Yes, Real Work is Done to Accelerate a Car; Pseudowork as Real "Work"

Richard Kaufman

Abstract

The work-energy theorem states that the work done on a system is equal to the change in translational and rotational kinetic energy of the system. However, there are cases in which the work-energy theorem provides for so-called paradoxes. In these situations, there is no work done by an external static force (which does not act through a distance) and yet the kinetic energy of the system has increased. To help patch up the discrepancy between both sides of the work-energy equation, the literature discusses "pseudowork", which is identified as not real work. Although the literature states or implies that energy within a system is directly transformed into kinetic energy, we argue that real work must actually be performed to increase the kinetic energy. That is, we argue that real work is performed because energy within a system is transformed into work done on the system. In these cases, a system interacts with an external object (with sufficient inertia to remain static) through equal and opposite forces. Although an external static force does not act through a distance at the system's boundary, the external static force is transferred within the system, imparting an unbalanced force that acts through a distance to perform external work within the system itself. This is somewhat similar to the way in which the external force of gravity can act within a system to perform work, where the force of gravity is taken to act at the center of mass.

I. Introduction

Reese [1] discusses two "paradoxical" physics examples regarding work and energy that form the basis of this paper. In this section, we briefly describe these examples, which have also been discussed by Penchina [2] and others [3,4,5]. In subsequent sections, we elaborate on work and pseudowork. Finally, we argue that these cases show pseudowork as real work. That is, real work must be done to cause a change in kinetic energy. Rod Cross also argues that pseudowork is real work.[6] In the present paper we use Reese's two "paradoxical" examples to argue that energy within a system is used to perform work on the system through interaction with a high inertia external body. This interaction results in an external static external force that is transferred within the system to produce an unbalanced force that acts through a distance to perform work. Our aim is to provide clarity to students and teachers on why the change in kinetic energy requires real work, per the work-energy theorem.

In Reese's first paradoxical example, Case 1, a car accelerates from rest to a final velocity. During the acceleration, only the static force of friction acts on the bottom of the tire in the horizontal direction. It is stated:

> It is this force that causes the car to accelerate. However, since there is no motion at the point of application of the static frictional force of the road on the tires, the static frictional force does zero work. Each of the forces on the car does *zero work*.

(Emphasis in original)

The apparent paradox, which is also discussed by Kumar [7], arises since there is a change in kinetic energy without any work being done on the car.

In the second example, Case 2, you are imagined to be standing on skates on a horizontal surface. Your arms are bent so that you can push off of a vertical wall. As you extend your hands to push off the wall, there is a static normal force of the wall against your hands. It is stated:

> The normal force of the wall … does zero work, since the force exists only at the point of contact between you and the wall and there is no motion at the point of application of this normal force during the push-off.

Again, an apparent paradox between work and kinetic energy arises because of the Work-Energy (WE) theorem. This theorem [8] states that the total work (*i.e.*, force times displacement) done on a system by external forces is equal to the change in kinetic energy of the system: $W_{total} = \Delta KE$.

In both examples, the left side of the equation is zero (the static force does not act through a distance), but the right side of the equation is non-zero. Reese refers to the WE theorem as the CWE (Classical Work-Energy) theorem and states:

> One way to resolve or patch up … the CWE theorem for these cases is to imagine or invent a new kind of fictional work, called the **pseudowork** done by the static frictional force in Case 1 and the normal force of the wall in Case 2. In other words, separate the left-hand side of the CWE theorem, W_{total}, into two contributions – the *real work done* by forces and *pseudowork* done by forces that do no real work.
> (Emphasis in original)

In the next section, we will look further into work and pseudowork. However, we close this section with a statement from Reese:

> The source of the apparent pseudowork of these forces lies *within the system* itself. For the car, the energy that changes its kinetic energy ultimately comes from chemical reactions inside the engine. For the skater, the energy comes from biochemical reactions that drive the muscles.
> (Emphasis in original)

II. Work and Pseudowork in the Literature

Work is defined as a force acting over a distance. For example, in Case 1, the static frictional forces that act continuously at the intersection of a rolling tire and the road is not work since these forces do not move with the point of application.

In the last section, pseudowork was discussed. Penchina [2] coined the term pseudowork by stating:

> We define a quantity which we call the pseudowork of an object, obtained by taking the scalar product of the resultant external force with the translation vector (displacement) of the center of mass of the object.
>
> $$W_{ps} \equiv \text{Pseudowork} \equiv \int \mathrm{F_{ext}} \cdot \mathrm{ds_{cm}}$$
>
> Note that the external forces need not be applied to the center of mass, and the points of application of these external forces need not have the same displacement as the center of mass; this is not true work in the usual sense of the word.
>
> ...we have proved the theorem that
>
> $$\int \mathrm{F_{ext}} \cdot \mathrm{ds_{cm}} = \Delta\left(\tfrac{1}{2} M v_{\mathrm{cm}}^2\right),$$
>
> or
>
> $$W_{ps} \equiv \text{Pseudowork} = \Delta(\mathrm{KE_{cm}})$$
>
> i.e., *the pseudowork of the resultant external force exerted on an object equals the change in kinetic energy of translation of the center of mass of the object.* We call this theorem the *pseudowork-energy principle.*

In the following, we summarize points from the literature about pseudowork, the WE theorem, etc. It should be noted that these bullet points are included in multiple references.

