Gravitational Field Equations of the Theory of Self-Variation

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**Abstract.** The Theory of Self-Variation formulates gravity and electromagnetism with the same equations. These Equations concern the field created by the rest mass / electric charge of a particle. The central equation of the Theory relates three physical quantities, the rest mass or charge of the field source, the relative velocity of the field source to the observer, and the propagation velocity of the field relative to the observer. These velocities are directly related to the potential and intensity of the field measured by an observer. The first calculations give consistency of the Theory at the distance scales that we have observational data. Theory predicts increased stellar velocities on the outskirts of galaxies. It also predicts increased velocities of galaxies on the outskirts of galaxy clusters. In the frame of the Theory, the equations we present in this article apply to all interactions, not just gravity and electromagnetism. Further investigation of the equations will yield the complete, precise prediction of the Theory.

## 1. Introduction

The Self-Variation principle postulates that the rest masses and charges of the fundamental particles ("Self-Variating Charge $Q$") slowly increase (in absolute value) with time, while simultaneously radiating negative energy in the surrounding spacetime in order to balance the energy of their rest mass / charge increase. Through a series of mathematical calculations described in detail in [5], this axiom necessarily involves a modification of the electromagnetic potential. For comparison the classical electromagnetic Liénard–Wiechert potentials are

$$V_{LW} = \frac{q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)}$$

$$\mathbf{A}_{LW} = V\frac{\mathbf{u}}{c^2},$$

whereas the corresponding Self-Variation potentials are

$$V = \frac{\left(1-\frac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2} + \frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})q}{4\pi\varepsilon_0 c^3\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2},$$

$$\mathbf{A} = V\frac{\boldsymbol{\upsilon}}{c^2}.$$

The striking difference is that there is a second term in $V$, which is independent of the distance $r$, from the source charge and which has significant theoretical implications. Also the Equations for $\mathbf{A}$, are not identical. We shall examine below the implications in detail for the gravitational field. Notice that the field intensity resulting from either of the potentials (Liénard–Wiechert or Self-Variation's) depends on the distance $r$. It is also crucial to realize that the electromagnetic potential of the Self - Variation Theory gives the correct field intensity, whether we consider the charge $q$, to be constant or varying in time in accordance with the principle of the Self-Variation. In the case of constant charges the field intensity of the Self-Variation potentials is exactly the same as the one implied by the Liénard–Wiechert potentials. However for charges which vary in time while satisfying the Self-Variation principle, the field intensity implied by the Liénard–Wiechert potential differs in an essential way.

The Liénard–Wiechert potentials are compatible with the Lorentz–Einstein transformations for constant charges but not for charges that vary in time. In contrast the Self-Variation potentials are compatible with the Lorentz–Einstein transformations both for constant charges and for charges that vary in time. In this sense the Self-Variation potentials present a much more strict formulation of the field potentials.

Gravitational potential

The Self-Variation potential for the gravitational interaction is derived from the above Equations of the electromagnetic Self-Variation potential by substituting the charge $q$, with the rest mass $M$, of the source of the gravitational field hence, $\frac{q}{4\pi\varepsilon_0} \to -GM$, where $G$, is the constant of gravity and by substituting the acceleration $\boldsymbol{\alpha}$ of the particle in the electromagnetic field, with the intensity $\mathbf{g}$ of the gravitational field, hence, $\boldsymbol{\alpha} \to \mathbf{g}$. Also notice that now $\boldsymbol{\upsilon}$, represents the speed of propagation of the gravitational field, hence we must substitute the speed of light in vacuum $c$, with the speed of propagation of the gravitational field $\upsilon$, hence, $c \to \upsilon$. These substitutions lead to the corresponding gravitational potentials of the Self-Variation,

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} - \frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}, \tag{1}$$

$$\mathbf{A} = V\frac{\boldsymbol{\upsilon}}{\upsilon^2}.$$

where $\mathbf{u}$, is the velocity of the rest mass $M$ relative to the observer, and $r$, is the distance from the rest mass $M$. Deriving the gravitational potentials in this way, implies that there is a gravitational analog to the magnetic field $\mathbf{B}$, and has units $s^{-1}$ (see [5], Equations (3) – (6)). Notice that in the limit case where the speed of propagation of the gravitational field approaches infinity, $\upsilon \to \infty$, we get the limit potential

$$V = -\frac{GM}{r},$$

which is no other than the one of classical mechanics which assumed instant action of gravity at distance $r$.

