

# Proof of Fermat's Last Theorem for odd primes

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**ABSTRACT.** It was already proved right that  $x^n + y^n = z^n$ , ( $n > 2$ ) has no solutions in positive integers which we called Fermat's Last Theorem (FLT) by Andrew Wiles. But his proof would be impossible in the 17<sup>th</sup> century. Since Fermat showed he proved  $n$ =even by leaving proof for  $n$ =4, many people have tried to prove odd primes. I took the idea from Euler proof and proved in case of  $n$ =odd primes by simple method.

## 1. Introduction

Pierre de Fermat claimed he had proof that no three positive integer  $x$ ,  $y$  and  $z$  satisfy the equation  $x^n + y^n = z^n$  for  $n$  greater than 2 which we called Fermat's Last Theorem (FLT). For about 3 century, many people have tried to prove FLT. Finally, FLT was proved right by Andrew Wiles in 1995. However, the proof of Wiles is a modern math that is difficult to understand and complex. So, some people, who think Fermat prove FLT by himself, still believe elementary method exist. We know that FLT can be proved by proving  $n$ =odd primes because case of  $n$ =even proved by Fermat himself. I got an idea to solve for  $n$ =odd primes from Euler's proof. In this paper, I proved the case of  $n$ =odd primes by simple method.

## 2. Proof for $n=3$

$$x^3 + y^3 = z^3, (x < y < z)$$

Where  $x, y$  and  $z$  = positive integer, relatively prime

This equation can be classified into three categories as follows.

Case 1.  $(x, y, z) = (\text{even}, \text{odd}, \text{odd})$

Case 2.  $(x, y, z) = (\text{odd}, \text{even}, \text{odd})$

Case 3.  $(x, y, z) = (\text{odd}, \text{odd}, \text{even})$

Case 1.  $(x, y, z) = (\text{even}, \text{odd}, \text{odd})$

Let  $y=(u-v)$ ,  $z=(u+v)$ .

Assume  $u$  and  $v$  are not relatively prime.

Let  $u=fU$ ,  $v=fV$ .

$$y = f(U - V)$$

$$z = f(U + V)$$

But this contradicts because  $y$  and  $z$  are relatively prime.

So,  $u$  and  $v$  are relatively prime.

Also  $u$  and  $v$  are opposite parity because  $y$  and  $z$  are odd.

$$x^3 = (u + v)^3 - (u - v)^3$$

$$x^3 = 2v(v^2 + 3u^2)$$

Assume  $u$ =even,  $v$ =odd.

$v^2 + 3u^2$  is odd.

Assume  $u$ =odd,  $v$ =even.

$v^2 + 3u^2$  is odd.

So,  $v^2 + 3u^2$  is always odd.

Thus the greatest common factor of  $2v$  and  $v^2 + 3u^2$  is odd.

Assume the common factor is odd except 1 and 3.

Let  $v = fV$  and  $v^2 + 3u^2 = fN$ .

$$3u^2 = f(N - fV^2)$$

$u$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v$  are relatively prime.

So, the greatest common factor of  $2v$  and  $v^2 + 3u^2$  is either 1 or 3.

Assume the greatest common factor of  $2v$  and  $v^2 + 3u^2$  is 1.

It is possible that  $2v = p^3$  and  $v^2 + 3u^2 = q^3$ .

Where  $p$  and  $q$  are relatively prime

$$x^3 = p^3q^3$$

Assume the greatest common factor of  $2v$  and  $v^2 + 3u^2$  is 3.

Let  $v=3r$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v$  are relatively prime.

$$x^3 = 6r(9r^2 + 3u^2)$$

$$x^3 = 18r(u^2 + 3r^2)$$

The greatest common factor of  $18r$  and  $u^2 + 3r^2$  must be 1.

It is possible that  $18r = p'^3$  and  $u^2 + 3r^2 = q'^3$ .

$$x^3 = p'^3q'^3$$

Assume  $z=y+i$ .

From  $y=(u-v)$ ,  $z=(u+v)$ ,

$$z - y = (u + v) - (u - v) = 2v$$

$$2v = i$$

Assume  $y=k+i$ .

$k$  is odd because  $y$  is odd and  $i$  is even.

Assume  $y$  and  $i$  are not relatively prime.

Let  $y=fY$ ,  $i=fI$

$$z = y + i$$

$$z = f(Y + I)$$

But this contradicts because  $y$  and  $z$  are relatively prime.

So,  $y$  and  $i$  are relatively prime.

Assume  $k$  and  $y$  are not relatively prime.

Let  $k=fK$ ,  $y=fY$ .

$$i = y - k$$

$$i = f(Y - K)$$

But this contradicts because  $y$  and  $i$  are relatively prime.

So,  $k$  and  $y$  are relatively prime.

Let  $k=(u'-v)$ ,  $y=(u'+v)$ .

Assume  $u'$  and  $v$  are not relatively prime.

Let  $u'=fU'$ ,  $v=fV$

$$k = f(U' - V)$$

$$y = f(U' + V)$$

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $u'$  and  $v$  are relatively prime.

Also  $u'$  and  $v$  are opposite parity because  $k$  and  $y$  are odd.

$$y^3 - k^3 = (u' + v)^3 - (u' - v)^3$$

$$y^3 - k^3 = 2v(v^2 + 3u'^2)$$

Assume  $u'$ =even,  $v$ =odd.

$v^2 + 3u'^2$  is odd.

Assume  $u'$ =odd,  $v$ =even.

$v^2 + 3u'^2$  is odd.

So,  $v^2 + 3u'^2$  is always odd.

Thus the greatest common factor of  $2v$  and  $v^2 + 3u'^2$  is odd.

Assume the common factor is odd except 1 and 3.

Let  $v=fV$  and  $v^2 + 3u'^2=fN$ .

$$3u'^2 = f(N - fV^2)$$

$u'$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u'$  and  $v$  are relatively prime.

So, the greatest common factor of  $2v$  and  $v^2 + 3u'^2$  is either 1 or 3.

Assume the greatest common factor of  $2v$  and  $v^2 + 3u'^2$  is 1,

It is possible that  $2v = p^3$  and  $v^2 + 3u'^2 = \gamma$ .

$$y^3 - k^3 = \gamma p^3$$

Assume  $y^3 = \gamma y'$ ,  $k^3 = \gamma k'$  or  $y^3 = \alpha p^3$ ,  $k^3 = \beta p^3$ .