- Pseudowork is not real work but is sometimes referred to as work-like. [2,4, 9,10]
- The WE theorem equates the total *external* work done to the kinetic energy change of the body. [11]
- The WE theorem is applicable to point particles, or ideal rigid bodies, on which a force acts. [3,5,9,11,12]
- For an object, pseuodowork is sometime referred to as center-of-mass work or as the result of multiplying forces by the center-of-mass displacement rather than their individual displacements. [4,5,9,10,11]
- The pseudowork energy equation from Penchina has also been referred to as the center-of-mass (CM) *equation* to highlight that center-of-mass quantities are considered rather than real work and energy. [4,13]
- When static forces (*i.e.*, forces where there is no displacement) are applied at the boundary of a system, then no real work (zero-work) is generated. [3,4,5,12]

- Static forces may be required in order for an object to accelerate (such as in the cases described above), but these forces do no real work. [2,3,5]
- Pseudowork can unfortunately, inadvertently, or "accidentally" give the correct "answer" by considering a stationary frictional force to do work through a distance. [2,4,5]
- The WE theorem describes mechanical work-energy that is often used in mechanics courses. The full first law of thermodynamics (FLT) may include energy terms as such as radiation, etc, that are not included in the CM equation or WE theorem. The CM and WE can differ when a force does not act through the center-of-mass, and this can occur for deformable systems and rotating rigid systems. [13]
- A separate analysis can include situations for non-inertial frames. This has been coined the invariant first law of thermodynamics (IFLT). [13] However, we are not considering these non-inertial cases in this paper.
- Other cases of pseudowork are discussed, such as a person jumping off the ground [3,4], which is somewhat equivalent to a skater pushing off a wall.
- It has been noted that only a few textbooks even discuss the cases of zero-work forces. [3]

We note here that, contrary to the bullets above, there are also examples in the literature where work (or power) is calculated as if an external static force is used to accelerate a car [14], or work is done by an external static normal force when a car crashed/crumples into a tree [15].

Some references in the literature state, suggest, or imply that energy within a system is (directly) converted to kinetic energy of the system without any external work being done. For a Case 1 example using the car, Arons [5] states:

> … the frictional force that accelerates a car through action on the tires is a zero-work force (in the absence of slipping), and, although it accelerates the car, it does no work. The kinetic energy imparted to the car originates in the chemical processes taking place in the engine. Similar statements can be made about the actions and energy transformations involved in walking or running.

Similarly, Sherwood [4] states that "…the friction force exerted by the road … does *no* work, and the car's kinetic energy comes from the burning of gasoline, not the road." (emphasis in original).
Mungan [16] refers to Case 2 in an endnote and states:

> … if a girl on roller skates pushes off from a rigid wall, she gains kinetic energy. But the wall has not done work on the girl in the sense of transferring energy to her! Her gain in kinetic energy is at the expense of internal chemical energy from her muscles.

This last point for the skater is also stated by Sherwood [13] who references Bernard: "…the skater's increased kinetic energy comes at the expanse of internal energy."

Moreover, the transformation of internal energy to kinetic energy is sometimes attributed to *internal* (not external) forces and work for Case 1. Arons [3] states:

> …f_{dr} denotes the frictional force exerted by the road on the driving wheels… Although f_{dr} is the accelerating force, it is a zero-work force… The main point to bring out, however, is that chemical internal energy, initially resident in the fuel, is transformed through work done by *internal* forces, into translational kinetic energy of the car, internal kinetic energy of moving parts, thermal energy increases, etc. and that no external work is done on the system despite the existence of an external accelerating force.
> (Emphasis in original)

For an example using Case 2 of the figure skater, Arons [3] says:

> The normal force $\overline{N}$ exerted by the wall on the skater is again a zero-work force, and the source of the kinetic energy of the skater is the chemical internal energy change mediated by the work done by unquantifiable internal forces and displacements.

Although Arons [3] acknowledges that work is done within a system, he states that it is performed "by unquantifiable internal forces and displacements." In the next section, we will further discuss the work done within a system.

An article by Kumar [7], referred to previously, states that, "If there is no external force, the system as a whole does not move i.e. its centre of mass is stationary." It further states, "A point particle will move from rest or change its velocity only if there is a net external force on it; and *it changes its energy only if the external force does work on it*." (Emphasis in original)

While we agree with these statements from Kumar, his later statements might seem contradictory:

> …the kinetic energy of the centre of mass may change though no work is done externally. In this case, it is the internal work that changes the kinetic energy of the centre of mass as well as the kinetic energy of relative motion. This is the situation of the car accelerating on a road, with its wheels perfectly rolling.