Like the corresponding Equation for electromagnetism, Equation (1) refers to the gravitational field created by the rest mass of a particle. By taking into account the distribution of particles in spacetime we get the gravitational field on a macroscopic scale.

**2. Potential, propagation speed and intensity of the gravitational field caused by a single rest mass $M$**

The differential equations we get from Equation (1) depend on the direction of the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$. In this article we study in detail one of these cases. Considering that the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions, $\boldsymbol{\upsilon} = \upsilon \frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g \frac{\mathbf{r}}{r}$, $\boldsymbol{\upsilon} \cdot \mathbf{g} = -\upsilon g$, where $\upsilon = \|\boldsymbol{\upsilon}\|$ and $g = \|\mathbf{g}\|$. Then

$$\frac{d\boldsymbol{\upsilon}}{dt} = \frac{d\upsilon}{dt}\frac{\mathbf{r}}{r} + \upsilon \frac{d}{dt}\left(\frac{\mathbf{r}}{r}\right) = \mathbf{g} = -g\frac{\mathbf{r}}{r}. \tag{2}$$

Then from Equation (1) we have

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{\upsilon^2}}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{GM}{\upsilon^2}\frac{g}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}. \tag{3}$$

The gravitational field intensity $\mathbf{g}(r)$ is given by

$$\mathbf{g}(r) = -\nabla V(r) = -\frac{dV}{dr}\frac{\mathbf{r}}{r}. \tag{4}$$

From Equation (2) in [5], we have that, $dr = -cdw$. However using the symbols of the current article this Equation is written as, $dr = -\upsilon dt$. From Equation (2) we get $\frac{d\upsilon}{dt} = -g$. Combining these Equations we get

$$\upsilon \frac{d\upsilon}{dr} = g = \frac{dV}{dr}, \tag{5}$$

where with $t$, we have denoted the time of the observer. From Equation (5) we have,

$$\upsilon^2 = \sigma + 2V \tag{6}$$

where $\sigma$ is a constant. In this article we study the case $\sigma \neq 0$.

From Equations (3) and (5) we get,

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{\upsilon^2}}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\upsilon^2}\frac{GM}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{dV}{dr}. \tag{7}$$

Then, from Equations (6) and (7) we obtain the differential equation for the potential,

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{\upsilon^2}}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\sigma + 2V}\frac{GM}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{dV}{dr} \tag{8}$$

and the differential equation for the speed of propagation of the gravitational field,

$$\upsilon^2 - \sigma = -\frac{2GM}{r}\frac{1-\dfrac{u^2}{\upsilon^2}}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2} + \frac{1}{\upsilon^2}\frac{GM}{\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}\frac{d\upsilon^2}{dr}\,. \tag{9}$$

Equation (9) relates three physics quantities, the rest mass $M$ of the field source, the velocity $\mathbf{u}$ of the field source relative to the observer, and the speed of propagation $\boldsymbol{\upsilon}$ of the field relative to the observer. The fact that this equation relates only these three physics quantities makes it fundamental to the gravitational interaction. For the observer, the properties of spacetime depend on the rest mass $M$ .

In the electromagnetic interaction the Self-Variation Theory predicts two independent pairs of potentials. One gives the electromagnetic field of the moving electric charge and depends on the velocity $\mathbf{u}$ of the charge (see [5], Equations (16) and (17)),

$$V_u = \frac{\left(1-\dfrac{u^2}{c^2}\right)q}{4\pi\varepsilon_0 r\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_u = V_u\frac{\boldsymbol{\upsilon}}{c^2}\,.$$

The other gives the electromagnetic radiation emitted by the electric charge and depends on the acceleration $\mathbf{a}$ of the charge (see [5], Equations (18) and (19)),

$$V_\alpha = \frac{(\boldsymbol{\upsilon}\cdot\boldsymbol{\alpha})q}{4\pi\varepsilon_0 c^3\left(1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}\right)^2}$$

$$\mathbf{A}_a = V_a\frac{\boldsymbol{\upsilon}}{c^2}\,.$$

As a consequence of the substitution $\mathbf{a}\rightarrow\mathbf{g}$, this separation cannot be made in the gravitational interaction. The intensity of the gravitational field, the acceleration of gravity $\mathbf{g}$ is related to both the

$$-\frac{GM}{r}\frac{1-\frac{u^2}{\upsilon^2}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

term and the

$$-\frac{GM}{\upsilon^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{\left(1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{\upsilon^2}\right)^2}$$

term in the second part of Equation (1). In addition to Equation (4), the potential and the gravitational field strength are also related to each other through Equation (1).

