No Collatz Conjecture integer series have looping

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Abstract

If the series of Collatz Conjecture integer has looping in it, it is sure the members of the loop cannot reach to value 1. Here it is proven that the possibility of looping is zero except one.

1. Introduction

Procedure of Collatz Conjecture is recognized as following operations.

It starts with positive odd integer n_1.

It continues following calculation up to $n_i = 1$.

- Compute $n = 3 \times n_{i-1} + 1$. (1)
- n is divided by 2, m_{i-1} times until it becomes positive odd integer.

$$n_i = \frac{n}{2^{m_{i-1}}} \quad (2)$$

2. Looping

Collatz conjecture procedure is represented as follow.

$$(3((3((3 \times n + 1)/2^{m_1}) + 1)/2^{m_2}) + 1)/2^{m_3} \cdots$$

$$= \frac{3^i n}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-1}}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-2}}{2^{m_2+\cdots+m_i}} + \cdots + \frac{3^1}{2^{m_{i-1}+m_i}} + \frac{3^0}{2^{m_i}} \quad (3)$$

n: starting positive odd integer

If this procedure has looping, following equation is satisfied.

$$\frac{3^i n}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-1}}{2^{m_1+m_2+\cdots+m_i}} + \frac{3^{i-2}}{2^{m_2+\cdots+m_i}} + \cdots + \frac{3^1}{2^{m_{i-1}+m_i}} + \frac{3^0}{2^{m_i}} = n. \quad (4)$$

$$\left(2^{m_1+m_2+\cdots+m_i} - 3^i\right)n = 3^{i-1} + 2^{m_1} \cdot 3^{i-2} + \cdots + 2^{m_1+m_2+\cdots+m_{i-2}} \cdot 3^1 + 2^{m_1+m_2+\cdots+m_{i-1}} \cdot 3^0. \quad (5)$$

Characteristics of right side (5) are followings.

It is expanded using power of 3 from 3^0 to 3^{i-1} term. (a)

Coefficient for each power of 3 is represented as one bit of binary (ex. $2^{m_1+m_2+\cdots+m_{i-2}}$), also each power of 3 and its coefficient has following relation.

$2^{m_1+m_2+\cdots+m_{i-j}} \cdot 3^{j-1}$ j: integer, $1,2,3 \cdots i$, $m_0 = 0$. (b)

Existence of positive odd integer n solution for (5) means that there is looping.

We investigate whether such n solution could exist or not.

Left side of (5) could be expanded and become same format as right side. (6) defines m for it.

$$\mathrm{m} = (m_1 + m_2 + \cdots + m_i)/i \tag{6}$$

Left side of (5) becomes (7).

$$(2^{mi} - 3^i)n = (2^m - 3)(3^{i-1} + 2^m \cdot 3^{i-2} + \cdots + 2^{m(i-2)} \cdot 3^1 + 2^{m(i-1)} \cdot 3^0)n. \tag{7}$$

Then (5) becomes (8).

$$\begin{aligned}&(2^m - 3)(3^{i-1} + 2^m \cdot 3^{i-2} + \cdots + 2^{m(i-2)} \cdot 3^1 + 2^{m(i-1)} \cdot 3^0)n \\ &= 3^{i-1} + 2^{m_1} \cdot 3^{i-2} + \cdots + 2^{m_1+m_2+\cdots+m_{i-2}} \cdot 3^1 + 2^{m_1+m_2+\cdots+m_{i-1}} \cdot 3^0\end{aligned} \tag{8}$$

Both sides (8) have following form.

$$\sum_{i=0}^{i-1} \alpha_i 3^i \tag{9}$$

$$\alpha_i; These\ have\ no\ factor\ of\ 3. \tag{10}$$

$$i; 0,1,2, \ldots\ldots$$

No term $\alpha_n 3^n$ of left side cannot be represented by linear combination of right-side terms $\beta_i 3^i$ not including the term $i = n$, that is,

$$\alpha_n 3^n \neq \sum_{i=0}^{i \neq n} \beta_i 3^i \tag{11}$$

Proof of this is;

About right side of (11), its formula is (12) for the terms $i < n$.

$$\sum_{i=0}^{n-1} \beta_i 3^i = \beta_0 3^0 + \beta_1 3^1 + \cdots + \beta_{n-1} 3^{n-1} \tag{12}$$

About right side of (11), its formula is (13) for the terms $i > n$.

$$\sum_{i=n+1}^{i} \beta_i 3^i = (\beta_{n+1} 3^1 + \beta_{n+2} 3^2 + \cdots) 3^n \tag{13}$$

Therefore, right side formula of (11) for $i \neq n$ is (14).

$$\sum_{i=0}^{i \neq n} \beta_i p^i = (\beta_0 3^0 + \beta_1 3^1 + \cdots + \beta_{n-1} 3^{n-1}) + (\beta_{n+1} 3^1 + \beta_{n+2} 3^2 + \cdots) 3^n \tag{14}$$

In general, following formula (15) cannot be deformed to (16).

$$\gamma_0 + 3^l \gamma_1 \qquad \gamma_0 \text{ has no factor of 3.} \tag{15}$$

$$3(\gamma_0' + 3^{l-1} \gamma_1) \tag{16}$$

Therefore, (12) cannot have factor 3. Also (13) have no factor 3^n but 3^{n+1}.

Then (14) have no factor of 3.

Therefore, the unequal equation (11) is satisfied.

Therefore, in order (8) is satisfied, every both side coefficients should be same for each term which has same power of 3. Then it makes following equations.

$$(2^m - 3)n = 1 \tag{17}$$

$$(2^m - 3)n \cdot 2^m = 2^{m_1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$(2^m - 3)n \cdot 2^{m(i-1)} = 2^{m_1+m_2+\cdots+m_{i-1}}$$

In the case that m is integer, considering n is positive odd integer, we can get (18) on the first equation of (17).

$m = 2, n = 1$ (18)

Resolving all equations sequentially, we can get result (19).

$m_1 = m_2 = \cdots = m_i = m = 2$ (19)

In the case that m is not integer, if the first equation of (17) is true, resolving all equations sequentially, we can get result (20).

$m_1 = m_2 = \cdots = m_i = m$ (20)

But (20) is contradict with the assumption m is not integer. This means if m is not integer, the first equation of (17) cannot be satisfied.

Therefore, solution can be gotten only when m is integer. And its solution is (18) and (19).

This means that this looping is only one member looping or self-looping when n=1.

Therefore, n=1 can be terminal point of Collatz Conjecture operation.

3. Consideration

No looping proof in this report could be used with *1 and *2 which investigate Collatz Conjecture Space. These show that the space expectation value of 2^{m_i} in (4) is $2^2 = 4$.

Also, no looping report could be used with *3 which investigates the series of Collatz Conjecture integer.

Therefore, these combinations show Collatz Conjecture is correct.

*1) viXra:2204.0151

*2) viXra:2304.0182

*3) viXra:2302.0015