Approaching the value of vacuum permittivity using vacuum ether dipoles concept

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Abstract

General relativity is in conflict with quantum mechanics. It has been shown by Broekaert that it is possible to reproduce the general relativity weak field tensor theory by using vacuum refractive index (formally square root of product vacuum permeability and permittivity) depending on surrounding masses. The refractive index of vacuum, is able to change in the vicinity of big masses according to Dicke's formula (taking into account gravitational lensing), also with strong electric field like Schwinger effect and with strong magnetic field like the case of birefringence induced by neutron stars on X-rays. It could be interesting to link these effects of quantum mechanics and gravity using a 3D Euclidean space and considering only variation of permeability and permittivity of the vacuum. By doing so, we take the idea of the quantum ether that can propagate electromagnetism and gravitation. The quantum ether is described by Fleming as a dipole sea, here we will use a sort of Dirac's sea model of coupled fermions with opposite spin-momentum (that looks like helicity).

1 Introduction

Broekaert [1] found an equivalence between gravitational field-dependent vacuum refractive index and weak field general relativity. The refractive index, or more generally the couple permittivity/permeability can fluctuate also with electric field like in the Schwinger effect [2] and with magnetic field with the X-ray polarization near to a neutron star [3]. This supposes that the vacuum can be polarized [4, 5, 6, 7, 8, 9] and contain sort of dipoles. Directly inspired from Fleming's insights [10], I would like to consider here that the vacuum is made by dipoles (like the Dirac's sea). Each dipoles of particles have a dipole of charges (opposite charges), a magnetic dipole (so a spin for each of them) and an other concept of masses dipoles (opposite masses: one positive, the other negative). The purpose of this article is to approximately derive the value of the vacuum permittivity constant from first principles using a field equation (inspired from the Dirac equation) and the dipole electric polarizability expression.

ϵ_0	vacuum permittivity $\simeq 8.854 \times 10^{-12}$ F/m
$\hbar$	reduced Planck constant $\simeq 1.055 \times 10^{-34}$ J/s
q	electron charge $\simeq 1.602 \times 10^{-19}$ C
α	fine structure constant $\simeq 1/137.036$
c	speed of light $\simeq 2.998 \times 10^{8}$ m/s

Table 1: Used physical constants

2 Equation for vacuum dipoles

2.1 Particles pair definition

Let us first define the 4x1 spinor $\mathbf{\Psi}$ as:

$$\mathbf{\Psi} = \begin{pmatrix} \mathbf{\Phi} \\ \boldsymbol{\chi} \end{pmatrix} \tag{1}$$

with $\mathbf{\Phi}$ and $\boldsymbol{\chi}$ are 2x1 spinor. Physically $\mathbf{\Phi}$ and $\boldsymbol{\chi}$ are two particles with spin 1/2 which formed the dipole. The normalization condition applies:

$$\langle \mathbf{\Phi} | \mathbf{\Phi} \rangle = \langle \boldsymbol{\chi} | \boldsymbol{\chi} \rangle = 1 \tag{2}$$

Then we identify each spinor as:

- $\mathbf{\Phi}$: localized particle of charge q, mass $-m$, spin momentum $-1/2$ and negative energy $-E$ (supposed to be a negative mass positron)

- $\boldsymbol{\chi}$: localized particle of charge $-q$, mass m, spin momentum $1/2$ and positive energy E (supposed to be an electron)

We can re-express these properties by saying that we have two particles with opposite charge and spin-momentum (we will see the corresponding operator in the next section). Moreover, since vacuum is able to transport any wavelength or photon frequency, we suppose that the energy level of dipole should be continue and equally distributed.

2.2 Vacuum dipole equation

Instead of using Dirac equation, suited for "free fermions" (in case of non interaction potential they are free particles), we will use another equation with different symmetries. We stance previously that particles should have opposite mass, spin, energy and charge. We choose to define the operator $\widehat{D}_v$ as:

$$\widehat{D}_v = \hbar c \widehat{\mathbf{\Omega}}.\mathbf{\nabla} + \left(mc^2 + i\hbar\partial_t\right) \widehat{\Omega_4} \tag{3}$$

with:

$$\widehat{\mathbf{\Omega}} = \begin{pmatrix} 0 & \widehat{\boldsymbol{\sigma}} \\ \widehat{\boldsymbol{\sigma}} & 0 \end{pmatrix} \tag{4}$$

$$\widehat{\Omega_4} = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix} \tag{5}$$

