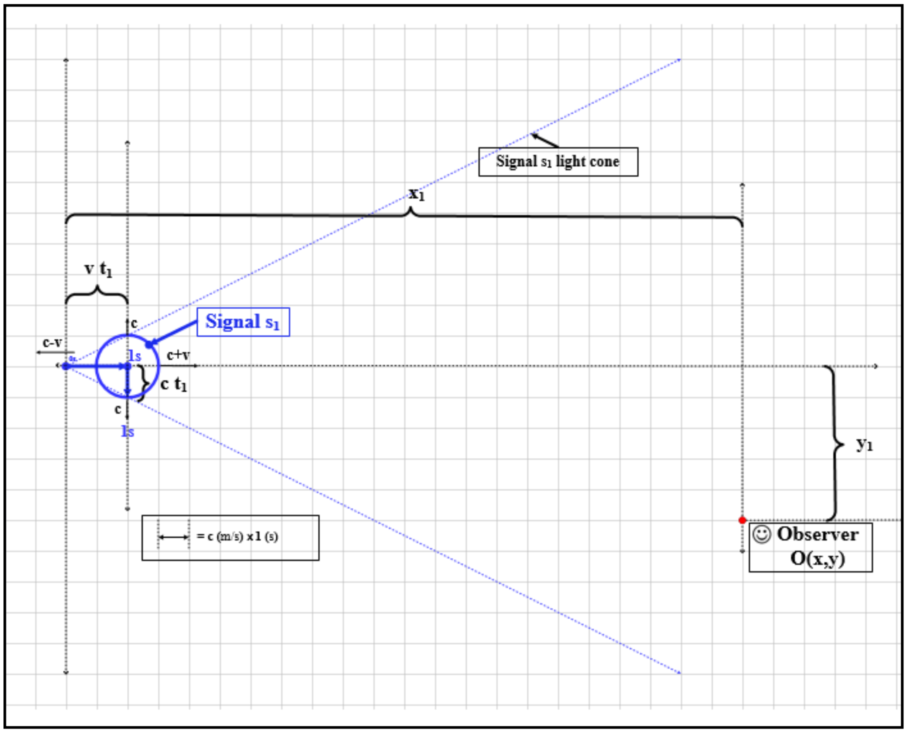
1 Edward Henry Dowdye, Jr., Extinction Shift Principle, "Under the Electrodynamics of Galilean Transformations", Third Edition 2012, p. 26

Transverse relative time shift - Geometry when v = 2c

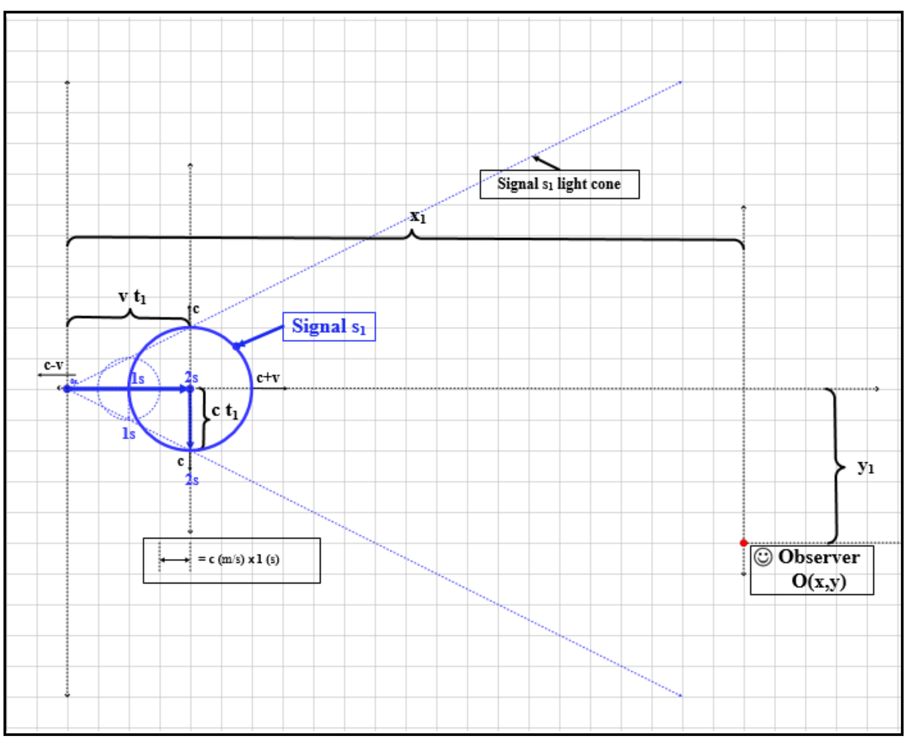
Let S travel with velocity 2c (m/s) emit a signal s_1 at time equal to 0 (s) as it crosses the origin going in the x direction. Let there also be an observer O at coordinate x, y or O(x,y), where x = 22c (m) and y = 5c (m). The Observer coordinates are in meters (m). At 1 (s) S will have traveled the distance $6x10^8$ (m). The signal s_1 would have traveled $3x10^8$ (m) in the -y direction and $9x10^8$ (m) on the x-axis, [2c (m/s) + 1c (m/s)][1 (s)] = 3c (m), see geometry in Figure 2.1.