**3. Gravitational interaction of two bodies**

We study the case where a body of rest mass $m$ moves in the gravitational field of a stationary body of rest mass $M >> m$.

In polar coordinates $(r,\theta)$, the orbits $r = r(\theta)$ of the body of rest mass $m$ is given by the solution of the system of equations,

$$L = mr^2\frac{d\theta}{dt} = constant \tag{10}$$

$$\ddot{r} - r\left(\dot{\theta}\right)^2 = -g(r) \tag{11}$$

and

$$mV(r) + K = E = constant\ . \tag{12}$$

In these Equations $t$ is the time of the observer, $L$ and $K$ the angular momentum and the kinetic energy of the body of rest mass $m$, $\ddot{r} = \frac{d^2r}{dt^2}$, $\dot{\theta} = \frac{d\theta}{dt}$ and $E$ the mechanical energy of the system of the two bodies.

From the solution of the differential Equation (8) we get the potential $V(r)$. Then, from Equation (5) we get the intensity $g(r)$ of the field. Alternatively, from the solution of the differential equation (9) we obtain the speed of propagation of the field $\upsilon(r)$. Then, from equation (6) we get the potential $V(r)$.

In the solutions of the differential equations (8) or (9), an integration constant $k$ is introduced. Knowing the value of this constant we have the exact prediction of the Theory of Self-Variation for the gravitational field created by the rest mass of a particle. In a first approach, the value of $k$ can be estimated from the already known observational data (see, [1] – [3] and [6] – [15]). For such a measurement, at the macrocosmic scale the distribution of particles in spacetime must be taken into account and not only their total rest mass. The Theory's prediction of the velocities of stars in the outskirts of galaxies and the velocities of galaxies in the outskirts of galaxy clusters can be made using appropriate mathematical models on the respective distance scales.

The solutions given by differential equations (8) and (9) depend on the inner product $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}$. If $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}=0$ we get the simplest possible solutions. We study such solutions in the next sections 4 and 5. In section 6 we study the case $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}\neq 0$.

**4. Circular orbits**

In the system of bodies in the previous section, if the body of rest mass $m$ moves with velocity $\mathbf{u}$ relative to the body of rest mass $M$, then the body of rest mass $M$ moves with velocity $-\mathbf{u}$ relative to the body of rest mass $m$.

The inner product $\mathbf{u}\cdot\boldsymbol{\upsilon}$ in the differential Equations (8) and (9) has the consequence that complex calculations are required for their solution. These calculations are simplified in the case of circular orbits. The case of circular orbits is a first approach to the conclusions drawn from the Equations we present in this article.

For the body of rest mass $m$ we have $\frac{d\boldsymbol{\upsilon}}{dt}=\frac{d\mathbf{u}}{dt}=\mathbf{g}$. In the case of circular orbit it is

$$\mathbf{u}\cdot\mathbf{g}=\mathbf{0} \tag{13}$$

and taking into consideration that the vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions we have

$$\mathbf{u}\cdot\boldsymbol{\upsilon}=\mathbf{0}. \tag{14}$$

Now we have

$$\frac{du^2}{dt}=\frac{d\mathbf{u}^2}{dt}=2\mathbf{u}\frac{d\mathbf{u}}{dt}=2\mathbf{u}\cdot\mathbf{g}$$

and with Equation (13) we get

$$\frac{du^2}{dt}=0, \tag{15}$$

where $u=\|\mathbf{u}\|$. Therefore the velocity $u$ is constant in the circular orbit.

From Equations (9) and (14) we have

$$\upsilon^2-\sigma=-\frac{2GM}{r}\left(1-\frac{u^2}{\upsilon^2}\right)+\frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr}. \tag{16}$$

Let

$$x=\frac{\sigma}{GM}r \tag{17}$$

and let

$$f(x)=\frac{\upsilon^2(x)}{c^2}. \tag{18}$$

From Equations (16), (17) and (18) we get the following differential equation,

$$\frac{df}{dx}-\frac{c^2}{\sigma}f^2+f-\frac{2}{x}\left(f-\frac{u^2}{c^2}\right)=0\,. \tag{19}$$

In Equation (19) it is

$$0\le\frac{u^2}{c^2}<1\,. \tag{20}$$

The solution we get from the differential equation (19) depends on the value of the quotient $\frac{u^2}{c^2}$. In the solution of the differential equation, the integration constant $k$ is introduced. Then solving the differential Equation (19), the velocity $\upsilon=\upsilon(x)$ is given by Equation (18).