with I_2 the 2×2 identity matrix and $\widehat{\boldsymbol{\sigma}}$ the Pauli vector. The $\widehat{\sigma}_i$ are the three Pauli matrices ($i = \{1, 2, 3\}$) such as:

$$\{\widehat{\sigma}_i, \widehat{\sigma}_j\} = \widehat{\sigma}_i\widehat{\sigma}_j + \widehat{\sigma}_j\widehat{\sigma}_i = 2\delta_{i,j} I_2 \tag{6}$$

with $\delta_{i,j}$ the Kronecker symbol. The omega matrices have the following properties (i and j can take the values $\{1, 2, 3\}$):

$$\left\{\widehat{\Omega}_i, \widehat{\Omega}_4\right\} = 0 \tag{7}$$

$$\left\{\widehat{\Omega}_i, \widehat{\Omega}_j\right\} = 2\delta_{i,j} I_4 \tag{8}$$

$$\widehat{\Omega}_4^2 = -I_4 \tag{9}$$

Now we define the vacuum dipole equation as:

$$\widehat{D}_v \boldsymbol{\Psi} = 0 \tag{10}$$

If we square the operator we have:

$$\left[\hbar^2 c^2 \Delta - (mc^2 + i\hbar\partial_t)^2\right] \boldsymbol{\Psi} = 0 \tag{11}$$

and neglecting the mass term (we will see that this term can take any value greater than 0 to compute the vacuum permittivity) we obtain ($m \to 0$):

$$\left(\Delta + \frac{\partial_t^2}{c^2}\right) \boldsymbol{\Psi} \simeq 0 \tag{12}$$

Which is a sort of standing wave equation, that could make sens: the photon being the traveling wave in vacuum and the dipole are the medium, that should be "motionless" (we are talking about the vacuum). We can see it also as a Wick rotated D'Alembert wave equation. Then if we stance:

$$\boldsymbol{\Psi}(\boldsymbol{r}, t) = \boldsymbol{\Psi}(\boldsymbol{r}) e^{-iEt/\hbar} \tag{13}$$

We obtain the stationary vacuum dipole equation:

$$\left[\hbar c \widehat{\boldsymbol{\Omega}}.\boldsymbol{\nabla} + \left(mc^2 + E\right) \widehat{\Omega_4}\right] \boldsymbol{\Psi}(\boldsymbol{r}) = 0 \tag{14}$$

and in explicit matrix form:

$$\left[\hbar c \begin{pmatrix} 0 & \widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} \\ \widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} & 0 \end{pmatrix} + \left(mc^2 + E\right) \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}\right] \begin{pmatrix} \boldsymbol{\Phi} \\ \boldsymbol{\chi} \end{pmatrix} = 0 \tag{15}$$

or in 2 lines:

$$\left(\hbar c \widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} + mc^2 + E\right) \boldsymbol{\chi} = 0 \tag{16}$$

$$\left(\hbar c \widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} - mc^2 - E\right) \boldsymbol{\Phi} = 0 \tag{17}$$

These 2 last equations represent opposite "spin-momentum" ($\widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla}$ is proportional to spin-momentum operator). We can see here a similarity with Gersten equation for the photon [11]. This configuration of particles is interesting and allows continuous bound states.

2.2.1 Addition of an electrostatic potential between the 2 particles

We stance that $\boldsymbol{\Phi}$ and $\boldsymbol{\chi}$ are two interacting opposite charged particles (of charge $\pm q$) and are attracted by their own electrostatic potential V:

$$V = -\frac{q^2}{4\pi\epsilon_0 r} \tag{18}$$

with q the elementary charge, ϵ_0 the vacuum permittivity and r being the distance between $\boldsymbol{\Phi}$ and $\boldsymbol{\chi}$: we interpret this here as the center to center distribution distance. We rewrite equation (14) with this co-potential by doing the substitution $E \to E + V(r)$:

$$\left[-\hbar c\widehat{\boldsymbol{\Omega}}.\boldsymbol{\nabla} + \left(mc^2 + E + V(r)\right)\widehat{\Omega_4}\right]\boldsymbol{\Psi}(\boldsymbol{r}) = 0 \tag{19}$$

and in explicit form:

$$\left(\hbar c\widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} + mc^2 + E + V(r)\right)\boldsymbol{\chi} = 0 \tag{20}$$

$$\left(\hbar c\widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} - mc^2 - E - V(r)\right)\boldsymbol{\Phi} = 0 \tag{21}$$