Figure 2.1: S with velocity 2c (m/s), time = 1 (s).



In 2 seconds S will have traveled to 4c (m). Signal s_1 would have traveled 2c in the -y direction and 8c (m) along the x axis, see Figure 2.2.

Figure 2.2: S with velocity 2c (m/s), time = 2 (s).



In 3 seconds S will have traveled to 6c (m). Signal s1 would have traveled 3c in the -y direction and 9c (m) along the x axis, see Figure 2.3.

Sometime after 7 seconds but before 8 seconds the s_1 would arrive at O in t_1 seconds, see Figure 3.

Figure 2.3: S with velocity 2c (m/s), time = 3 (s).

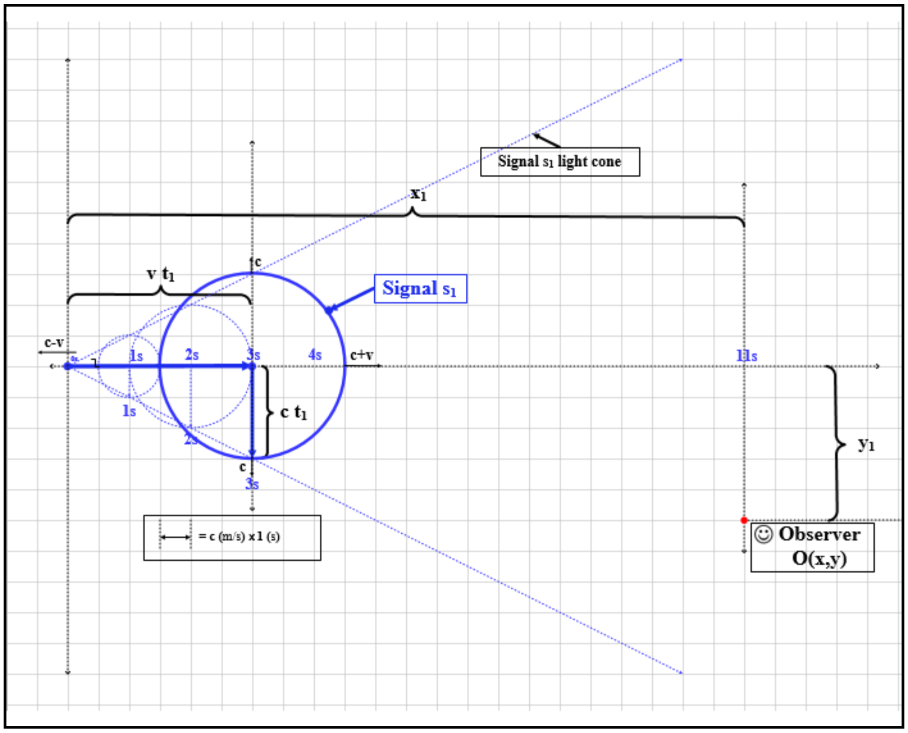
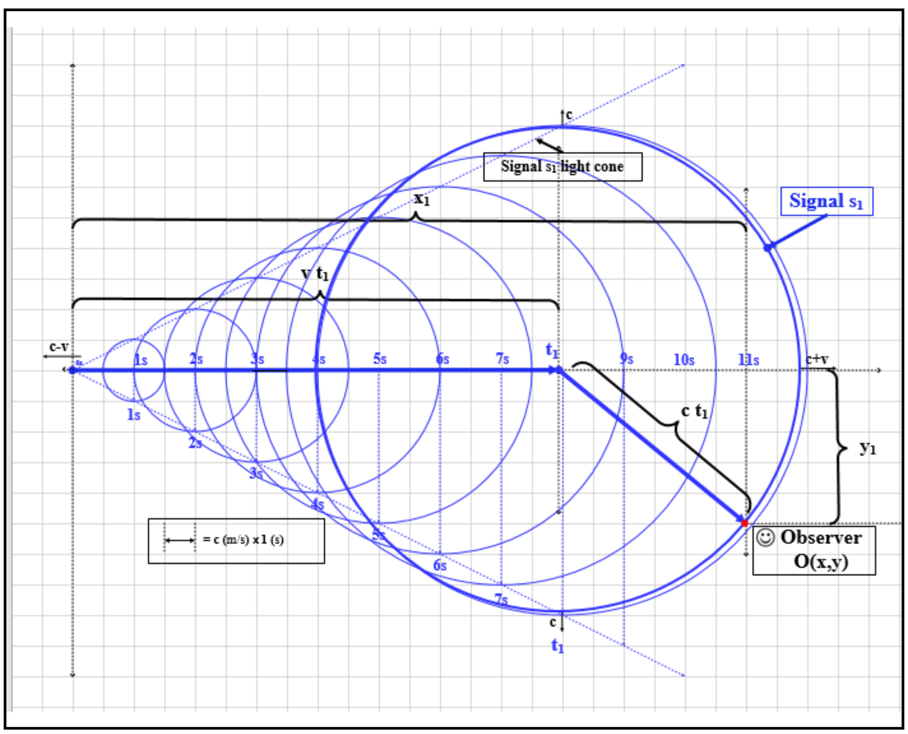


Figure 3: S with velocity 2c (m/s), time = t_1.



