On a Possible Explanation of the Increased Star Velocities at the outskirts of Galaxies

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Abstract. Self-Variation Theory predicts increased stellar velocities on the outskirts of galaxies. The same Equation also predicts increased velocities of galaxies on the outskirts of galaxy clusters. Comparison with observational data requires the measurement of a constant of physics that appears in the gravitational field equations.

1. Introduction

With the substitutions $-GM \to \dfrac{q}{4\pi\varepsilon_0}$ and $\mathbf{g} \to \boldsymbol{\alpha}$ in the macroscopic electromagnetic potential of the Theory of Self-Variation (see, [4]) we get the corresponding potential V of the gravitational interaction,

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{c^2}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{GM}{c^3}\frac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}}. \tag{1}$$

In this Equation G is the constant of gravity, M the mass-source of the gravitational field, r the distance from the mass M, $\mathbf{u}$ the speed with which M moves, $\boldsymbol{\upsilon}$ the speed with which the cause of the field moves and $\mathbf{g}$ the intensity of the field.

The vector $\boldsymbol{\upsilon}$ is given by the equations $\boldsymbol{\upsilon} = c\dfrac{\mathbf{r}}{r}$, where c is the speed of light in vacuum (see [4], Fig. 1). The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ may have either the same direction or opposite directions.

2. The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have opposite directions

In the case that vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have same opposite directions, $\dfrac{\boldsymbol{\upsilon}\cdot\mathbf{g}}{c} = -g = -\|\mathbf{g}\|$ from Equation (1) we get,

$$V = -\frac{GM}{r}\frac{1-\dfrac{u^2}{c^2}}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} - \frac{GM}{c^2}\frac{g}{1-\dfrac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} \tag{2}$$

and by symbolizing

$$\alpha = \frac{c^2}{GM} \tag{3}$$

we get the following equation,

$$V = -\frac{GM}{r} - \frac{g}{\alpha}\,. \tag{4}$$

From equations (4) and

$$\mathbf{g}(r) = -\nabla V(r) = -\frac{dV}{dr}\frac{\mathbf{r}}{r} \tag{5}$$

we get

$$V = -\frac{GM}{r} - \frac{dV}{\alpha dr}$$

and equivalently we obtain

$$V + \frac{dV}{\alpha dr} = -\frac{GM}{r}\,. \tag{6}$$

The potential given by Equation (6) is the same either we consider the mass M to be constant or to vary strictly as predicted by the principle of self-variation. Thus we solve the differential Equation (6) for constant mass M (see, [1]).

By symbolizing

$$x = \alpha r, r = \frac{x}{\alpha}, \tag{7}$$

from Equation (6) we get

$$V + \frac{dV}{dx} = -\frac{GM\alpha}{x}$$

and with Equation (3) we get

$$V + \frac{dV}{dx} = -\frac{c^2}{x}$$

and by symbolizing

$$\frac{V(x)}{c^2} = f(x) \tag{8}$$

we obtain

$$f'(x) + f(x) = -\frac{1}{x}\,. \tag{9}$$

From Equation (9) we get

$$e^x f'(x) + e^x f(x) = -\frac{e^x}{x}$$

and equivalently we get

$$\left(e^{x} f\left(x\right)\right)' = -\frac{e^{x}}{x}$$

and equivalently we get

$$e^{x} f\left(x\right) = k - \int \frac{e^{x}}{x} dx$$

and finally we obtain

$$f\left(x\right) = k \cdot e^{-x} - \sum_{n=1}^{\infty} \frac{\left(n-1\right)!}{x^{n}}, \qquad (10)$$

where k is a constant.

From Equations (8) and (10) we get

$$V\left(x\right) = kc^{2} \cdot e^{-x} - c^{2} \sum_{n=1}^{\infty} \frac{\left(n-1\right)!}{x^{n}}. \qquad (11)$$

From Equation (11) and transformation (7) we obtain the function $V = V\left(r\right)$ as given by the following equation,

$$V\left(r\right) = kc^{2} \cdot e^{-\alpha r} - \sum_{n=1}^{\infty} \frac{c^{2} \cdot \left(n-1\right)!}{\alpha^{n} r^{n}}. \qquad (12)$$

From Equations (12) and (5) we obtain the function $\mathbf{g} = \mathbf{g}\left(r\right)$ as given by the following equation,

$$\mathbf{g} = \mathbf{g}\left(r\right) = -\left(-k\alpha c^{2} \cdot e^{-\alpha r} + \sum_{n=1}^{\infty} \frac{c^{2} \cdot n!}{\alpha^{n} r^{n+1}}\right)\frac{\mathbf{r}}{r}. \qquad (13)$$