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $\gamma$  must be  $s^3$ .

$$y^3 - k^3 = s^3 p^3$$

Where  $s$  and  $p$  are relatively prime

Let  $y^3 = c^3$ ,  $k^3 = b^3$ ,  $s^3 p^3 = a^3$

$$a^3 + b^3 = c^3$$

But this equation contradicts by the method of infinite descent.

Assume the greatest common factor of  $2v$  and  $v^2 + 3u'^2$  is 3.

Let  $v=3r$ .

$u'$  and  $r$  are relatively prime because  $u'$  and  $v$  are relatively prime.

$$y^3 - k^3 = 6r(9r^2 + 3u'^2)$$

$$y^3 - k^3 = 18r(u'^2 + 3r^2)$$

The greatest common factor of  $18r$  and  $u'^2 + 3r^2$  must be 1.

It is possible that  $18r = p'^3$  and  $u'^2 + 3r^2 = \gamma'$ .

$$y^3 - k^3 = \gamma'p'^3$$

Assume  $y^3 = \gamma\gamma'$ ,  $k^3 = \gamma'k'$  or  $y^3 = \alpha'p'^3$ ,  $k^3 = \beta'p'^3$

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $\gamma'$  must be  $s'^3$ .

$$y^3 - k^3 = s'^3p'^3$$

Where  $s'$  and  $p'$  are relatively prime

Let  $y^3 = c^3$ ,  $k^3 = b^3$ ,  $s'^3p'^3 = a^3$ .

$$a^3 + b^3 = c^3$$

But this equation contradicts by the method of infinite descent.

Case 2.  $(x, y, z) = (\text{odd}, \text{even}, \text{odd})$

Let  $x=(u-v)$ ,  $z=(u+v)$ .

Case 2 has no solution because it can be proved in the same form as Case 1.

Case 3.  $(x, y, z) = (\text{odd}, \text{odd}, \text{even})$

Let  $x=(u-v)$ ,  $y=(u+v)$ .

Assume  $u$  and  $v$  are not relatively prime.

Let  $u=fU$ ,  $v=fV$ .

$$x = f(U - V)$$

$$y = f(U + V)$$

But this contradicts because  $x$  and  $y$  are relatively prime.

So,  $u$  and  $v$  are relatively prime.

Also  $u$  and  $v$  are opposite parity because  $x$  and  $y$  are odd.

$$z^3 = (u + v)^3 + (u - v)^3$$

$$z^3 = 2u(u^2 + 3v^2)$$

Assume  $u=\text{even}$ ,  $v=\text{odd}$ .

$u^2 + 3v^2$  is odd.

Assume  $u=\text{odd}, v=\text{even}$ .

$u^2 + 3v^2$  is odd.

So,  $u^2 + 3v^2$  is always odd.

Thus the greatest common factor of  $2u$  and  $u^2 + 3v^2$  is odd.

Assume the common factor is odd except 1 and 3.

Let  $u=fU$  and  $u^2 + 3v^2=fN$ .

$$3v^2 = f(N - fU^2)$$

$u$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v$  are relatively prime.

So, the greatest common factor of  $2u$  and  $u^2 + 3v^2$  is either 1 or 3.

Assume the greatest common factor of  $2u$  and  $u^2 + 3v^2$  is 1.

It is possible that  $2u = p^3$  and  $u^2 + 3v^2 = q^3$ .

Where  $p$  and  $q$  are relatively prime

$$z^3 = p^3q^3$$

Assume the greatest common factor of  $2u$  and  $u^2 + 3v^2$  is 3.

Let  $u=3r$ .

$v$  and  $r$  are relatively prime because  $u$  and  $v$  are relatively prime.

$$z^3 = 6r(9r^2 + 3v^2)$$

$$z^3 = 18r(v^2 + 3r^2)$$

The greatest common factor of  $18r$  and  $v^2 + 3r^2$  must be 1.

It is possible that  $18r = p'^3$  and  $v^2 + 3r^2 = q'^3$ .

$$z^3 = p'^3q'^3$$

Assume  $x+y=i$ .

From  $x=(u-v), y=(u+v)$ ,

$$x + y = (u + v) + (u - v) = 2u$$

$$2u = i$$

Assume  $x+k=i$ .

$k$  is odd because  $x$  is odd and  $i$  is even.

Assume  $x$  and  $i$  are not relatively prime.

Let  $x=fX$ ,  $i=fI$ .

$$y = i - x$$

$$y = f(I - X)$$

But this contradicts because  $x$  and  $y$  are relatively prime.

So,  $x$  and  $i$  are relatively prime.

Assume  $x$  and  $k$  are not relatively prime.

Let  $x=fX$ ,  $k=fK$ .

$$i = x + k$$

$$i = f(X + K)$$

But this contradicts because  $x$  and  $i$  are relatively prime.

So,  $x$  and  $k$  are relatively prime.

Let  $x=(u-v')$ ,  $k=(u+v')$ .

Assume  $u$  and  $v'$  are not relatively prime.

Let  $u=fU$ ,  $v'=fV'$ .

$$x = f(U - V')$$

$$k = f(U + V')$$

But this contradicts because  $x$  and  $k$  are relatively prime.

So,  $u$  and  $v'$  are relatively prime.

Also  $u$  and  $v'$  are opposite parity because  $x$  and  $k$  are odd.

$$x^3 + k^3 = (u - v')^3 + (u + v')^3$$

$$x^3 + k^3 = 2u(u^2 + 3v'^2)$$

Assume  $u$ =even,  $v'$ =odd.

$u^2 + 3v'^2$  is odd.

Assume  $u$ =odd,  $v'$ =even.

$u^2 + 3v'^2$  is odd.

So,  $u^2 + 3v'^2$  is always odd.

Thus the greatest common factor of  $2u$  and  $u^2 + 3v'^2$  is odd.

Assume the common factor is odd except 1 and 3.

Let  $u=fU$  and  $u^2 + 3v'^2=fN$ .

$$3v'^2 = f(N - fU^2)$$

$u$  and  $v'$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v'$  are relatively prime.

So, the greatest common factor of  $2u$  and  $u^2 + 3v'^2$  is either 1 or 3.