Determining t_1

The transient time t_1 for the light pulse to reach $O(x_1, y_1)$ can be determined from geometry, see Figure 4. The velocity of the Source S is v (m/s). The distance traveled by S in t_1 (s) equals vt_1 (m). The distance s_1 travels radially about S is ct_1 (m).

The velocity of the Source S is v (m/s). The distance traveled by the S in t_1 (s) is vt_1 (m). The distance the signal s_1 travels radially about S is ct_1.

Angles A, B, D have corresponding sides a, b, d.

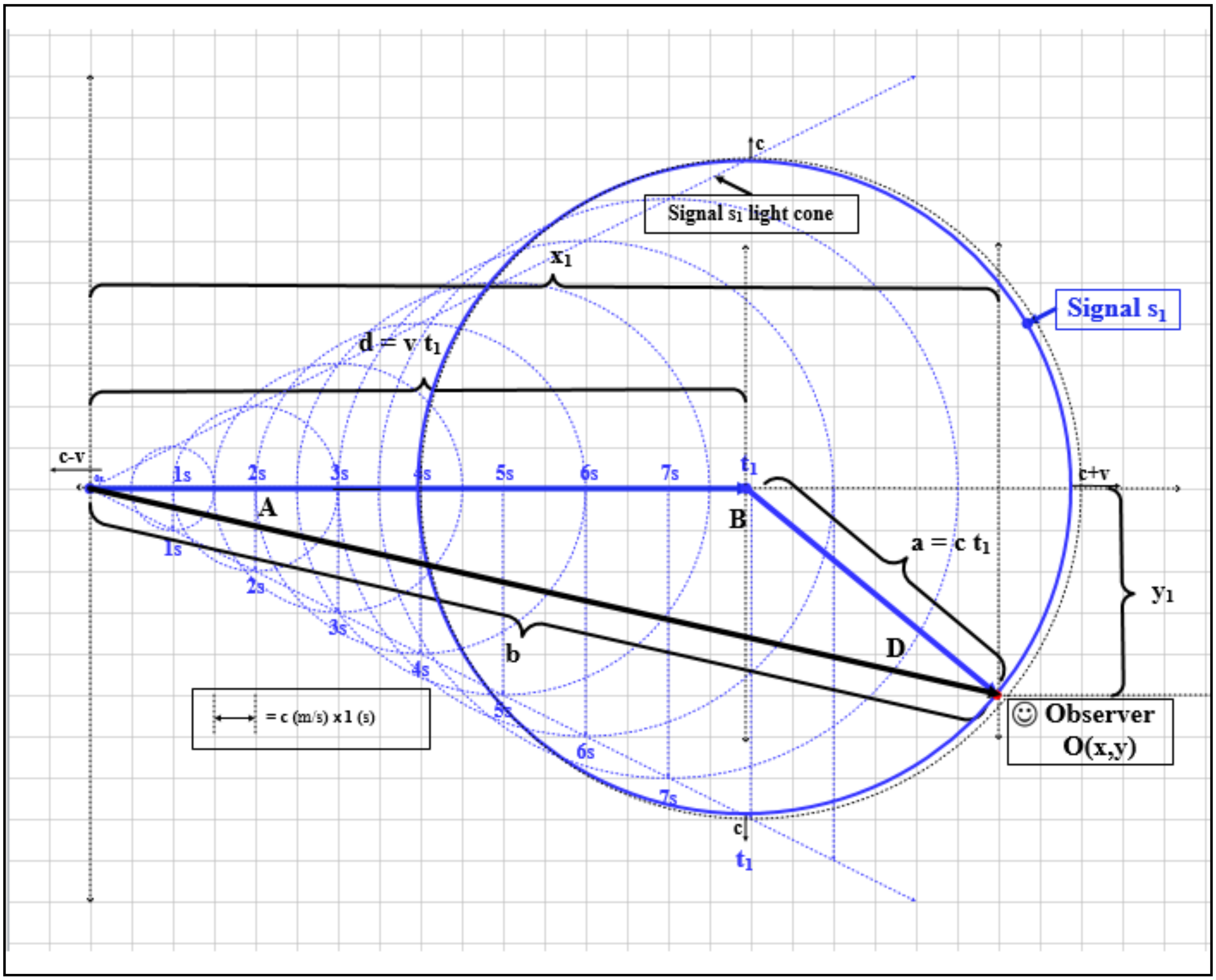
$$a = ct_1 \tag{3}$$

$$b^2 = x_1^2 + y_1^2 \tag{4}$$

$$d = vt_1 \tag{5}$$

$$\cos(A) = \frac{x_1}{b} = \frac{x_1}{\sqrt{x_1^2 + y_1^2}} \tag{6}$$

Figure 4: Geometry used to determine equation for t_1.



$$a^2 = b^2 + d^2 - 2bd\cos(A) \quad (7)$$

From the law of cosines[2] we get (7). Combine (3)(4)(5)(6) into (7) to get (8) were the t, x, y subscripts are dropped. Solve[3] for t_1 (9).