From equations (6) and (18) we get the field potential as a function of $x$,

$$V(x)=-\frac{\sigma}{2}+\frac{c^2}{2}f(x)\,. \tag{21}$$

From equations (21) and Transformation (17) we get the field potential as a function of $r$.

From Equation (4) we have

$$\mathbf{g}(r)=-\frac{dV}{dr}\frac{\mathbf{r}}{r}=-\frac{dV}{dx}\frac{dx}{dr}\frac{\mathbf{r}}{r}$$

and with the Transformation (17) we have

$$\mathbf{g}(x)=-\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (6) we have

$$\mathbf{g}(x)=-\frac{\sigma}{2GM}\frac{d\upsilon^2}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (18) we have

$$\mathbf{g}(x)=-\frac{\sigma c^2}{2GM}\frac{df}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (19) we obtain the intensity $\mathbf{g}$ of the field as a function of $x$,

$$\mathbf{g}(x)=-\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma}f^2(x)-f(x)+\frac{2}{x}\left(f(x)-\frac{u^2}{c^2}\right)\right)\frac{\mathbf{r}}{r}\,. \tag{22}$$

The functions $\upsilon=\upsilon(r)$, $V=V(r)$ and $\mathbf{g}=\mathbf{g}(r)$ are obtained from Equations (18), (21) and (22) through the Transformation (17).

As a consequence of Equation (19) the velocity $u$ and the gravitational field are mutually dependent. This is a clear conclusion of the Theory, which has its origin in Equation (1). The solution of the differential Equation (19) gives pairs $\left(f(x),u\right)$. Included in these solutions are velocities $u$ that correspond to the increased velocities of stars in the outskirts of galaxies and the increased velocities of galaxies in the outskirts of galaxy clusters.

**5. Potential, propagation speed, and intensity of the field induced by a stationary rest mass relative to an observer**

If the observer and the source of the field $M$ are stationary between them, $\mathbf{u}=\mathbf{0}$, then from Equations (8) and (9) we have

$$V=-\frac{GM}{r}+\frac{GM}{\sigma+2V}\frac{dV}{dr} \tag{23}$$

and

$$\upsilon^2-\sigma=-\frac{2GM}{r}+\frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr}. \tag{24}$$

From Equation (24), again applying Transformations (17) and (18) we get,

$$\frac{df}{dx}-\frac{c^2}{\sigma}f^2+f-\frac{2f}{x}=0. \tag{25}$$

Solving (25) for $f$, we have

$$f(x)=\frac{\sigma}{c^2}\frac{x^2}{ke^x+x^2+2x+2} \tag{26}$$

where $k$, is the integration constant. Then from (18), (26) we have

$$\upsilon^2(x)=\sigma\frac{x^2}{ke^x+x^2+2x+2}. \tag{27}$$

Finally applying the Transformation (17) to (27) we get the speed of propagation of the gravitational field, as derived from the Self-Variation gravitational potential, with respect to $r$,

$$\upsilon^2(r)=\sigma\frac{a^2r^2}{ke^{ax}+a^2r^2+2ar+2}, \tag{28}$$

where $a=\dfrac{\sigma}{GM}$. (29)

From Equations (6) and (27) we obtain the gravitational potential with respect to $x$,

$$V(x)=-\sigma\frac{ke^x+2x+2}{2\left(ke^x+x^2+2x+2\right)}. \tag{30}$$

Then from Equations (30) and Transformation (17) we obtain the gravitational potential with respect to $r$,

$$V(r)=-\sigma\frac{ke^{ar}+2ar+2}{2\left(ke^{ar}+a^2r^2+2ar+2\right)}. \tag{31}$$