Assume the greatest common factor of  $2u$  and  $u^2 + 3v'^2$  is 1.

It is possible that  $2u = p^3$  and  $u^2 + 3v'^2 = \gamma$ .

$$x^3 + k^3 = \gamma p^3$$

Assume  $x^3 = \gamma x'$ ,  $k^3 = \gamma k'$  or  $x^3 = \alpha p^3$ ,  $k^3 = \beta p^3$ .

But this contradicts because  $x$  and  $k$  are relatively prime.

So,  $\gamma$  must be  $s^3$ .

$$x^3 + k^3 = s^3 p^3$$

Where  $s$  and  $p$  are relatively prime

Assume the greatest common factor of  $2u$  and  $u^2 + 3v'^2$  is 3.

Let  $u=3r$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v'$  are relatively prime.

$$x^3 + k^3 = 6r(9r^2 + 3v'^2)$$

$$x^3 + k^3 = 18r(v'^2 + 3r^2)$$

The greatest common factor of  $18r$  and  $v'^2 + 3r^2$  must be 1.

It is possible that  $18r = p'^3$  and  $u'^2 + 3r^2 = \gamma'$ .

$$x^3 + k^3 = \gamma' p'^3$$

Assume  $x^3 = \gamma' x'$ ,  $k^3 = \gamma' k'$  or  $x^3 = \alpha' p'^3$ ,  $k^3 = \beta' p'^3$ .

But this contradicts because  $x$  and  $k$  are relatively prime.

So,  $\gamma'$  must be  $s'^3$ .



$$x^3 + k^3 = s'^3 p'^3$$

Where  $s'$  and  $p'$  are relatively prime.

$$\text{Let } x^3 = a^3, k^3 = b^3, p'^3 s'^3 = c^3$$

$$a^3 + b^3 = c^3$$

But this equation contradicts by the method of infinite descent.

Therefore, there are no positive integers in case of  $n=3$  since all of Case 1, 2 and 3 are contradiction.

### 3. Proof for $n=5$

$$x^5 + y^5 = z^5, (x < y < z)$$

Where  $x, y$  and  $z =$  positive integer, relatively prime

This equation can be classified into three categories as follows.

Case 1.  $(x, y, z) = (\text{even}, \text{odd}, \text{odd})$

Case 2.  $(x, y, z) = (\text{odd}, \text{even}, \text{odd})$

Case 3.  $(x, y, z) = (\text{odd}, \text{odd}, \text{even})$

Case 1.  $(x, y, z) = (\text{even}, \text{odd}, \text{odd})$

$$\text{Let } y=(u-v), z=(u+v).$$

Assume  $u$  and  $v$  are not relatively prime.

$$\text{Let } u=fU, v=fV.$$

$$y = f(U - V)$$

$$z = f(U + V)$$

But this contradicts because  $y$  and  $z$  are relatively prime.

So,  $u$  and  $v$  are relatively prime.

Also  $u$  and  $v$  are opposite parity because  $y$  and  $z$  are odd.

$$x^5 = (u + v)^5 - (u - v)^5$$

$$x^5 = 2v(v^4 + 5(u^4 + 2u^2v^2))$$

Assume  $u=\text{even}, v=\text{odd}$ .

$v^4 + 5(u^4 + 2u^2v^2)$  is odd.

Assume  $u=\text{odd}, v=\text{even}$ .

$v^4 + 5(u^4 + 2u^2v^2)$  is odd.

So,  $v^4 + 5(u^4 + 2u^2v^2)$  is always odd.

Thus the greatest common factor of  $2v$  and  $v^4 + 5(u^4 + 2u^2v^2)$  is odd.

Assume the common factor is odd except 1 and 5.

Let  $v = fV$  and  $v^4 + 5(u^4 + 2u^2v^2) = fN$ .

$$f^4V^4 + 5(u^4 + 2u^2f^2V^2) = fN$$

$$5u^4 = f(N - f^3V^4 - 2u^2fV^2)$$

$u$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v$  are relatively prime.

So, the greatest common factor of  $2v$  and  $v^4 + 5(u^4 + 2u^2v^2)$  is either 1 or 5.

Assume the greatest common factor of  $2v$  and  $v^4 + 5(u^4 + 2u^2v^2)$  is 1.

It is possible that  $2v = p^5$  and  $v^4 + 5(u^4 + 2u^2v^2) = q^5$ .

Where  $p$  and  $q$  are relatively prime

$$x^5 = p^5q^5$$

Assume the greatest common factor of  $2v$  and  $v^4 + 5(u^4 + 2u^2v^2)$  is 5.

Let  $v=5r$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v$  are relatively prime.

$$x^5 = 10r(5^4r^4 + 5(u^4 + 2u^25^2r^2))$$

$$x^5 = 50r(5^3r^4 + (u^4 + 2u^25^2r^2))$$

The greatest common factor of  $50r$  and  $5^3r^4 + (u^4 + 2u^25^2r^2)$  must be 1.

It is possible that  $50r = p'^5$  and  $5^3r^4 + (u^4 + 2u^25^2r^2) = q'^5$ .

$$x^5 = p'^5q'^5$$

Assume  $z=y+i$ .

From  $y=(u-v)$ ,  $z=(u+v)$ ,

$$z - y = (u + v) - (u - v) = 2v$$

$$2v = i$$

Assume  $y=k+i$ .

$k$  is odd because  $y$  is odd and  $i$  is even.

Assume  $y$  and  $i$  are not relatively prime.

Let  $y=fY, i=fI$

$$z = y + i$$

$$z = f(Y + I)$$

But this contradicts because  $y$  and  $z$  are relatively prime.

So,  $y$  and  $i$  are relatively prime.

Assume  $k$  and  $y$  are not relatively prime.

Let  $k=fK, y=fY$ .

$$i = y - k$$

$$i = f(Y - K)$$

But this contradicts because  $y$  and  $i$  are relatively prime.

So,  $k$  and  $y$  are relatively prime.

Let  $k=(u'-v), y=(u'+v)$ .

Assume  $u'$  and  $v$  are not relatively prime.

Let  $u'=fU', v=fV$

$$k = f(U' - V)$$

$$y = f(U' + V)$$

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $u'$  and  $v$  are relatively prime.