$$0 = -(ct_1)^2 + (x^2 + y^2) + (vt_1)^2 - 2\sqrt{x^2+y^2}\,vt_1\frac{x}{\sqrt{x^2+y^2}} \quad (8)$$

$$t_1 = \frac{-vx \pm \sqrt{c^2x^2 + c^2y^2 - v^2y^2}}{c^2 - v^2} \quad (9)$$

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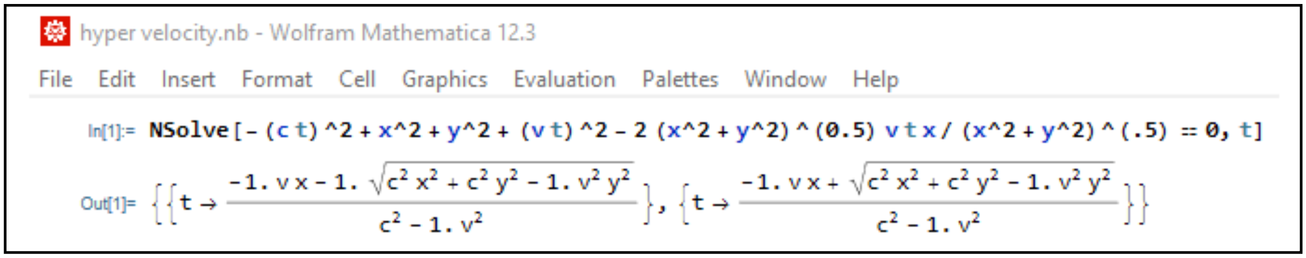
[2] Standard Mathematical Tables, 27th Edition, CRC PRESS, Copyright 1984, p. 144

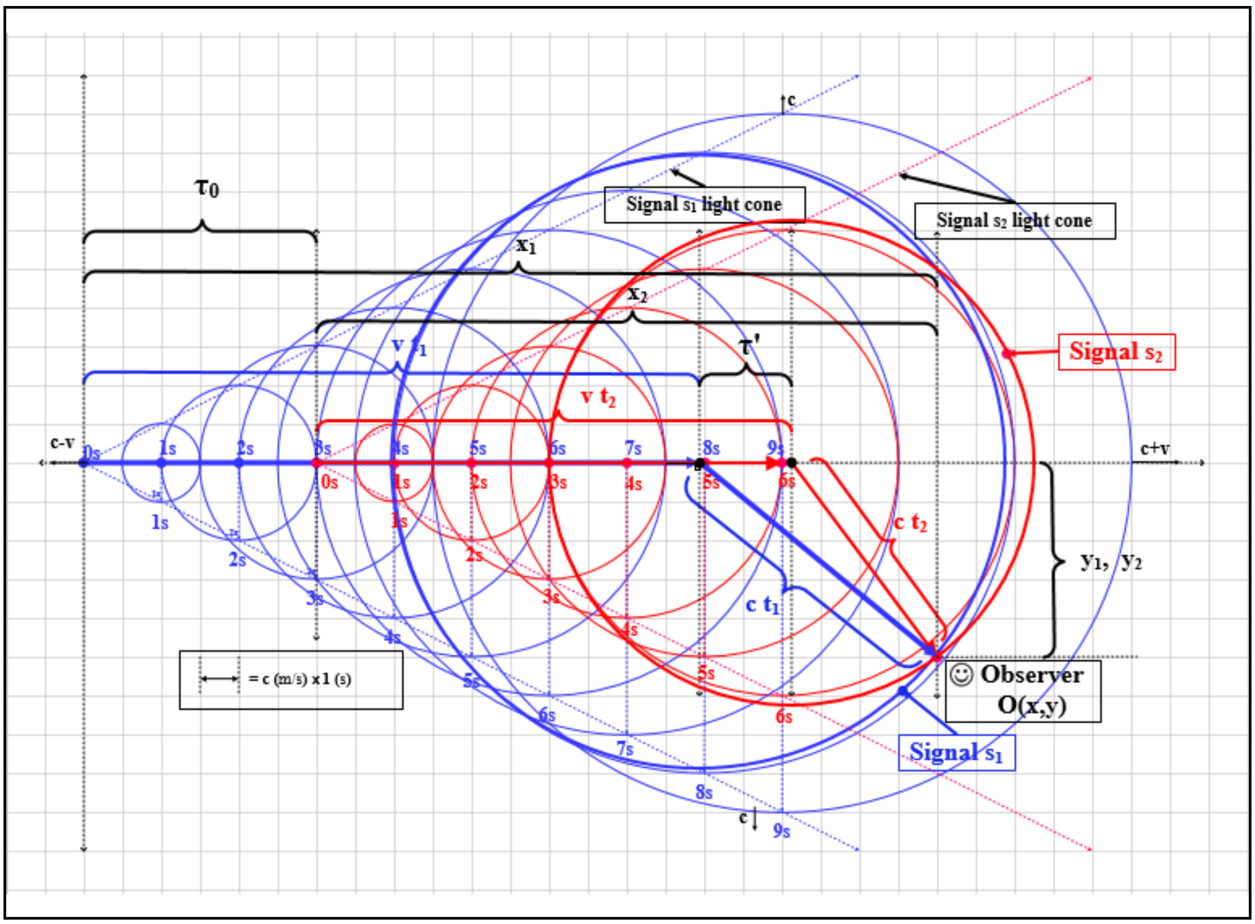
[3]Mathematica 12.3.1 Kernel for Microsoft Windows (64-bit), Copyright 1988-2021 Wolfram Research, Inc.

Figure 5: Mathematica solution t as a function of v, x, y, c.