The gravitational field intensity $\mathbf{g}$ is calculated as follows. From Equation (4) and Transformation (17) we get

$$\mathbf{g}(x) = -\frac{\sigma}{GM}\frac{dV(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (6) we get,

$$\mathbf{g}(x) = -\frac{\sigma}{2GM}\frac{d\upsilon^2(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equation (18) we get,

$$\mathbf{g}(x) = -\frac{\sigma c^2}{2GM}\frac{df(x)}{dx}\frac{\mathbf{r}}{r}$$

and with Equations (25) we get,

$$\mathbf{g}(x) = -\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma}f^2(x) - f(x) + \frac{2}{x}f(x)\right)\frac{\mathbf{r}}{r}$$

and with (26) we obtain,

$$\mathbf{g}(x) = -\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma}f^2(x) - f(x) + \frac{2}{x}f(x)\right)\frac{\mathbf{r}}{r} = -\frac{\sigma^2}{2GM}\frac{2kxe^x - kx^2e^x + 2x^2 + 4x}{\left(ke^x + x^2 + 2x + 2\right)^2}\frac{\mathbf{r}}{r}. \tag{32}$$

The function $f = f(x)$ is given by Equation (26). From Transformation (17) and Equations (26) and (32) we get the field intensity $\mathbf{g} = \mathbf{g}(r)$ as a function of $r$.

We made the substitution $\frac{q}{4\pi\varepsilon_0} \to -GM$ (and not $\frac{q}{4\pi\varepsilon_0} \to GM$ ) in order for the gravitational interaction to be attractive. However, this is not achieved. From Equation (32) it follows that there are values of $k$ and $x$ for which gravity is repulsive. Consequently, the general case of the gravitational interaction is obtained by substituting $\frac{q}{4\pi\varepsilon_0} \to \pm GM$. Equation (1) is common to electromagnetism and gravity. Through the transformations

$\frac{q}{4\pi\varepsilon_0} \leftrightarrow \pm GM$, $\mathbf{a} \leftrightarrow \mathbf{g}$ and $c \leftrightarrow \upsilon$

we pass from one interaction to another.

For $\mathbf{u} = \mathbf{0}$ and $\frac{q}{4\pi\varepsilon_0} \to +GM$ the equivalents of Equations (24), (25) and (26) are

$$\upsilon^2 - \sigma = \frac{2GM}{r} - \frac{GM}{\upsilon^2}\frac{d\upsilon^2}{dr},$$

$$\frac{df}{dx} + \frac{c^2}{\sigma}f^2 - f - \frac{2f}{x} = 0,$$

$$f(x)=\frac{\sigma}{c^2}\frac{x^2}{ke^{-x}+x^2-2x+2}.$$

Then we get

$$\upsilon^2(x)=\sigma\frac{x^2}{ke^{-x}+x^2-2x+2}, \tag{33}$$

$$V(x)=-\sigma\frac{ke^{-x}-2x+2}{2\left(ke^{-x}+x^2-2x+2\right)} \tag{34}$$

and

$$\mathbf{g}(x)=-\frac{\sigma^2}{2GM}\frac{-2kxe^{-x}-kx^2e^{-x}+2x^2-4x}{\left(ke^{-x}+x^2-2x+2\right)^2}\frac{\mathbf{r}}{r}. \tag{35}$$

By comparing the triads of Equations ((27), (30), (32)) and ((33), (34), (35)) the similarities and differences of the $\frac{q}{4\pi\varepsilon_0}\to -GM$ and $\frac{q}{4\pi\varepsilon_0}\to +GM$ substitutions emerge.

As a consequence of the equality $-\frac{GM}{x}=\frac{GM}{-x}$, the triads of Equations ((27), (30), (32)) and ((33), (34), (35)) are related through the $x\leftrightarrow -x$ transformation. Through this transformation one triad arises from the other. From Equation (1) it follows that for the two substitutions, this symmetry also exists in the case $\mathbf{u}\neq\mathbf{0}$. Therefore, for each solution $\upsilon(x)$, $V(x)$, $g(x)$ of the Equations of the field we also get its "complementary" solution through the transformation $x\to -x$.

The case $\mathbf{u}=\mathbf{0}$ is the simplest. In addition, it gives exact solutions of $\upsilon$, $V$ and $\mathbf{g}$. Therefore, the initial investigation of the equations of this article can be done for all possible substitutions with $\mathbf{u}=\mathbf{0}$.