Also  $u'$  and  $v$  are opposite parity because  $k$  and  $y$  are odd.

$$y^5 - k^5 = (u' + v)^5 - (u' - v)^5$$

$$y^5 - k^5 = 2v(v^4 + 5(u'^4 + 2u'^2v^2))$$

Assume  $u'$ =even,  $v$ =odd.

$v^4 + 5(u'^4 + 2u'^2v^2)$  is odd.

Assume  $u'$ =odd,  $v$ =even.

$v^4 + 5(u'^4 + 2u'^2v^2)$  is odd.

So,  $v^4 + 5(u'^4 + 2u'^2v^2)$  is always odd.

Thus the greatest common factor of  $2v$  and  $v^4 + 5(u'^4 + 2u'^2v^2)$  is odd.

Assume the common factor is odd except 1 and 5.

Let  $v = fV$  and  $v^4 + 5(u'^4 + 2u'^2v^2) = fN$ .

$$f^4V^4 + 5(u'^4 + 2u'^2f^2V^2) = fN$$

$$5u'^4 = f(N - f^3V^4 - 2u'^2fV^2)$$

$u'$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u'$  and  $v$  are relatively prime.

So, the greatest common factor of  $2v$  and  $v^4 + 5(u'^4 + 2u'^2v^2)$  is either 1 or 5.

Assume the greatest common factor of  $2v$  and  $v^4 + 5(u'^4 + 2u'^2v^2)$  is 1,

It is possible that  $2v = p^5$  and  $v^4 + 5(u'^4 + 2u'^2v^2) = \gamma$ .

$$y^5 - k^5 = \gamma p^5$$

Assume  $y^5 = \gamma y'$ ,  $k^5 = \gamma k'$  or  $y^5 = \alpha p^5$ ,  $k^5 = \beta p^5$ .

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $\gamma$  must be  $s^5$ .

$$y^5 - k^5 = s^5 p^5$$

Where  $s$  and  $p$  are relatively prime

Let  $y^5 = c^5$ ,  $k^5 = b^5$ ,  $s^5 p^5 = a^5$

$$a^5 + b^5 = c^5$$

But this equation contradicts by the method of infinite descent.

Assume the greatest common factor of  $2v$  and  $v^4 + 5(u'^4 + 2u'^2v^2)$  is 5.

Let  $v=5r$ .

$u'$  and  $r$  are relatively prime because  $u'$  and  $v$  are relatively prime.

$$y^5 - k^5 = 10r(5^4r^4 + 5(u'^4 + 2u'^25^2r^2))$$

$$y^5 - k^5 = 50r(5^3r^4 + (u'^4 + 2u'^25^2r^2))$$

The greatest common factor of  $50r$  and  $5^3r^4 + (u'^4 + 2u'^25^2r^2)$  must be 1.

It is possible that  $50r = p'^5$  and  $5^3r^4 + (u'^4 + 2u'^25^2r^2) = \gamma'$ .

$$y^5 - k^5 = \gamma' p'^5$$

Assume  $y^5 = \gamma'y'$ ,  $k^5 = \gamma'k'$  or  $y^5 = \alpha'p'^5$ ,  $k^5 = \beta'p'^5$

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $\gamma'$  must be  $s'^5$ .

$$y^5 - k^5 = s'^5 p'^5$$

Where  $s'$  and  $p'$  are relatively prime.

Let  $y^5 = c^5$ ,  $k^5 = b^5$ ,  $s'^5 p'^5 = a^5$ .

$$a^5 + b^5 = c^5$$

But this contradicts by the method of infinite descent.

Case 2.  $(x, y, z) = (\text{odd}, \text{even}, \text{odd})$

Let  $x=(u-v)$ ,  $z=(u+v)$ .

Case 2 has no solution because it can be proved in the same form as Case 1.

Case 3.  $(x, y, z) = (\text{odd}, \text{odd}, \text{even})$

Let  $x=(u-v)$ ,  $y=(u+v)$ .

Assume  $u$  and  $v$  are not relatively prime.

Let  $u=fU$ ,  $v=fV$ .

$$x = f(U - V)$$

$$y = f(U + V)$$

But this contradicts because  $x$  and  $y$  are relatively prime.

So,  $u$  and  $v$  are relatively prime.

Also  $u$  and  $v$  are opposite parity because  $x$  and  $y$  are odd.

$$z^5 = (u + v)^5 + (u - v)^5$$

$$z^5 = 2u(u^4 + 5(v^4 + 2v^2u^2))$$

Assume  $u=\text{even}$ ,  $v=\text{odd}$ .

$u^4 + 5(v^4 + 2v^2u^2)$  is odd.

Assume  $u=\text{odd}$ ,  $v=\text{even}$ .

$u^4 + 5(v^4 + 2v^2u^2)$  is odd.

So,  $u^4 + 5(v^4 + 2v^2u^2)$  is always odd.

Thus the greatest common factor of  $2u$  and  $u^4 + 5(v^4 + 2v^2u^2)$  is odd.

Assume the common factor is odd except 1 and 5.

Let  $u = fU$  and  $u^4 + 5(v^4 + 2v^2u^2) = fN$ .

$$f^4U^4 + 5(v^4 + 2v^2f^2U^2) = fN$$

$$5v^4 = f(N - f^3U^4 - 2v^2fU^2)$$

$u$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v$  are relatively prime.

So, the greatest common factor of  $2u$  and  $u^4 + 5(v^4 + 2v^2u^2)$  is either 1 or 5.

Assume the greatest common factor of  $2u$  and  $u^4 + 5(v^4 + 2v^2u^2)$  is 1.

It is possible that  $2u = p^5$  and  $u^4 + 5(v^4 + 2v^2u^2) = q^5$ .

Where  $p$  and  $q$  are relatively prime

$$z^5 = p^5q^5$$

Assume the greatest common factor of  $2v$  and  $v^4 + 5(u^4 + 2u^2v^2)$  is 5.

Let  $u=5r$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v$  are relatively prime.

$$x^5 = 10r(5^4r^4 + 5(v^4 + 2v^25^2r^2))$$

$$x^5 = 50r(5^3r^4 + (v^4 + 2v^25^2r^2))$$

The greatest common factor of  $50r$  and  $5^3r^4 + (v^4 + 2v^25^2r^2)$  must be 1.