From Equations (13) and (7) we obtain the function $\mathbf{g} = \mathbf{g}\left(x\right)$ as given by the following equation,

$$\mathbf{g} = \mathbf{g}\left(x\right) = -\left(-k\alpha c^{2} \cdot e^{-x} + \sum_{n=1}^{\infty} \frac{\alpha c^{2} \cdot n!}{x^{n+1}}\right)\frac{\mathbf{r}}{r}. \qquad (14)$$

From Equations (14) and (13) we obtain,

$$g = -k\alpha c^{2} \cdot e^{-x} + \sum_{n=1}^{\infty} \frac{\alpha c^{2} \cdot n!}{x^{n+1}} = -k\alpha c^{2} \cdot e^{-\alpha r} + \sum_{n=1}^{\infty} \frac{c^{2} \cdot n!}{\alpha^{n} r^{n+1}}. \qquad (15)$$

From the inequality $g\left(r\right) = \left\|\mathbf{g}\left(r\right)\right\| \geq 0$ and equation (15) we get,

$$k \cdot e^{-\alpha r} \leq \sum_{n=1}^{\infty} \frac{n!}{\left(\alpha r\right)^{n+1}}. \qquad (16)$$

From Equations (3) and (4) we get,

$$V(r)+\frac{GM}{c^2}g(r)=-\frac{GM}{r}. \quad (17)$$

Equation (17) relates the potential $V(r)$ and the intensity $g(r)=\|\mathbf{g}(r)\|$ of the gravitational field.

3. Axial symmetry

We apply the Gravitational Field Equations of section 2. In order to avoid complex calculations (which can be done in cases where it is essential) we present their simplest application. We present the case where a star moves in a circular orbit on the outskirts of a galaxy or a galaxy moves in a circular orbit on the outskirts of a galaxy cluster with speed U. In this case the following equation applies,

$$\mathrm{U}^2=g\cdot r=\frac{g\cdot x}{\alpha}. \quad (18)$$

The mass M of a circular disk of radius r, thickness b and constant density ρ is $M=\pi b\rho r^2$. Therefore, from Equations (3) and (7) we get $x=\frac{c^2}{G\pi b\rho r}$. Symbolizing

$$\sigma=\frac{c^2}{G\pi b\rho} \quad (19)$$

we get,

$$\alpha=\frac{\sigma}{r^2} \quad (20)$$

and

$$x=\frac{\sigma}{r}. \quad (21)$$

From Equations (15) and (20), (21) we get,

$$g(r)=\frac{\sigma c^2}{r^2}\left(-k\cdot e^{-\frac{\sigma}{r}}+\sum_{n=1}^{\infty}n!\left(\frac{r}{\sigma}\right)^{n+1}\right) \quad (22)$$

and with Equation (18) we obtain,

$$\mathrm{U}^2=\frac{\sigma c^2}{r}\left(-k\cdot e^{-\frac{\sigma}{r}}+\sum_{n=1}^{\infty}n!\left(\frac{r}{\sigma}\right)^{n+1}\right). \quad (23)$$

For the Newtonian speed v is,

$$v^2=G\pi b\rho r. \quad (24)$$

In Equation (23) is,

$$\frac{r}{\sigma}=\frac{G\pi b\rho r}{c^2}=\frac{v^2}{c^2}<<1.$$

Thus, for $n=1$ we take the approach,

$$\mathrm{U}^2=\frac{\sigma c^2}{r}\left(-k\cdot e^{-\frac{\sigma}{r}}+\left(\frac{r}{\sigma}\right)^2\right).$$

In this Equation it is $\mathrm{U}>v$ if $k=-\mu^2$,

$$\mathrm{U}^2=\frac{\sigma c^2}{r}\left(\mu^2\cdot e^{-\frac{\sigma}{r}}+\left(\frac{r}{\sigma}\right)^2\right). \tag{25}$$

From Equations (25), (19) and (24) we obtain,

$$\mathrm{U}^2=\frac{\mu^2c^4}{G\pi b\rho r}e^{-\frac{\sigma}{r}}+G\pi b\rho r=\frac{\mu^2c^4}{v^2}e^{-\frac{c^2}{v^2}}+v^2. \tag{26}$$

Comparison with observational data (see, [1] - [3] and [5] – [12]) requires the measurement of the physical constant μ.