As a consequence of the equivalence

$$\mathbf{u}\cdot\boldsymbol{\upsilon}=0\Leftrightarrow\begin{cases}\mathbf{u}=\mathbf{0}\\ or\\ \mathbf{u}\perp\boldsymbol{\upsilon}\end{cases},$$

the cases $\mathbf{u}=\mathbf{0}$ and $\mathbf{u}\perp\boldsymbol{\upsilon}$ (circular orbits) are given by Equations (19) and (18), (21), (22),

$$\frac{df}{dx}-\frac{c^2}{\sigma}f^2+f-\frac{2}{x}\left(f-\frac{u^2}{c^2}\right)=0,$$

$$\upsilon^2(x)=c^2f(x),$$

$$V(x)=-\frac{\sigma}{2}+\frac{c^2}{2}f(x),$$

$$\mathbf{g}(x) = -\frac{\sigma c^2}{2GM}\left(\frac{c^2}{\sigma} f^2(x) - f(x) + \frac{2}{x}\left(f(x) - \frac{u^2}{c^2}\right)\right)\frac{\mathbf{r}}{r}.$$

Then, through the transformation $x \to -x$ we obtain the complementary solution.

From equations (27) and (33) it follows that there are values of $k$ and $x$ for which $\upsilon > c$. They are the values of $k$ and $x$ for which gravity is repulsive. Since the propagation speed of the field can take values greater than $c$, we conclude that the gravitational interaction is not accompanied by a particle, analogous to the photon of the electromagnetic interaction. This is impossible according to Special Relativity. However, $\upsilon$ is the propagation speed of the field relative to the observer. Thus the question arises whether the observer can attribute the gravitational interaction to a particle, for the values of $x$ for which $\upsilon(x) < c$. From the investigation of the Equations so far, there is no answer to this question.

**6. The general case $\mathbf{u} \cdot \boldsymbol{\upsilon} \neq 0$**

From Equation (9) and transformation (17) we get the following equation,

$$\left(\frac{\upsilon^2}{\sigma} - 1\right)\left(1 - \frac{\mathbf{u} \cdot \boldsymbol{\upsilon}}{\upsilon^2}\right)^2 x = -2 + 2\frac{u^2}{\upsilon^2} + \frac{1}{\upsilon^2}\frac{x d\upsilon^2}{dx}. \qquad (36)$$

From equation (36) and the transformation $x \to -x$ we get the complementary equation

$$-\left(\frac{\upsilon^2}{\sigma} - 1\right)\left(1 - \frac{\mathbf{u} \cdot \boldsymbol{\upsilon}}{\upsilon^2}\right)^2 x = -2 + 2\frac{u^2}{\upsilon^2} + \frac{1}{\upsilon^2}\frac{x d\upsilon^2}{dx}. \qquad (37)$$

For the case we studied ($\boldsymbol{\upsilon} = \upsilon\frac{\mathbf{r}}{r}$ and $\mathbf{g} = -g\frac{\mathbf{r}}{r}$), (36) and (37) are the general Equations. Corresponding equations are obtained for all possible combinations in the directions of vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$. The resulting solutions of these differential equations depend on the inner product $\mathbf{u} \cdot \frac{\boldsymbol{\upsilon}}{\upsilon^2}$.

In the differential equations (16) and (24) the unknown function is the $\upsilon^2$. Thus we made the transformation (18). In the differential equations (36) and (37) the unknown function is the $\upsilon$. Thus we make the transformation

$$\upsilon(x) = cf(x) \qquad (38)$$

and get,

$$\left(\frac{c^2}{\sigma} f^2 - 1\right)\left(1 - \frac{u\cos\theta}{c}\frac{1}{f}\right)^2 x = -2 + 2\frac{u^2}{c^2}\frac{1}{f^2} + \frac{2}{f}\frac{xdf}{dx}, \qquad (39)$$

$$-\left(\frac{c^2}{\sigma} f^2 - 1\right)\left(1 - \frac{u\cos\theta}{c}\frac{1}{f}\right)^2 x = -2 + 2\frac{u^2}{c^2}\frac{1}{f^2} + \frac{2}{f}\frac{xdf}{dx}, \qquad (40)$$

where $\theta$ is the angle of the vectors $\mathbf{u}$ and $\boldsymbol{\upsilon}$.

The differential Equations (38), (39) give triads of solutions $\left(f\left(x\right),u,\theta\right)$. Included in these solutions are velocities $u$ that correspond to the increased velocities of stars in the outskirts of galaxies and the increased velocities of galaxies in the outskirts of galaxy clusters.

