On the fundamental relation between universal gravitational constant G and Coulomb's constant k

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Abstract

In this brief paper we relate the universal gravitational constant G and Coulomb's constant k to the volumes of subatomic particles. We define a new characteristic of subatomic particles, quantum volume, which varies in an inverse proportion to the mass of the subatomic particle. As an immediate corollary, we propose an explanation to the Proton's radius puzzle that reconciles the various seemingly contradictory results obtained, checking our postulates with a prediction of upper and lower bounds for the electron's radius which is consistent with the current experimental bounds.

1 Introduction

According to Newton's law of universal gravitation, the gravitational force between two masses m_1 and m_2 separated by a distance r is:

$$F_g = G\frac{m_1 m_2}{r^2}$$

Where G is the universal gravitational constant, whose value is:

$$G = 6.674 \times 10^{-11}\frac{N \cdot m^2}{kg^2}$$

According to Coulomb's law, the electric force between two charges q_1 and q_2 separated by a distance r is:

$$F_e = k\frac{q_1 q_2}{r^2}$$

Where k is the constant of Coulomb's law, which depends on the medium in which the charges are located. For vacuum, its value is:

$$k = 8.988 \times 10^{9}\frac{N \cdot m^2}{C^2}$$

The ratio between k and G (from now on, electric-gravitational ratio E_g) is obtained by dividing both expressions:

$$E_g = \frac{k}{G} = \frac{8.988 \times 10^{9}\frac{N \cdot m^2}{C^2}}{6.674 \times 10^{-11}\frac{N \cdot m^2}{kg^2}} = 1.346 \times 10^{20}\frac{kg^2}{C^2} \tag{1}$$

2 The "happy coincidence" of quantum volume ratio and electric-gravitational ratio

In this section, we will define a new measure of the size of subatomic particles that we denominate quantum volume. We assume that the quantum volume of the proton and electron can be approximated from their radii, using the sphere formula:

$$V = \frac{4}{3}\pi r^3$$

where V is the volume and r is the radius.

The radius of the proton is measured by the charge radius, which is defined as the distance from the center of the electric charge distribution to the point where the electric potential drops to half its maximum value. The most recent value of the proton charge radius, obtained by the Muon-Proton Scattering Experiment (MUSE) experiment in 2019, is 0.833 ± 0.010 femtometers (1 fm $= 10^{-15}$ m)[6].

The electron radius is measured by the classical radius, which is defined as the distance from the center of the electric charge distribution to the point where the kinetic energy is equal to the potential energy. The value of the classical electron radius, obtained by classical electromagnetic theory, is $2.8179 \cdot 10^{-15}$ m.[2] However, this value is not compatible with quantum theory, which predicts that the electron is a point particle without spatial extension. Concretely, observation of a single electron in a Penning trap shows the upper limit of the particle's radius is in the order of 10^{-22} meters.[1] Therefore, the experimental upper limit for the electron radius is used, which is 10^{-22} m.

We can calculate the quantum volume ratio denoted as Q_v as a measure of the relative size of these subatomic particles. Estimating the quantum volumes from the radii stated before, and dividing those quantum volumes, we obtain that:

$$Q_v = \frac{V_p}{V_e} = \frac{r_p^3}{r_e^3} = \frac{(0.833 \times 10^{-15})^3}{(10^{-22})^3} = 5.92 \times 10^{20}$$

It can be seen that the quantum volume ratio is of the same order of magnitude as the ratio between Coulomb constant k and the gravitational constant G, since both are of the order of 10^{20}.

In the next section, we theorize on the possibility that this fact is not a "happy coincidence".

3 Main postulates and corollaries

3.1 Existence of a quantum density equilibrium

The first main postulate of this paper is that the electric-gravitational ratio is indeed the quantum volume ratio:

$$E_g = \frac{k}{G} = \frac{V_p}{V_e} = Q_v$$

In a more intuitive way,

$$\frac{k}{V_p} = \frac{G}{V_e} \tag{2}$$

From Newton's Law and Coulomb's Law expressions, we can consider G and k respectively as unit measures of the capacity to exert gravitational force per unit of mass (from now on, gravitational power), and the capacity to exert electrostatic force per unit of charge (from now on, electrostatic power). Therefore, if we define quantum electric density as the electrostatic power per quantum volume of some subatomic particle, and quantum gravitational density as the gravitational power per quantum volume of some subatomic particle, this relationship states that the quantum electric density of the proton is equiparable to the quantum gravitational density of the electron.