It is possible that  $50r = p'^5$  and  $5^3r^4 + (v^4 + 2v^25^2r^2) = q'^5$ .

$$x^5 = p'^5q'^5$$

$v$  and  $r$  are relatively prime because  $u$  and  $v$  are relatively prime.

Assume  $x+y=i$ .

From  $x=(u-v)$ ,  $y=(u+v)$ ,

$$x + y = (u + v) + (u - v) = 2u$$

$$2u = i$$

Assume  $x+k=i$ .

k is odd because x is odd and i is even.

Assume x and i are not relatively prime.

Let  $x=fX$ ,  $i=fi$ .

$$y = i - x$$

$$y = f(I - K)$$

But this contradicts because x and y are relatively prime.

So, x and i are relatively prime.

Assume x and k are not relatively prime.

Let  $x=fX$ ,  $k=fK$ .

$$i = x + k$$

$$i = f(X + K)$$

But this contradicts because x and i are relatively prime.

So, x and k are relatively prime.

Let  $x=(u-v')$ ,  $k=(u+v')$ .

Assume u and v' are not relatively prime.

Let  $u=fU$ ,  $v'=fV'$ .

$$x = f(U - V')$$

$$k = f(U + V')$$

But this contradicts because x and k are relatively prime.

So, u and v' are relatively prime.

Also u and v' are opposite parity because x and k are odd.

$$x^5 + k^5 = (u - v')^5 + (u + v')^5$$

$$x^5 + k^5 = 2u(u^4 + 5(v'^4 + 2v'^2u^2))$$

Assume  $u=\text{even}$ ,  $v'=\text{odd}$ .

$u^4 + 5(v'^4 + 2v'^2u^2)$  is odd.

Assume  $u=\text{odd}$ ,  $v'=\text{even}$ .

$u^4 + 5(v'^4 + 2v'^2u^2)$  is odd.

So,  $u^4 + 5(v'^4 + 2v'^2u^2)$  is always odd.

Thus the greatest common factor of  $2u$  and  $u^4 + 5(v'^4 + 2v'^2u^2)$  is odd.

Assume the common factor is odd except 1 and 5.

Let  $u = fU$  and  $u^4 + 5(v'^4 + 2v'^2u^2) = fN$ .

$$f^4U^4 + 5(v'^4 + 2v'^2f^2U^2) = fN$$

$$5v'^4 = f(N - f^3U^4 - 2v'^2fU^2)$$

$u$  and  $v'$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v'$  are relatively prime.

So, the greatest common factor of  $2u$  and  $u^4 + 5(v'^4 + 2v'^2u^2)$  is either 1 or 5.

Assume the greatest common factor of  $2u$  and  $u^4 + 5(v'^4 + 2v'^2u^2)$  is 1.

It is possible that  $2u = p^5$  and  $u^4 + 5(v'^4 + 2v'^2u^2) = \gamma$ .

$$x^5 + k^5 = \gamma p^5$$

Assume  $x^5 = \gamma x'$ ,  $k^5 = \gamma k'$  or  $x^5 = \alpha p^5$ ,  $k^5 = \beta p^5$ .

But this contradicts because  $x$  and  $k$  are relatively prime.

So,  $\gamma$  must be  $s^5$ .

$$x^5 + k^5 = s^5 p^5$$

Where  $s$  and  $p$  are relatively prime

Assume the greatest common factor of  $2u$  and  $u^4 + 5(v'^4 + 2v'^2u^2)$  is 5.

Let  $u=5r$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v'$  are relatively prime.

$$x^5 = 10r(5^4r^4 + 5(v'^4 + 2v'^25^2r^2))$$

$$x^5 = 50r(5^3r^4 + (v'^4 + 2v'^25^2r^2))$$

The greatest common factor of  $50r$  and  $5^3r^4 + (v'^4 + 2v'^25^2r^2)$  must be 1.

It is possible that  $50r = p'^5$  and  $5^3r^4 + (v'^4 + 2v'^25^2r^2) = \gamma'$ .

$$x^5 = \gamma' p'^5$$

Assume  $x^5 = \gamma' x'$ ,  $k^5 = \gamma' k'$  or  $x^5 = \alpha' p'^5$ ,  $k^5 = \beta' p'^5$ .

But this contradicts because  $x$  and  $k$  are relatively prime.



So,  $\gamma'$  must be  $s'^5$ .

$$x^5 + k^5 = s'^5 p'^5$$

Where  $s'$  and  $p'$  are relatively prime.

$$\text{Let } x^5 = a^5, k^5 = b^5, p'^5 s'^5 = c^5$$

$$a^5 + b^5 = c^5$$

But this equation contradicts by the method of infinite descent.

Therefore, there are no positive integers in case of  $n=5$  since all of Case 1, 2 and 3 are contradiction.

#### 4. Proof for $n=\text{odd primes}$

$$x^n + y^n = z^n, (x < y < z)$$

Where  $x, y$  and  $z =$  positive integer, relatively prime

This equation can be classified into three categories as follows.

Case 1.  $(x, y, z) = (\text{even}, \text{odd}, \text{odd})$

Case 2.  $(x, y, z) = (\text{odd}, \text{even}, \text{odd})$

Case 3.  $(x, y, z) = (\text{odd}, \text{odd}, \text{even})$

Case 1.  $(x, y, z) = (\text{even}, \text{odd}, \text{odd})$

Let  $y=(u-v), z=(u+v)$ .

Assume  $u$  and  $v$  are not relatively prime.

Let  $u=fU, v=fV$ .

$$y = f(U - V)$$

$$z = f(U + V)$$

But this contradicts because  $y$  and  $z$  are relatively prime.

So,  $u$  and  $v$  are relatively prime.