2.2.2 Resolution of the equation with zero angular momentum

First let us divide by $\hbar c$ and rewrite equations (20) as:

$$\left(\widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} + k_0 + k + v(r)\right)\boldsymbol{\chi} = 0 \tag{22}$$

$$\left(\widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} - k_0 - k - v(r)\right)\boldsymbol{\Phi} = 0 \tag{23}$$

with $k = E/(\hbar c)$, $k_0 = mc/\hbar$ and $v(r) = -\alpha/r$ with:

$$\alpha = \frac{q^2}{4\pi\epsilon_0\hbar c} \tag{24}$$

the fine structure constant. In spherical coordinates we can write the scalar product of operator:

$$\widehat{\boldsymbol{\sigma}}.\boldsymbol{\nabla} = \widehat{\sigma}_r\nabla_r + \widehat{\sigma}_\theta\nabla_\theta + \widehat{\sigma}_\varphi\nabla_\varphi \tag{25}$$

We can write the gradient as:

$$\boldsymbol{\nabla} = \boldsymbol{e_r}\partial_r - \boldsymbol{e_r}\wedge(\boldsymbol{e_r}\wedge\boldsymbol{\nabla}) \tag{26}$$

and using the definition of angular momentum $\boldsymbol{L} = -i\hbar\boldsymbol{r}\wedge\boldsymbol{\nabla}$:

$$\boldsymbol{\nabla} = \boldsymbol{e_r}\partial_r - i\frac{\boldsymbol{e_r}}{\hbar r}\wedge\boldsymbol{L} \tag{27}$$

Now we stance:

$$\boldsymbol{L} = 0 \tag{28}$$

then we obtain the couple of differential equations:

$$\left(\widehat{\sigma}_r\partial_r + k_0 + k + v(r)\right)\boldsymbol{\chi} = 0 \tag{29}$$

$$\left(\widehat{\sigma}_r\partial_r - k_0 - k - v(r)\right)\boldsymbol{\Phi} = 0 \tag{30}$$

We define the 2x1 spinor $\boldsymbol{Y_\lambda}$ as an eigenvector of $\widehat{\sigma}_r$ (they are the "radial spin states"):

$$\widehat{\sigma}_r \boldsymbol{Y_\lambda} = \lambda \boldsymbol{Y_\lambda} \tag{31}$$

By the definition of Pauli matrices (${\widehat{\sigma}_i}^2 = I_2$ and ${\widehat{\sigma}_i}^T = \widehat{\sigma}_i$) we have $\det(\widehat{\sigma}_r) = -1$ and $\mathrm{Tr}(\widehat{\sigma}_r) = 0$ so we deduce the eigenvalues of $\widehat{\sigma}_r$:

$$\lambda = \pm 1 \tag{32}$$

Now we express $\boldsymbol{\Phi}$ and $\boldsymbol{\chi}$ as:

$$\boldsymbol{\Phi} = f_k(r)\boldsymbol{Y_{\lambda_1}} \tag{33}$$
$$\boldsymbol{\chi} = f_k(r)\boldsymbol{Y_{\lambda_2}} \tag{34}$$

The equations (29) become:

$$\left(\lambda_1 \partial_r + k_0 + k + v(r)\right) f_k(r)\boldsymbol{Y_{\lambda_1}} = 0 \tag{35}$$
$$\left(\lambda_2 \partial_r - k_0 - k - v(r)\right) f_k(r)\boldsymbol{Y_{\lambda_2}} = 0 \tag{36}$$

Then the equations are identical if:

$$\lambda_1 = 1 \tag{37}$$
$$\lambda_2 = -1 \tag{38}$$

That confirms that if $\boldsymbol{\Phi}$ and $\boldsymbol{\chi}$ have opposite (radial) spin then we have the single equation:

$$\left[k + k_0 + v(r)\right] f_k(r) + \partial_r f_k(r) = 0 \tag{39}$$

We can solve it immediately:

$$f_k(r) = N e^{-(k_0+k)r - \int v(r)dr} \tag{40}$$

Because $v = -\alpha/r$ we can integrate easily to obtain:

$$f_k(r) = N r^\alpha e^{-(k_0+k)r} \tag{41}$$

The normalization condition:

$$\int_0^{+\infty} r^2 f(r)^2 dr = 1 \tag{42}$$

provides an equation for N:

$$N^2 \int_0^{+\infty} r^{2(1+\alpha)} e^{-2(k_0+k)r} dr = 1 \tag{43}$$

Using the identity:

$$\int_0^{+\infty} r^n e^{-ar} dr = \frac{\Gamma(n+1)}{a^{n+1}} \tag{44}$$

with $a > 0$ and $\Gamma(x)$ the Euler gamma function. So we have:

$$N = \frac{[2(k_0 + k)]^{\alpha+3/2}}{\sqrt{\Gamma(2\alpha+3)}} \tag{45}$$

We obtain for $f_k(r)$:

$$f_k(r) = \frac{[2(k_0 + k)]^{\alpha+3/2}}{\sqrt{\Gamma(2\alpha+3)}} r^\alpha e^{-(k_0+k)r} \tag{46}$$