4. Spherical symmetry

The mass M of a sphere of radius r and constant density ρ is $M=\frac{4\pi\rho}{3}r^3$. Therefore, from Equations (3) and (7) we get $x=\frac{3c^2}{4G\pi\rho r^2}$. Symbolizing

$$\sigma=\frac{3c^2}{4G\pi\rho} \tag{27}$$

we get

$$\alpha=\frac{\sigma}{r^3} \tag{28}$$

and

$$x=\frac{\sigma}{r^2}. \tag{29}$$

From Equations (15) and (28), (29) we get

$$g(r)=\frac{\sigma c^2}{r^3}\left(-k\cdot e^{-\frac{\sigma}{r^2}}+\sum_{n=1}^{\infty}n!\left(\frac{r^2}{\sigma}\right)^{n+1}\right) \tag{30}$$

and with Equation (18) we obtain,

$$\mathrm{U}^2=\frac{\sigma c^2}{r^2}\left(-k\cdot e^{-\frac{\sigma}{r^2}}+\sum_{n=1}^{\infty}n!\left(\frac{r^2}{\sigma}\right)^{n+1}\right). \tag{31}$$

For the Newtonian speed ν is,

$$\nu^2 = \frac{G\pi b\rho}{3} r^2 . \tag{32}$$

In Equation (31) is,

$$\frac{r^2}{\sigma} = \frac{4G\pi b\rho r^2}{3c^2} = \frac{\nu^2}{c^2} << 1 .$$

Thus, for $n = 1$ we take the approach,

$$\mathrm{U}^2 = \frac{\sigma c^2}{r^2}\left(-k \cdot e^{-\frac{\sigma}{r^2}} + \left(\frac{r^2}{\sigma}\right)^2\right).$$

In this Equation it is $\mathrm{U} > \nu$ if $k = -\mu^2$,

$$\mathrm{U}^2 = \frac{\sigma c^2}{r^2}\left(\mu^2 \cdot e^{-\frac{\sigma}{r^2}} + \left(\frac{r^2}{\sigma}\right)^2\right). \tag{33}$$

From Equations (33), (27) and (32) we obtain,

$$\mathrm{U}^2 = \frac{3\mu^2 c^4}{4G\pi b\rho r^2} e^{-\frac{\sigma}{r^2}} + \frac{4G\pi b\rho r^2}{3} = \frac{\mu^2 c^4}{\nu^2} e^{-\frac{c^2}{\nu^2}} + \nu^2 . \tag{34}$$

Comparison with observational data (see, [1] – [3] and [5] – [12]) requires the measurement of the physical constant μ.

5. The vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have same direction

In the case that vectors $\boldsymbol{\upsilon}$ and $\mathbf{g}$ have same direction, $\frac{\boldsymbol{\upsilon} \cdot \mathbf{g}}{c} = g$ from Equation (1) we get,

$$V = -\frac{GM}{r}\frac{1-\frac{u^2}{c^2}}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} + \frac{GM}{c^2}\frac{g}{1-\frac{\mathbf{u}\cdot\boldsymbol{\upsilon}}{c^2}} . \tag{35}$$

If $\mathbf{u} = \mathbf{0}$, from Equation (35) we get

$$V(r) = -\frac{GM}{r} + \frac{GM}{c^2} g(r) . \tag{36}$$

This equation is the corresponding of (17).

From Equation (36), repeating the proof procedure of section 2 we get the following equations.

$$V(x) = Kc^2 \cdot e^x - c^2 \sum_{n=1}^{\infty} (-1)^n \frac{(n-1)!}{x^n} ,$$

$$V(r) = Kc^2 \cdot e^{\alpha r} - \sum_{n=1}^{\infty} (-1)^n \frac{c^2 \cdot (n-1)!}{\alpha^n r^n},$$

$$\mathbf{g} = \mathbf{g}(r) = -\left(K\alpha c^2 \cdot e^{\alpha r} + \sum_{n=1}^{\infty} (-1)^n \frac{c^2 \cdot n!}{\alpha^n r^{n+1}} \right) \frac{\mathbf{r}}{r},$$

$$\mathbf{g} = \mathbf{g}(x) = -\left(K\alpha c^2 \cdot e^{x} + \sum_{n=1}^{\infty} (-1)^n \frac{\alpha c^2 \cdot n!}{x^{n+1}} \right) \frac{\mathbf{r}}{r},$$

$$g = -K\alpha c^2 \cdot e^{x} - \sum_{n=1}^{\infty} (-1)^n \frac{\alpha c^2 \cdot n!}{x^{n+1}} = -K\alpha c^2 \cdot e^{\alpha r} - \sum_{n=1}^{\infty} (-1)^n \frac{c^2 \cdot n!}{\alpha^n r^{n+1}},$$

$$K \cdot e^{\alpha r} \leq \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{(\alpha r)^{n+1}}.$$

where K is a constant. The equations for speed υ are analogous to those in sections 3 and 4.

6. Conclusion

Through the Equations of the Theory of Self-Variation we can investigate the gravitational field. Also, we can compare the results of this investigation with other theories, such as the General Theory of Relativity.

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