The input and output to Mathematica is in Figure 5. Now t_1 is the time at which O observes s_1. We use the $t_1(+)$ solution to calculate t_1. Let S emit a signal s_2 at time t equal to 3 (s). s_2 time of flight to O is t_2, some time after 6 (s). See Figure 6 for geometry of s_1

Figure 6: for geometry of s_1 and s_2.



and s_2. Now the total time from the initiation of s_1, time = 0 (s), to the reception of s_2 at O is τ_0 plus t_2 which is also equal to the total time of flight of t_1 of s_1 to the reception at O plus τ' (10)(11).

$$\tau_0 + t_2 = t_1 + \tau' \quad (10) \qquad \tau' = \tau_0 + t_2 - t_1 \quad (11)$$

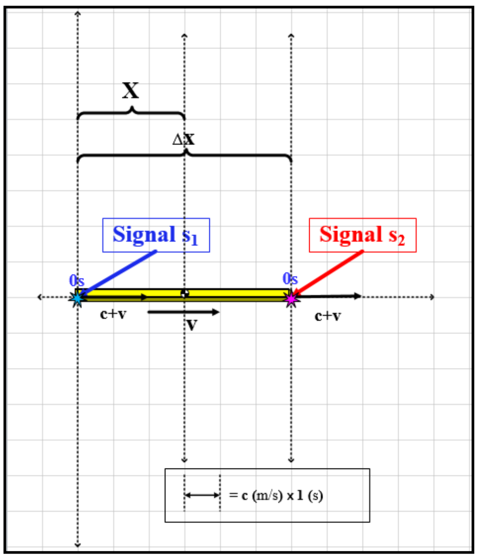
The ratio τ'/τ_0 is the *transverse relative time shift* factor observed by O at O(x,y).

Transverse relative distance

At time t_1 S location is vt_1 and O receives s_1. At time $(\tau_0 + t_2)$ S location is $v(\tau_0 + t_2)$ when O receives s_2. Now τ' (s) times the Source S velocity is equal to the actual distance traveled by S when O receives the t_1 and t_2 signal. The actual signal t_1 was emitted at t_0 where time is 0 seconds, the coordinate origin. S travels to $v\tau_0$ (m) in t_2 (s). What is interesting is that O perceives that S has traveled $v\tau'$ (m), a relative distance shift x'(12). Where x_0 (m) is the actual distance traveled by S with velocity v in τ_0 (s) (13).

$$x' = v\tau' \quad (12) \qquad x_0 = v\tau_0 \quad (13)$$

Figure 7: Sources S_1 and S_2 moving with velocity v=2c (m/s) emit signals s_1 and s_2 at time t=0 seconds.



Transverse relative length

Let there be two Sources S_1 and S_2 moving with velocity v and separated by distance $v\tau_0$ (m), see Figure 7. S_1 and S_2 move to the right as if there were a virtual rod of length Δx between them. Let X be the distance from the origin to the rods center at time t=0 (s). S_1 and S_2 emit signals s_1 and s_2 at time $t_1 = t_2 = 0$ (s), v = 2c (m/s).

In 1 (s) the rod would have moved to position 2c (m) and s1 and s2 would have expanded by 1c in radius as shown in Figure 7.1. In 2 (s) the rod would be in position shown in Figure 7.2, in 3 (s) , Figure 7.3, in 4 (s) Figure 7.4 and in 7 seconds Figure 7.7.

Figure 7.1: s_1, s_2 and rod position at time t = 1 (s).

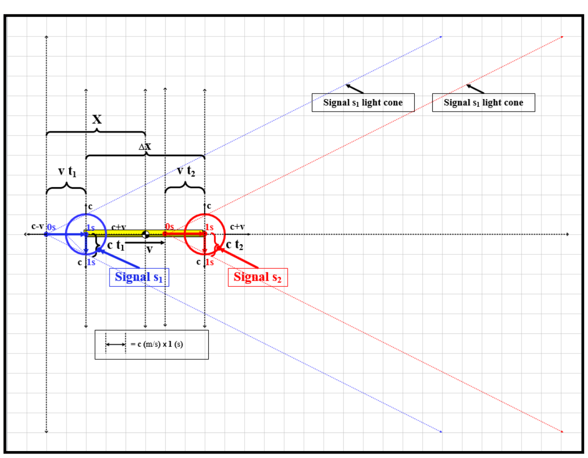


Figure 7.2: s_1, s_2 and rod position at time t = 2 (s).

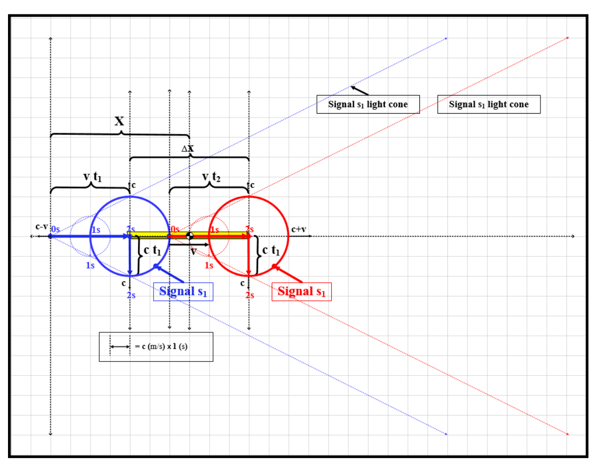


Figure 7.3: s_1, s_2 and rod position at time t = 3 (s).