3.2 Quantum volume variability and its limits

Previous subsection result suggest that subatomic particles have some sort of "internal" equilibrium. From now on, we denote this equilibrium as quantum density equilibrium.

Our second main postulate is that quantum volume is not static once reached a minimum size threshold. For sufficiently small particles, this fundamental measure is "flexible", in the sense that particles accommodate their quantum volume to maintain their quantum density equilibrium when they get affected by different interactions or exogenous factors. We state that the classic volume could be the maximum quantum volume that some subatomic particle can reach, but its quantum volume can be reduced if necessary to reach the quantum density equilibrium; and the potential reduction that quantum volume can suffer is a function of the mass of the particle.

Let us define the maximum quantum radius $Max(r)$ as the threshold where quantum volume (flexible) starts to behave as classic volume (static). We state that

$$Max(r) = Q_v \cdot \hbar = 1.418 \cdot 10^{-14}$$

Where $\hbar$ is the reduced Planck's constant, the quantum of angular momentum in quantum mechanics.

Let us define the quantum mass m_b as the expected mass that would have some subatomic particle with $Max(r)$, and consider this value as the basic unit of quantum mass. As we have subatomic particles (proton and neutron) with radius of the same order of magnitude as $Max(r)$, we can guess that some subatomic particle with $Max(r)$ will have approximately the same density as protons and neutrons.

We estimate that the maximum quantum radius $Max(r)$ of some subatomical particle s is inversely proportional to its quantum mass squared. Therefore, if we denote the quantum radius of some subatomic particle s as r_s, and the classical radius (maximum quantum radius) of that particle as $Max(r_s)$, we have that

$$Max(r_s) \geq r_s \geq \frac{Max(r_s) \cdot m_s^2}{m_b^2} \tag{3}$$

3.3 Corollary: an explanation to the Proton's radius puzzle

The proton radius puzzle is an unanswered problem in physics relating to the size of the proton.[4] Historically, the proton charge radius was measured by two independent methods, which converged to a value of about 0.877×10^{-15}. This value was challenged by a 2010 experiment using a third method, which produced a radius about 4% smaller than this, at 0.842×10^{-15}.[5] New experimental results reported in the autumn of 2019 agree with the smaller measurement, as does a re-analysis of older data published in 2022.[7] While some believe that this difference has been resolved, this opinion is not yet universally held.[3]

3.3.1 Checking the theory validity

From the Proton's radius puzzle experiments results, we can guess that the maximum quantum radius of a proton, denoted as $Max(r_p)$, is more or less equal to 0.88×10^{-15} m, whereas the most recent experiments have shown that the quantum radius of a proton can be as small as 0.823×10^{-15} m. If we assume those values to be close to $Max(r_p)$ and $Min(r_p)$, we have that

$$m_b^2 = \frac{0.88 \times 10^{-15} \cdot (1.6726 \times 10^{-27})^2}{0.823 \times 10^{-15}}$$

$$m_b = 1.729 \times 10^{-27} \tag{4}$$

If the measures of the Proton's radius are correct in all the experiments performed, and we have derived correctly m_b from those measures, and our theory is correct, we can predict with high accuracy the bounds for the electron's radius, which should be compatible with quantum theory and consistent with the most recent experimental upper limits.

Assuming as $Max(r_e)$ the classical radius of the electron, we have that

$$2.818 \times 10^{-15} \leq r_e \leq \frac{2.818 \times 10^{-15} \cdot (9.1 \times 10^{-31})^2}{(1.729 \times 10^{-27})^2}$$

$$2.818 \times 10^{-15} \leq r_e \leq 7.8 \times 10^{-22} \tag{5}$$

This upper bound is consistent with the experimental upper limits of electron's radius.[1] This result yields credibility to our theory, which may serve both as an explanation to the Proton's radius puzzle, and as a useful tool to establish upper and lower bounds of the volume of subatomic particles, as well as to open new avenues of research to enhance our comprehension of the quantum world.

4 Final Remarks

Our postulates point towards some deep connection between the gravitational force and the electric force at a subatomic level, which forces sufficiently small particles to "adapt" internally their volume in order to reach an equilibrium. However, a more robust, general and complete theory that develops further our postulates and its consequences would be required, as well as further experiments to validate our proposed theory.

I want to specially thank my caring wife Elena for supporting me throughout this marvellous journey of free-time researching and learning during this last eight years. And "*I praise you, Father, Lord of Heaven and Earth, because you have hidden these things from the wise and learned, and revealed them to little children*" (Matthew 11, 25).

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