Solution of a Five Degree Equation

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Abstract

In this article, I solve the general equation of degree 5 of the form:

$$x^5 + 9c^2x^3 + (4ab + 24\alpha\beta - 24\alpha^2 c)x^2 - (72\alpha\beta c - 36\beta^2)x + 36abc^2 = 0$$

For this I used Mathematics that I invented. The method I invented allows me to solve All equations of degrees greater than 4 , as well as equations of the form: $x^n + mx + p = 0$

The principle, is to find for all these equations, an equation of degree three (3)

Wish has at least one solution in common with those of degree greater than 4.

With my method of course this is possible.

Attached resolution of a general equation of degree 5.

Five degré équation

$$x^5+9c^2x^3+(4ab+24\alpha\beta-24\alpha^2c)x^2-(72\alpha\beta c-36\beta^2)x+36abc^2=0......(I)$$

At least one of the solutions of this equation belongs to the solutions of the 3rd degree equation: $x^3-4\alpha^2x+4ab=0........(II)$

With conditions on α and β ; see system 3(a) and 4(a)

Or to solve the equation: $x^5+mx^3+px^2+qx+\lambda=0$.

By identification with the equation (I)

$$9c^2=m\rightarrow c=\frac{\pm\sqrt{m}}{3}.........(1)$$;

$$36abc^2=\lambda \longrightarrow ab=\frac{\lambda}{4m}.......(2)$$

$$4ab+24\alpha\beta-24c\alpha^2=p\rightarrow 24\alpha\beta\pm\frac{24\alpha^2\sqrt{m}}{3}=-4ab+p$$

$$24\alpha\beta\pm8\sqrt{m}\alpha^2=-\frac{\lambda}{m}+p\rightarrow 3\alpha\beta\pm\sqrt{m}\alpha^2=\frac{mp-\gamma}{8m}.....(3)$$

$$-72\alpha\beta c+36\beta^2=q\rightarrow\frac{-2(\pm)\sqrt{m}}{3}\alpha\beta+\beta^2=\frac{q}{36}........(4)$$

$$\left\{\begin{array}{l} 3\alpha\beta\pm\sqrt{m}\alpha^2=\dfrac{mp-\lambda}{8m}\ \text{.....}\ (3)\text{a} \\ \dfrac{-2(\pm)\sqrt{m}}{3}\alpha\beta+\beta^2=\dfrac{q}{36}\ ...(4)\text{a}. \end{array}\right.$$

by eliminating $\alpha\beta$ we obtain :

$$-\frac{2m}{9}\alpha^2+\beta^2=\frac{2(\pm\sqrt{m}}{72}(\frac{mp-\lambda}{m})+\frac{q}{36}$$. by multiplying by α^2

$$-\frac{2m}{3}\alpha^4+3\alpha^2\beta^2=\left[\frac{2(\pm\sqrt{m})}{3}\left(\frac{mp-\lambda}{8m}\right)+\frac{q}{12}\right]\alpha^2$$. by (A)

$$3\alpha\beta=\mp\sqrt{m}\alpha^2+\frac{mp-\lambda}{8m}$$

$$9\alpha^2\beta^2=m\alpha^4\mp\frac{\sqrt{m}(mp-\lambda)}{4m}\alpha^2+\left(\frac{mp-\lambda}{8m}\right)^2$$ we replace in (A)

Bisquare equation, we find α^2, we have ab, so we can solve the 5th degree equation with the algebraic radicals.

Note 1: if we replace in the equation (I), $x^3=4\alpha^2x-4ab$, we obtain the equation of the 4th degree…. . (III)

$$x^5+9c^2(4\alpha^2x-4ab)+(4ab+24\alpha\beta-24\alpha^2c)x^2-(72\alpha\beta c-36\beta^2)x+36abc^2=0......(I)$$
$$x^5+(4ab+24\alpha\beta-24\alpha^2c)x^2-(72\alpha\beta c-36\beta^2-36\alpha^2c^2)x=0$$
$$x^4+(4ab+24\alpha\beta-24\alpha^2c)x-(72\alpha\beta c-36\beta^2-36\alpha^2c^2)=0$$

$$x^4+(-4ab-24\alpha\beta+24\alpha^2c)x-(72\alpha\beta c-36\beta^2+36\alpha^2ab)=0.........(III)$$

Note 2: with this method we can solve a 4th degree equation and another of the 5th degree at the same time.

I have the general method for solving equations of any degree n.

Exemples:

(1) $$\begin{cases} x=4;a=-3;b=4;c=2;\alpha=-1;\beta=\dfrac{-2}{3} \\ x^5+36x^3-80x^2-80x-1728=0 \end{cases}$$

(2) $$\begin{cases} x=3;a=-2;b=3;c=-3;\alpha=\dfrac{-1}{2};\beta=2 \\ x^5+81x^3-30x^2-72x-1944=0 \end{cases}$$

$$\begin{cases} x=-5;a=4;b=5;c=1;\alpha=\dfrac{3}{2};\beta=4 \end{cases}$$

(3) $$x^5+9x^3+170x^2+144x+720=0$$

(4) $$\begin{cases} x=-5; a=4; b=5; c=-3; \alpha=\frac{3}{2}; \beta=-2 \\ x^5+81x^3+170x^2-504x+6480=0 \end{cases}$$

(5) $$\begin{cases} x=-2; a=5; b=-6; c=\frac{1}{3}; \alpha=4; \beta=4 \\ x^5+x^3+136x^2+192x-120=0 \end{cases}$$

(6) $$\begin{cases} x=\frac{1}{2}; a-3; b=\frac{-5}{4}; c=\frac{-1}{6}; \alpha=\frac{-11}{4}; \beta=\frac{11}{12} \\ x^5+\frac{1}{4}x^3-\frac{61}{4}x^2+\frac{15}{4}=0 \end{cases}$$

(7) $$\begin{cases} x=-\frac{1}{3}; a-\frac{2}{3}; b=\frac{1}{3}; c=\frac{1}{9}; \alpha=\frac{-5}{6}; \beta=\frac{-5}{27} \\ x^5+\frac{1}{9}x^3+\frac{52}{54}x^2-\frac{8}{81}=0 \end{cases}$$