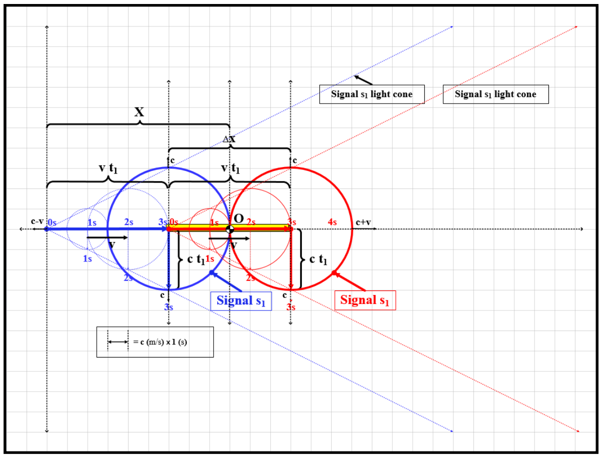
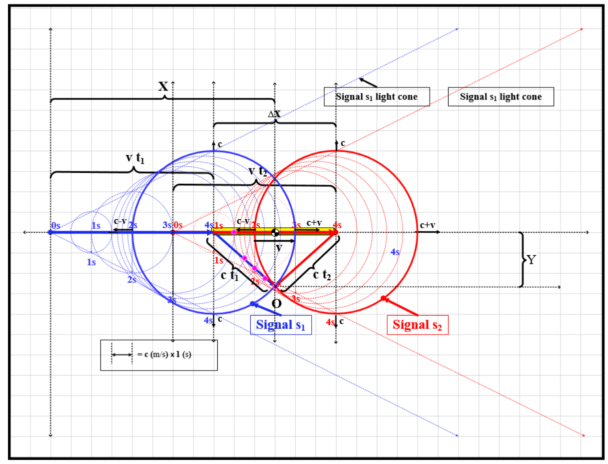


Figure 7.4: s_1, s_2 and rod position at time t = 4(s).