Also  $u$  and  $v$  are opposite parity because  $y$  and  $z$  are odd.

$$x^n = (u + v)^n - (u - v)^n$$

$$x^n = (u^n + C_1 u^{n-1} v^1 + C_2 u^{n-2} v^2 + \dots + C_{n-2} u^2 v^{n-2} + C_{n-1} u^1 v^{n-1} + v^n) - (u^n - C_1 u^{n-1} v^1 + C_2 u^{n-2} v^2 + \dots - C_{n-2} u^2 v^{n-2} + C_{n-1} u^1 v^{n-1} - v^n)$$

Where  $C = \{C_1, C_2, \dots, C_{n-2}, C_{n-1}\} = \left\{ \frac{n}{1}, \frac{n(n-1)}{1 \cdot 2}, \dots, \frac{n(n-1) \dots 4 \cdot 3}{1 \cdot 2 \dots (n-3)(n-2)}, \frac{n(n-1) \dots 2 \cdot 1}{1 \cdot 2 \dots (n-2)(n-1)} \right\}$

$$x^n = 2v(v^{n-1} + n(u^{n-1} + C'_3 u^{n-3}v^2 + \dots + C'_{n-2} u^2 v^{n-3}))$$

Where  $C' = \{C'_1, C'_2, \dots, C'_{n-2}, C'_{n-1}\} = \left\{ \frac{1}{1}, \frac{(n-1)}{1 \cdot 2}, \dots, \frac{(n-1) \dots 4 \cdot 3}{1 \cdot 2 \dots (n-3)(n-2)}, \frac{(n-1) \dots 2 \cdot 1}{1 \cdot 2 \dots (n-2)(n-1)} \right\}$

$$x^n = 2v(v^{n-1} + n(u^{n-1} + M))$$

Where  $M = C'_3 u^{n-3}v^2 + C'_5 u^{n-5}v^4 + \dots + C'_{n-4} u^4 v^{n-5} + C'_{n-2} u^2 v^{n-3}$

Assume  $u=\text{even}, v=\text{odd}$ .

$v^{n-1} + n(u^{n-1} + M)$  is odd.

Assume  $u=\text{odd}, v=\text{even}$ .

$v^{n-1} + n(u^{n-1} + M)$  is odd.

So,  $v^{n-1} + n(u^{n-1} + M)$  is always odd.

Thus the greatest common factor of  $2v$  and  $v^{n-1} + n(u^{n-1} + M)$  is odd.

Assume the common factor is odd except 1 and  $n$ .

Let  $v = fV$  and  $v^{n-1} + n(u^{n-1} + M) = fN$ .

$$f^{n-1}V^{n-1} + n(u^{n-1} + fM') = fN$$

Where  $fM' = f(C'_3 u^{n-3}f^1V^2 + C'_5 u^{n-5}f^3V^4 + \dots + C'_{n-4} u^4 f^{n-6}V^{n-5} + C'_{n-2} u^2 f^{n-4}V^{n-3})$

$$nu^{n-1} = f(N - f^{n-2}V^{n-1} - nM')$$

$u$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v$  are relatively prime.

So, the greatest common factor of  $2v$  and  $v^{n-1} + n(u^{n-1} + M)$  is either 1 or  $n$ .

Assume the greatest common factor of  $2v$  and  $v^{n-1} + n(u^{n-1} + M)$  is 1.

It is possible that  $2v = p^n$  and  $v^{n-1} + n(u^{n-1} + M) = q^n$ .

Where  $p$  and  $q$  are relatively prime

$$x^n = p^n q^n$$

Assume the greatest common factor of  $2v$  and  $v^{n-1} + n(u^{n-1} + M)$  is  $n$ .

Let  $v=nr$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v$  are relatively prime.

$$x^n = 2nv(v^{n-1} + n(u^{n-1} + M))$$

$$x^n = 2n^2v(n^{n-2}r^{n-1} + (u^{n-1} + M))$$

The greatest common factor of  $2n^2v$  and  $n^{n-2}r^{n-1} + (u^{n-1} + M)$  must be 1.

It is possible that  $2n^2v = p'^n$  and  $n^{n-2}r^{n-1} + (u^{n-1} + M) = q'^n$ .

$$x^n = p'^n q'^n$$

Assume  $z=y+i$ .

From  $y=(u-v)$ ,  $z=(u+v)$ ,

$$z - y = (u + v) - (u - v) = 2v$$

$$2v = i$$

Assume  $y=k+i$ .

$k$  is odd because  $y$  is odd and  $i$  is even.

Assume  $y$  and  $i$  are not relatively prime.

Let  $y=fY$ ,  $i=fl$

$$z = y + i$$

$$z = f(Y + l)$$

But this contradicts because  $y$  and  $z$  are relatively prime.

So,  $y$  and  $i$  are relatively prime.

Assume  $k$  and  $y$  are not relatively prime.

Let  $k=fK$ ,  $y=fY$ .

$$i = y - k$$

$$i = f(Y - K)$$

But this contradicts because  $y$  and  $i$  are relatively prime.

So,  $k$  and  $y$  are relatively prime.

Let  $k=(u'-v)$ ,  $y=(u'+v)$ .

Assume  $u'$  and  $v$  are not relatively prime.

Let  $u'=fU'$ ,  $v=fV$

$$k = f(U' - V)$$

$$y = f(U' + V)$$

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $u'$  and  $v$  are relatively prime.

Also  $u'$  and  $v$  are opposite parity because  $k$  and  $y$  are odd.

$$y^n - k^n = (u' + v)^n - (u' - v)^n$$

$$y^n - k^n = 2v(v^{n-1} + n(u'^{n-1} + l))$$

Where  $l = C'_3 u'^{n-3} v^2 + C'_5 u'^{n-5} v^4 + \dots + C'_{n-4} u'^4 v^{n-5} + C'_{n-2} u'^2 v^{n-3}$

Assume  $u'$ =even,  $v$ =odd.

$v^{n-1} + n(u'^{n-1} + l)$  is odd.

Assume  $u'$ =odd,  $v$ =even.

$v^{n-1} + n(u'^{n-1} + l)$  is odd.

So,  $v^{n-1} + n(u'^{n-1} + l)$  is always odd.

Thus the greatest common factor of  $2v$  and  $v^{n-1} + n(u'^{n-1} + l)$  is odd.

Assume the common factor is odd except 1 and  $n$ .

Let  $v = fV$  and  $v^{n-1} + n(u'^{n-1} + l) = fN$ .

$$f^{n-1} V^{n-1} + n(u'^{n-1} + fl') = fN$$

Where  $fl' = f(C'_3 u'^{n-3} f^1 V^2 + C'_5 u'^{n-5} f^3 V^4 + \dots + C'_{n-4} u'^4 f^{n-6} V^{n-5} + C'_{n-2} u'^2 f^{n-4} V^{n-3})$

$$nu'^{n-1} = f(N - f^{n-2} V^{n-1} - nl')$$

$u'$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u'$  and  $v$  are relatively prime.