So we obtain for $\boldsymbol{\Phi}$ and $\boldsymbol{\chi}$:

$$\boldsymbol{\Phi} = f_k(r)\boldsymbol{Y_1} \tag{47}$$
$$\boldsymbol{\chi} = f_k(r)\boldsymbol{Y_{-1}} \tag{48}$$

Remark:

The expression of the scalar wave function (46) can be greatly simplified in ($\alpha << 1$):

$$f_k(r) \simeq \frac{[2(k_0 + k)]^{3/2}}{\sqrt{\Gamma(3)}} e^{-(k_0+k)r} \tag{49}$$

We see that it is not the electrostatic potential that participate to the bounding of the dipole, it is the radial spin-momentum operator $\widehat{\sigma}_r \partial_r$. So this should be a spin interaction between the two particles that keep them closer instead of the electrostatic potential, which can be neglected since its contribution is really weak.

3 Volumic polarizability of vacuum dipoles

3.1 Electrodynamics

In classical electrodynamics, we define the displacement field $\boldsymbol{D}$ in vacuum by (linear approximation):

$$\boldsymbol{D} \simeq \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \tag{50}$$

with $\boldsymbol{E}$ an applied electric field and $\boldsymbol{P}$ the vacuum dipole density. We stance that $\boldsymbol{D}$ is zero we have:

$$\epsilon_0 \boldsymbol{E} + \boldsymbol{P} = 0 \tag{51}$$

By definition we have the density dipole moment related to the dipole moment $\boldsymbol{p}$ of volume V by:

$$\boldsymbol{p} = \int_V \boldsymbol{P} dV \tag{52}$$

and the dipole moment is related to the electric field at first order by:

$$\boldsymbol{p} \simeq \alpha_0 \boldsymbol{E} \tag{53}$$

with α_0 the scalar polarizability (we suppose the vacuum is isotropic). Using equation (51), the definitions of dipole moment and polarizability we obtain:

$$\epsilon_0 \simeq -\frac{\alpha_0}{V} = -\alpha_v \tag{54}$$

Let us define then determine the value of α_v, the vacuum polarizability density [4], and check if it corresponds indeed to the vacuum permittivity constant.

3.2 Hydrogen polarizability as a definition starting point

For the hydrogen atom, the atomic polarizability α_H is defined as [12] (it is isotropic, so we choose z axis for commodity):

$$\alpha_H = -2q^2 \sum_{n=2}^{\infty} \frac{\langle n,1,0|z|1,0,0\rangle^2}{E_n - E_1} \tag{55}$$

with $E_n \simeq -\mathrm{Ry}/n^2$ (Ry is the Rydberg constant) and the notation $|n,l,m_l\rangle$ stance for principal, angular momentum and magnetic quantum numbers for the energy states of the hydrogen atom. We can check that for the hydrogen atom we have (using the spherical harmonics integrals):

$$\langle n,1,0|z|n',0,0\rangle = \frac{1}{\sqrt{3}}\langle n|r|n'\rangle \tag{56}$$

So we have:

$$\alpha_H = -\frac{2}{3}q^2 \sum_{n=2}^{\infty} \frac{\langle n|r|1\rangle^2}{E_n - E_1} \tag{57}$$

and we will take this definition as a starting point for vacuum polarizability density (because we have only radial states in our model, it is mandatory for us to consider directly the radial integral).

3.3 Dipole vacuum polarizability density

3.3.1 Dipole density of states

As we have continuous energy states for the dipole, we need to define a density of states (number of states per energy per volume). Using the expression of the momentum operator, we have (we use the approximate expression (49) of f_k):

$$\langle k|\hbar\boldsymbol{\nabla}|k\rangle = -\hbar \int_0^{+\infty} r^2 f_k(r)\boldsymbol{\nabla} f_k(r)dr \simeq \hbar(k+k_0)\boldsymbol{e_r} = \left(\frac{E}{c} + mc\right)\boldsymbol{e_r} \tag{58}$$

with $m \to 0$ then we obtain:

$$\langle k|\hbar\boldsymbol{\nabla}|k\rangle \simeq \frac{E}{c}\boldsymbol{e_r} \tag{59}$$