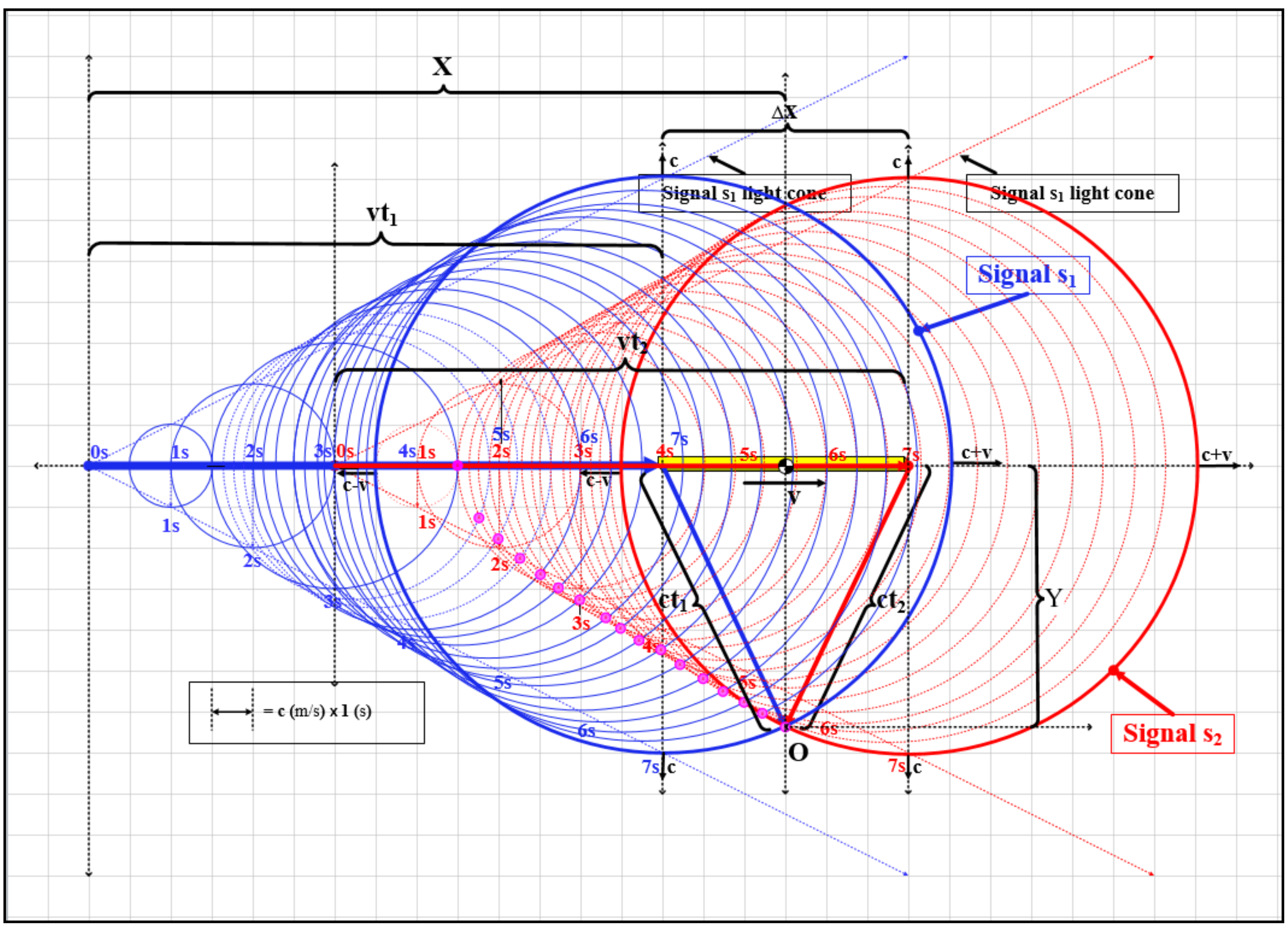


From geometry we see that observer O receives both signals s_1 and s_2 at the same t when located on X (14) and Y (15).

$$X = vt + \frac{1}{2}\Delta x \qquad (14)$$

$$Y = \sqrt{(ct)^2 - \frac{1}{4}\Delta x^2} \qquad (15)$$

Figure 7.7: s_1, s_2 and rod position at time t = 7(s).



Now let a rod of length rod with length Δx moving with velocity v emit signals s_1 and s_2 at time t = 0 (s) when s_1 is at the origin. Let observer O be at coordinates X, Y inside the

Figure 8: Rod of length rod with length Δx moving with velocity v emit signals s_1 and s_2 at time t = 0 (s) when s_1 is at the origin.

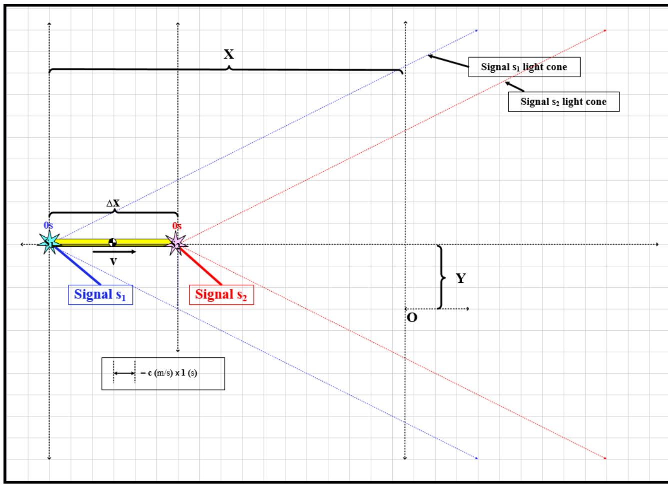
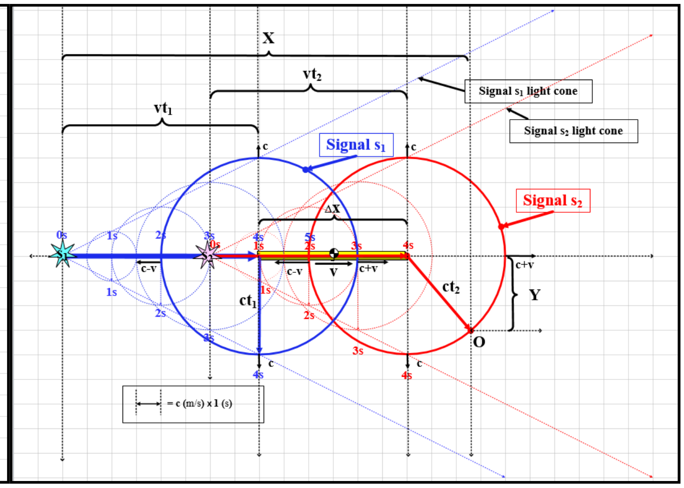


Figure 9: Rod of length rod with length Δx moving with velocity v emit signals s_1 and s_2 at time t = 0 (s) when s_1 is at the origin.



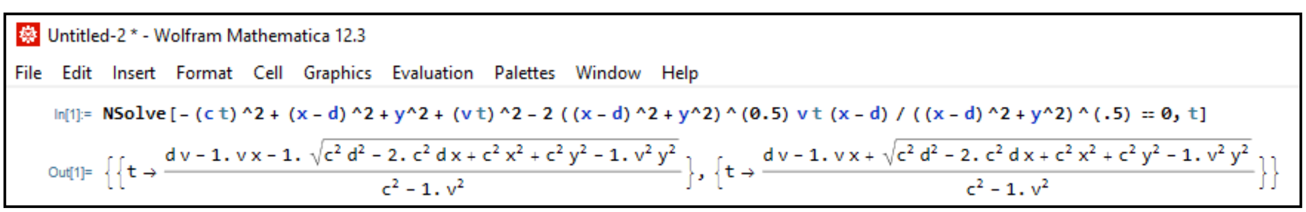
s2 light cone, see Figure 8. From geometry in Figure 9 substitute x-Δx for x in (8) to get (16). Solve (16) for t_1 to get (17) using Mathematic, Figure 10.

$$0 = -(ct_1)^2 + ((x-\Delta x)^2 + y^2) + (vt_1)^2 - 2\sqrt{(x-\Delta x)^2 + y^2}\, v t_1 \frac{x_1}{\sqrt{(x-\Delta x)^2 + y^2}}$$

(16)

$$t_2 = \frac{v\Delta x - vx \pm \sqrt{c^2\Delta x^2 - 2c^2\Delta x x + c^2x^2 + c^2y^2 - v^2y^2}}{c^2 - v^2} \quad (17)$$

Figure 10: Mathematica solution when O observes s_2 at t_2 as a function of v, Δx, x, y, c.