So, the greatest common factor of  $2v$  and  $v^{n-1} + n(u'^{n-1} + l)$  is either 1 or  $n$ .

Assume the greatest common factor of  $2v$  and  $v^{n-1} + n(u'^{n-1} + l)$  is 1,

It is possible that  $2v = p^n$  and  $v^{n-1} + n(u'^{n-1} + l) = \gamma$ .

$$y^n - k^n = \gamma p^n$$

Assume  $y^n = \gamma y'$ ,  $k^n = \gamma k'$  or  $y^n = \alpha p^n$ ,  $k^n = \beta p^n$ .

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $\gamma$  must be  $s^n$ .

$$y^n - k^n = s^n p^n$$

Where  $s$  and  $p$  are relatively prime

Let  $y^n = c^n, k^n = b^n, s^n p^n = a^n$

$$a^n + b^n = c^n$$

But this equation contradicts by the method of infinite descent.

Assume the greatest common factor of  $2v$  and  $v^{n-1} + n(u'^{n-1} + 1)$  is  $n$ .

Let  $v=nr$ .

$u'$  and  $r$  are relatively prime because  $u'$  and  $v$  are relatively prime.

$$y^n - k^n = 2nr(v^{n-1} + n(u'^{n-1} + 1))$$

$$y^n - k^n = 2n^2r(n^{n-2}r^{n-1} + (u'^{n-1} + 1))$$

The greatest common factor of  $2n^2r$  and  $n^{n-2}r^{n-1} + (u'^{n-1} + 1)$  must be 1.

It is possible that  $2n^2r = p'^n$  and  $n^{n-2}r^{n-1} + (u'^{n-1} + 1) = \gamma'$ .

$$y^n - k^n = \gamma'p'^n$$

Assume  $y^n = \gamma'y', k^n = \gamma'k'$  or  $y^n = \alpha'p'^n, k^n = \beta'p'^n$

But this contradicts because  $k$  and  $y$  are relatively prime.

So,  $\gamma'$  must be  $s'^n$ .

$$y^n - k^n = s'^n p'^n$$

Where  $s'$  and  $p'$  are relatively prime.

Let  $y^n = c^n, k^n = b^n, s'^n p'^n = a^n$ .

$$a^n + b^n = c^n$$

But this equation contradicts by the method of infinite descent.

Case 2.  $(x, y, z) = (\text{odd}, \text{even}, \text{odd})$

Let  $x=(u-v), z=(u+v)$ .

Case 2 has no solution because it can be proved in the same form as Case 1.

Case 3.  $(x, y, z) = (\text{odd}, \text{odd}, \text{even})$

Let  $x=(u-v), y=(u+v)$ .

Assume  $u$  and  $v$  are not relatively prime.

Let  $u=fU, v=fV$ .

$$x = f(U - V)$$

$$y = f(U + V)$$

But this contradicts because  $x$  and  $y$  are relatively prime.

So,  $u$  and  $v$  are relatively prime.

Also  $u$  and  $v$  are opposite parity because  $x$  and  $y$  are odd.

$$z^n = (u - v)^n + (u + v)^n$$

$$z^n = (u^n - C_1 u^{n-1} v^1 + C_2 u^{n-2} v^2 + \dots - C_{n-2} u^2 v^{n-2} + C_{n-1} u^1 v^{n-1} - v^n) + (u^n + C_1 u^{n-1} v^1 + C_2 u^{n-2} v^2 + \dots + C_{n-2} u^2 v^{n-2} + C_{n-1} u^1 v^{n-1} + v^n)$$

$$\text{Where } C = \{C_1, C_2, \dots, C_{n-2}, C_{n-1}\} = \left\{ \frac{n}{1}, \frac{n(n-1)}{1 \cdot 2}, \dots, \frac{n(n-1) \dots 4 \cdot 3}{1 \cdot 2 \dots (n-3)(n-2)}, \frac{n(n-1) \dots 2 \cdot 1}{1 \cdot 2 \dots (n-2)(n-1)} \right\}$$

$$x^n = 2u(u^{n-1} + n(v^{n-1} + C'_{n-3} v^{n-3} u^2 + \dots + C'_2 v^2 u^{n-3}))$$

$$\text{Where } C' = \{C'_1, C'_2, \dots, C'_{n-2}, C'_{n-1}\} = \left\{ \frac{1}{1}, \frac{(n-1)}{1 \cdot 2}, \dots, \frac{(n-1) \dots 4 \cdot 3}{1 \cdot 2 \dots (n-3)(n-2)}, \frac{(n-1) \dots 2 \cdot 1}{1 \cdot 2 \dots (n-2)(n-1)} \right\}$$

$$x^n = 2u(u^{n-1} + n(v^{n-1} + M))$$

$$\text{Where } M = C'_{n-3} v^{n-3} u^2 + C'_{n-5} v^{n-5} u^4 + \dots + C'_4 v^4 u^{n-5} + C'_2 v^2 u^{n-3}$$

Assume  $u = \text{even}, v = \text{odd}$ .

$$u^{n-1} + n(v^{n-1} + M) \text{ is odd.}$$

Assume  $u = \text{odd}, v = \text{even}$ .

$$u^{n-1} + n(v^{n-1} + M) \text{ is odd.}$$

So,  $u^{n-1} + n(v^{n-1} + M)$  is always odd.

Thus the greatest common factor of  $2u$  and  $u^{n-1} + n(v^{n-1} + M)$  is odd.

Assume the common factor is odd except 1 and  $n$ .

$$\text{Let } u = fU \text{ and } u^{n-1} + n(v^{n-1} + M) = fN.$$

$$f^{n-1} U^{n-1} + n(u^{n-1} + fM') = fN$$

$$\text{Where } fM' = f(C'_{n-3} v^{n-3} f^1 U^2 + C'_{n-5} v^{n-5} f^3 U^4 + \dots + C'_4 v^4 f^{n-6} U^{n-5} + C'_2 v^2 f^{n-4} U^{n-3})$$

$$nu^{n-1} = f(N - f^{n-2} U^{n-1} - nM')$$

$u$  and  $v$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v$  are relatively prime.

So, the greatest common factor of  $2u$  and  $u^{n-1} + n(v^{n-1} + M)$  is either 1 or  $n$ .

Assume the greatest common factor of  $2u$  and  $u^{n-1} + n(v^{n-1} + M)$  is 1.