After finding the function $f\left(x\right)$, the velocity $\upsilon\left(x\right)$ is given by equation (38). The field potential $V\left(x\right)$ is given by equation (6). The field strength is given by equation (4) written in the form $\mathbf{g}\left(x\right)=-\frac{\sigma}{GM}\frac{dV}{dx}\frac{\mathbf{r}}{r}$.

**7. The potentials $V$, $\mathbf{A}$**

The complete investigation of the Theory of Gravitation is done through Equation (1) and the pair of potentials $\left(V,\mathbf{A}\right)$, where

$$\mathbf{A}=V\frac{\boldsymbol{\upsilon}}{\upsilon}.$$

These potentials relate the gravitational field to spacetime, through the equations,

$$\mathbf{g}=-\nabla V-\frac{\partial\mathbf{A}}{\partial t},$$

$$\mathbf{B}=\nabla\times\mathbf{A},$$

where $\mathbf{B}$ is a gravitational proportional to the magnetic field and has units $s^{-1}$ (see [5], Equations (3) – (6)).

As a consequence of Self-Variation, at time $t$, the rest mass / electric charge located at point $P$ acts on point $A$ with the value it had at another point $E$ (see [5], Fig. 1). The intensity of the gravitational field given by the potentials $\left(V,\mathbf{A}\right)$ is the same whether we consider the rest mass $M$ to be constant or consider it to vary according to the Principle of Self-Variation. This property of the potentials of the Self-Variation Theory allows us to solve the differential Equations of the gravitational field by considering the rest mass $M$ as constant, as we did in the previous sections.

On the macroscopic scale it is clear that point $E$ refers to a previous, compared to $P$, position that the rest mass / electrical charge had. However, given that at time $t$ the rest mass / electric charge acts from point $P$ to point $A$, there is a set of possible points $E$. Unlike the macrocosm, at the microscopic scale there is no certainty about the location of point $E$.

**8. Discussion**

We have presented the Equations for Gravity as predicted by the Theory of Self-Variation. The substitution $\boldsymbol{\alpha}\rightarrow\mathbf{g}$, by which we get the Gravitational potential of Self-Variation from the corresponding Electromagnetic potential, is an idea belonging to Einstein. Without this substitution the Gravitational field of Self-Variation cannot arise. The Equations we have presented include Einstein's proposal for the equivalence of acceleration and gravity.

The potentials $\left(V,\mathbf{A}\right)$ apply to all distance scales. Taking into account the principle of Self-Variation, then it follows that the Equations of this article apply whether we consider the masses to be fixed or to consider them to be variable. The negative energy from the Self-Variation of the rest mass enters into these Equations. If we also take into account the principle of conservation of energy - momentum, then the Equations that justify the cosmological data arise.

In sections 4 and 5 we made clear assumptions about the inner product $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}$ in the applications of Equations (8) and (9). However, at the microscopic scale it is not clear what form the inner product $\mathbf{u}\cdot\frac{\boldsymbol{\upsilon}}{\upsilon^2}$ might take. The uncertainty of this form is added to the uncertainty of the location of point $E$ (see [5], Fig. 1). There are reasonable assumptions we can make on the macroscopic scale but cannot make on the microscopic scale.

Through the substitutions $\frac{q}{4\pi\varepsilon_0}\leftrightarrow\pm GM$ , $\mathbf{a}\leftrightarrow\mathbf{g}$ , $c\leftrightarrow\upsilon$ Equation (1) is common to gravity and electromagnetism. By doing all the combinations we get the possible equations of the gravitational interaction. A common characteristic of the resulting cases is that gravity is attractive or repulsive as the distance from the rest mass changes. The correlation of attraction / repulsion with the distance from the rest mass is a direct consequence of the Equations of this article.

If in Equation (1) we replace the rest mass with the "Self-Variating Charge $Q$" we get the "Unified Self-Variation Interaction". In gravitational interaction Self-Variating charge is the rest mass, $Q=M$ . In electromagnetic interaction the Self-Variating charge is the electric charge, $Q=q$ . An issue for investigation is the possible physics quantities $Q$ .

As a consequence of the Self-Variation of rest masses, spacetime contains negative energy. For the gravitational field of a particle this negative energy does not play an important role. However, as distance scales increase spacetime contains a large number of particles distributed over a negative energy background. At the macroscopic scale the gravitational interaction depends on the distribution of particles and negative energy. On the cosmological scale this combination makes the universe flat (see, [4]). A mathematical model for the Theory's predictions at the macrocosmic and cosmological scales is necessary.

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