Now for the moving rod O receives s_1 at t_1 after s_2 at t_2 therefore τ' is (18), τ_0 is 0 (s).

$$\tau' = \tau_0 + t_1 - t_2 = t_1 - t_2 \qquad (18)$$

Let a rod of length Δx = 3c (m), moving with velocity v = 2c (m/s) and Observer O at x = 16.5c (m), y = 3c (m), from (17) t_2 = 3.96 (s), (9) t_1 = 5.78 (s), and (18) τ' = 1.82 (s) (19).

$$\tau' = t_1 - t_2 = t_1 - t_2 = 5.78 - 3.96 = 1.82 \text{ (s)} \qquad (19)$$

From O calculates the rod length $\Delta x'$ to be 1.09x10[9] (m) (20). $\Delta x'$ is the "*transverse relative length*" observed and measured by 0.

$$\Delta x' = v\tau' = 2(3x10^8)(1.82) = 1.09x10^9 \text{ (m)} \qquad (20)$$

O calculates the per unit transverse relative length of the moving rod to be 0.6071 (21)

$$\frac{\text{Observer calculated rod length}}{\text{Actual rod length}} = \frac{v\tau'}{\Delta x} \; (21)$$

Observation

The ratio τ'/τ0 is the *transverse relative time shift factor* observed by O at O(x,y). The Observer must be downstream and inside of both the s_1 and s_2 light cones to observe both signals when the signal source S has any velocity v. If v is equal to 0 (m/s) then (8) reduces to (13) (14) the expected physics in Euclidean space with Galilean geometry.

$$0 = -(ct_1)^2 + (x^2 + y^2) + (vt_1)^2 - 2\sqrt{x^2 + y^2}\,vt_1\frac{x}{\sqrt{x^2 + y^2}} \qquad (8)$$

$$0 = -(ct_1)^2 + (x^2 + y^2) \qquad (13)$$

$$(ct_1)^2 = (x^2 + y^2) \qquad (14)$$

The calculations for the provided example are in Table 1. The procedure outlined in this work can be extended to estimate the incoming velocity of cosmic rays that initiate double pulses in detectors like Super-Kamiokande[4]. The velocity of light c must be adjusted to account for the properties of the light conducting medium.

[4] http://www-sk.icrr.u-tokyo.ac.jp/sk/index-e.html

Table 1: Calculations for time and distance quantities when v=2c.

Eq.	Signal	S_1	S_2	Notes
	c (m/s)	3.000E+08	3.000E+08	Velocity of light c when Source is at rest.
	v/c (m/s)	2.000E+00	2.000E+00	Ratio of Source velocity to that of c.
	v (m/s)	6.000E+08	6.000E+08	Velocity of Source.
	τ_0 (s)		3.000E+00	Time duration between s_1 and s_2 start events by Source traveling at velocity v.
	Δx (m) = 3c		1.800E+09	Rod length
	X (m) = 16.5c	4.950E+09	4.950E+09	Observer x-axis coordinate
	Y (m) = 3c	9.000E+08	9.000E+08	Observer y-axis coordinate
(17)	t_2+ (s) for rod	-	3.959E+00	Time (s) of receipt of s_2 by Observer in moving rod example
(9)	t_1+ (s) for rod	5.780E+00	-	Time (s) of receipt of s_1 by Observer in moving rod example
(19)	$\tau' = t_1 - t_2$ (s)	1.821E+00	-	τ_0 =0 since s_1 and s_2 are emitted at time t_0 = 0 seconds.
(20)	$\Delta x' = v\tau'$ (m)	1.093E+09	-	Observer calculated rod length knowing rod velocity and τ', a transverse relative length.
(21)	$v\tau'/\Delta x$	6.071E-01	-	Observer calculated per unit "transverse relative length" when velocity of rod is 2C and the observers position is x, y.

List of acronyms

c	Speed of light in meters per second, assumed $3x10^8$ (m/s)
m	Distance in meters (m)
O	Observer location X (m), Y (m)
S	Source for signal s_n
s	Time in seconds (s)
s_n	Signal s_n location at time t_n where **n** = 1,2...
t	Reference time in seconds for all space, typically time t = 0 (s) when source S passes the origin where X=Y=0 (m)
t_n	Time in seconds elapsed from initiation of signal s_n
τ_0	Time difference in seconds (s) when source **S** sends signals s_1 and s_2
τ'	Time difference in seconds (s) when observer **O** receives signals s_1 and s_2
v	Velocity of signal source in meters per second (m/s)
X	Observer **O** x-axis coordinate
x_n	Distance in meters (m) on x-axis location of signal s_n initiation to Observer **O** x-axis coordinate Δx Rod length in meters (m)
$\Delta x'$	Rod length in meters (m)
Y	Observer **O** y-axis coordinate in meters (m)
y_n	Observer **O** y-axis coordinate in meters (m)