It is possible that  $2u = p^n$  and  $u^{n-1} + n(v^{n-1} + M) = q^n$ .

Where  $p$  and  $q$  are relatively prime

$$z^n = p^n q^n$$

Assume the greatest common factor of  $2v$  and  $u^{n-1} + n(v^{n-1} + M)$  is  $n$ .

Let  $u=nr$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v$  are relatively prime.

$$x^n = 2nr(u^{n-1} + n(v^{n-1} + M))$$

$$x^n = 2n^2r(n^{n-2}r^{n-1} + (v^{n-1} + M))$$

The greatest common factor of  $2n^2r$  and  $n^{n-2}r^{n-1} + (v^{n-1} + M)$  must be 1.

It is possible that  $2n^2r = p'^n$  and  $n^{n-2}r^{n-1} + (v^{n-1} + M) = q'^n$ .

$$x^n = p'^n q'^n$$

Assume  $x+k=i$ .

$k$  is odd because  $x$  is odd and  $i$  is even.

Assume  $x$  and  $i$  are not relatively prime.

Let  $x=fX, i=fl$ .

$$y = i - x$$

$$y = f(l - X)$$

But this contradicts because  $x$  and  $y$  are relatively prime.

So,  $x$  and  $i$  are relatively prime.

Assume  $x$  and  $k$  are not relatively prime.

Let  $x=fX, k=fK$ .

$$i = x + k$$

$$i = f(X + K)$$

But this contradicts because  $x$  and  $i$  are relatively prime.

So,  $x$  and  $k$  are relatively prime.

Let  $x=(u-v'), k=(u+v')$ .

Assume  $u$  and  $v'$  are not relatively prime.

Let  $u=fU, v'=fV'$ .

$$x = f(U - V')$$

$$k = f(U + V')$$

But this contradicts because  $x$  and  $k$  are relatively prime.

So,  $u$  and  $v'$  are relatively prime.

Also  $u$  and  $v'$  are opposite parity because  $x$  and  $k$  are odd.

$$x^n + k^n = (u - v')^n + (u + v')^n$$

$$x^n + k^n = 2u(u^{n-1} + n(v'^{n-1} + l))$$

Where  $l = C'_{n-3}v'^{n-3}u^2 + C'_{n-5}v'^{n-5}u^4 + \dots + C'_4v'^4u^{n-5} + C'_2v'^2u^{n-3}$

Assume  $u=\text{even}, v'=\text{odd}$ .

$u^{n-1} + n(v'^{n-1} + l)$  is odd.

Assume  $u=\text{odd}, v'=\text{even}$ .

$u^{n-1} + n(v'^{n-1} + l)$  is odd.

So,  $u^{n-1} + n(v'^{n-1} + l)$  is always odd.

Thus the greatest common factor of  $2u$  and  $u^{n-1} + n(v'^{n-1} + l)$  is odd.

Assume the common factor is odd except 1 and  $n$ .

Let  $u = fU$  and  $u^{n-1} + n(v'^{n-1} + l) = fN$ .

$$f^{n-1}U^{n-1} + n(v'^{n-1} + fl') = fN$$

Where  $fl' = f(C'_{n-3}v'^{n-3}f^1U^2 + C'_{n-5}v'^{n-5}f^3U^4 + \dots + C'_4v'^4f^{n-6}U^{n-5} + C'_2v'^2f^{n-4}U^{n-3})$

$$nv'^{n-1} = f(N - f^{n-2}U^{n-1} - nl')$$

$u$  and  $v'$  have common factor of  $f$ .

But this contradicts because  $u$  and  $v'$  are relatively prime.

So, the greatest common factor of  $2u$  and  $u^{n-1} + n(v'^{n-1} + l)$  is either 1 or  $n$ .

Assume the greatest common factor of  $2u$  and  $u^{n-1} + n(v'^{n-1} + l)$  is 1.

It is possible that  $2u = p^n$  and  $u^{n-1} + n(v'^{n-1} + l) = \gamma$ .

$$x^n + k^n = \gamma p^n$$

Assume  $x^n = \gamma x' k^n = \gamma k'$  or  $x^n = \alpha p^n, k^n = \beta p^n$ .



But this contradicts because x and k are relatively prime.

So,  $\gamma$  must be  $s^n$ .

$$x^n + k^n = s^n p^n$$

Where s and p are relatively prime

Let  $x^n = a^n, k^n = b^n, s^n p^n = c^n$

$$a^n + b^n = c^n$$

But this equation contradicts by the method of infinite descent.

Assume the greatest common factor of  $2u$  and  $u^{n-1} + n(v'^{n-1} + 1)$  is n.

Let  $u=nr$ .

$u$  and  $r$  are relatively prime because  $u$  and  $v'$  are relatively prime.

$$x^n + k^n = 2nr(u^{n-1} + n(v'^{n-1} + 1))$$

$$x^n + k^n = 2n^2r(n^{n-2}r^{n-1} + (v'^{n-1} + 1))$$

The greatest common factor of  $2n^2r$  and  $n^{n-2}r^{n-1} + (v'^{n-1} + 1)$  must be 1.

It is possible that  $2n^2r = p'^n$  and  $n^{n-2}r^{n-1} + (v'^{n-1} + 1) = \gamma'$ .

$$x^n + k^n = \gamma' p'^n$$

Assume  $x^n = \gamma' x', k^n = \gamma' k'$  or  $x^n = \alpha' p'^n, k^n = \beta' p'^n$ .

But this contradicts because x and k are relatively prime.

So,  $\gamma'$  must be  $s'^n$ .

$$x^n + k^n = s'^n p'^n$$

Where  $s'$  and  $p'$  are relatively prime.

Let  $x^n = a^n, k^n = b^n, s'^n p'^n = c^n$

$$a^n + b^n = c^n$$

But this equation contradicts by the method of infinite descent.

Therefore, there are no positive integers in case of  $n=\text{odd}$  primes since all of Case 1, 2 and 3 are contradiction.

## 5. REFERENCES

- [1] Andrew Wiles. Modular Elliptic Curves and Fermat's Last Theorem [J]. *Annals of Mathematics* (second series), 1995.
- [2] Euler, L., *Elements of Algebra* (3rd ed.), London: Longman, pp. 399, 401–402, 1822.