like the photon (i.e $E = \hbar kc$ for a photon). The three-dimensional density of states is defined as:

$$\mathrm{DOS}(E) = 4\pi k^2 \frac{dk}{dE} \tag{60}$$

and knowing that $dE = \hbar c dk$ we have:

$$\mathrm{DOS}(E) = \frac{4\pi k^2}{\hbar c} \tag{61}$$

3.3.2 Definition of the dipole polarizability density

Thanks to the density of states, we can express the volumic polarizability of the dipole system using the Hydrogen polarizability (57) and taking into account that all energy transitions (continuous spectrum) are possible. That means all photons can be transmitted in the vacuum (and it is the case, as far as we know for the moment). We have to do some assumptions before to establish the vacuum polarizability density:

- We replace the discrete sum of energy transitions (between ground states $E = 0$ and $E > 0$ states) in equation (57) by an integral and a product by the density of states

- Regarding the linearity between the linear momentum and the energy, we choose the photon density of states to describe the dipole density of states as a natural choice

- The polarizability is defined assuming that the definition for the hydrogen atom (57), when we integrate the angular part, remains valid for our system

Now we define the vacuum polarizability density α_v as:

$$\alpha_v = -\frac{2}{3} q^2 \int_0^{+\infty} \mathrm{DOS}(E) \frac{\langle k|r|0\rangle^2}{E + mc^2} dE \tag{62}$$

So using equation (61) we have ($k = E/(\hbar c)$ and $k_0 = mc/\hbar$):

$$\alpha_v = -\frac{2^3 \pi q^2}{3\hbar c} \int_0^{+\infty} k^2 \frac{\langle k|r|0\rangle^2}{k + k_0} dk \tag{63}$$

with:

$$\langle k|r|0\rangle = \int_0^{+\infty} r^3 f_k(r) f_0(r) dr \tag{64}$$

3.3.3 Computation of the dipole polarizability density

Using approximate expression (49) the integral gives (we just use that $\alpha \simeq 1/137 << 1$):

$$\langle k|r|0\rangle \simeq \frac{[4(k_0 + k)k_0]^{3/2}}{\Gamma(3)} \int_0^{+\infty} r^3 e^{-(2k_0+k)r} dr \tag{65}$$

and we obtain:

$$\langle k|r|0\rangle = \frac{[4k_0(k_0+k)]^{3/2}}{(2k_0+k)^4}\frac{\Gamma(4)}{\Gamma(3)} \tag{66}$$

We square the expression to have:

$$\langle k|r|0\rangle^2 = 9\frac{[4k_0(k_0+k)]^3}{(2k_0+k)^8} \tag{67}$$

Inserting into equation (63) we get:

$$\alpha_v \simeq -3 \times \frac{2^9\pi q^2}{\hbar c}\int_0^{+\infty}\frac{k^2k_0^3(k+k_0)^2}{(2k_0+k)^8}dk \tag{68}$$

Then we pose $u = k/k_0$ and we have:

$$\alpha_v = -3 \times \frac{2^9\pi q^2}{\hbar c} I \tag{69}$$

with:

$$I = \int_0^{+\infty}\frac{u^2(u+1)^2}{(2+u)^8}du = \frac{1}{420} \tag{70}$$

Then we multiply α_v by $4\pi\alpha$ on the numerator and denominator to get ($\epsilon_0 = q^2/(4\pi\hbar c\alpha)$):

$$\alpha_v = -3 \times 2^{11}\pi^2\alpha I\epsilon_0 \simeq -1.05\epsilon_0 \tag{71}$$

which is an appreciable approximation within an error of less than 6%, we approach even closer when we do not neglect the fine structure constant in the computations.

4 Conclusion

With this model, including a sort of Dirac's sea fermionic dipoles with opposite energy, charge, mass and spin, we are able to approach the vacuum permittivity constant by simple computations. These dipoles are normalized (finite size in space) and have continuous energy values, which is in agreement with the fact that vacuum transmit equally all frequency. Vacuum dipoles are standing wave whether photons are traveling wave: there is an analogy between atomic oscillation and sound propagation in a material, vacuum dipoles being the medium of propagation. The mass is not well defined here, as in the case of a virtual particle, it is another subject to deep dive. Its value is however not necessary to characterize the vacuum properties (ϵ_0 and μ_0). We neglect also dipoles-dipoles interaction, that can be have a role in the permittivity value. Despite the fact that we cannot justify for the moment the form of the vacuum dipole equation, the advantage is that gives an approach to the permittivity value and bounded states (no infinite energy nor re-normalization).

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