Note how it was first stated that the velocity changes only if an *external* force does work, but later states that it is *internal* work that changes the kinetic energy of the center of mass. Both Arons and Kumar appear to attribute the change in kinetic energy to internal work within a system.

In the next section, we argue that Case 1 and Case 2 show pseudowork as real work.

III. Pseudowork as real work

In this paper, we focus on the Case 1 and Case 2 examples discussed previously and make the following points:

- Energy from within a system can be used to act against a high inertial body (*i.e.,* the earth) to generate an external static force that acts on the system itself.
- This external static force can be transferred within a system, where it is transformed into an unbalanced force that acts through a distance.
- The external force that acts through a distance within a system generates real work on the system itself. This real work can be performed within the system, although not necessarily at the center of mass itself (which is sometimes a point where there is no matter).
- That is, *external work can be performed within a system*, even when the energy for the work comes from the system itself.
- For rigid object or objects consisting of rigid parts with many particles, the center of mass is almost ubiquitously used in the application of the physics, unless specified otherwise.
- Pseudowork captures the real work done to an object in a manner similar to the way in which work done by gravity is usually taken to act at the center of mass.

For Case 1, the energy from the fuel or battery (of an electric car) is transformed into the rotation of a tire. The rotating tire applies a force on the road opposite to the displacement of the vehicle. The road causes an equal and opposite (*i.e.,* Newton's third law) static frictional force to be applied to the car resulting in its movement. This is due to the high inertia of the earth, which is so large that it does not move appreciably in reaction to the wheel (however, if the car was on a large treadmill, the treadmill would move in the opposite direction, and some energy would go into accelerating the treadmill belt and some energy into accelerating the car). For Case 2, the skater uses energy imparted by muscles to push against the wall. An equal and opposite static normal force pushes against the skater's hand. Again, the earth's mass is too large for the skater to move the earth appreciably.

The only external force that can cause the system to accelerate is from the external static force, as mentioned earlier in the literature. For Case 1, Figures 1 and 2 are adapted from Kaufman [17]. Figure 1 shows the external forces acting on a car with one drive wheel. Here there is one frictional force f of the road on the drive wheel, there are two normal forces designated with N, and the force of gravity, g, is shown acting on the center of mass of the car (the weight Mg is shown, where M is the mass of the car).

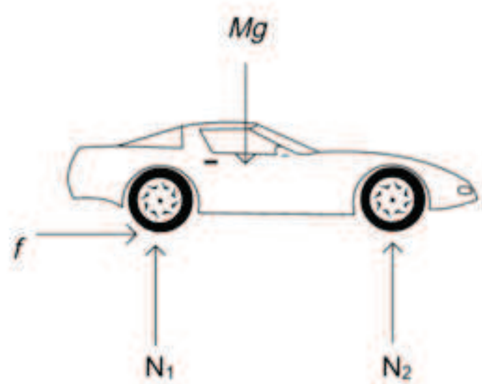

Figure 1: Accelerating car with one drive wheel.

The text for Figure 2 states:

> … the forces acting on the drive wheel: the force of friction, f, from the road, the force, f_1, of the car axle on the wheel, and the torque, t, due to the shaft that turns the wheel. The force of friction, f, is an action-reaction pair with the force of the wheel, f_{torque}, on the earth. The force f_{torque}, which is due to the torque and acts at the bottom of the tire, cannot be shown in [Fig. 2] since this would be overcounting the torque. Neither can f_{torque} be used to replace the torque, since a torque does not have a net force.[18]


Figure 2: The drive wheel.

Yet, forces are vectors that can be transmitted through an object. The external static force of friction is transferred through the wheel to the axle. At the axle, there are two forces acting, f and f_1. Since , $f > f_1$, there is an *unbalanced* force as the axle, and the car has acceleration a. That is, $f - f_1 = Ma$.

The use of unbalanced forces has previously appeared in the literature for problems like the accelerating car, although possibly in a different context that pits drag forces against the frictional forces at the wheels. For example, in a problem in a Dynamics textbook [19], a car is powered by an engine and is stated that:

> …the normal force $\mathbf{N}_C$ and the frictional force $\mathbf{F}_C$ represent the *resultant forces* of all four wheels. In particular, the unbalanced force drives or pushes the car *forward*. This effect is, of course, created by the rotating motion of the rear wheels on the pavement and is developed by the power of the engine.
> (Emphasis in original).

Case 1 is an example of a car that does not do a wheelie (the front of the car is not lifted off the ground with any rotational kinetic energy of the full car). Yet, it may not always be easy to determine where unbalanced forces act within every possible system. For example, in Case 2, the normal static force of the wall is transferred through the arms of the skater. This force is transmitted along the arms where it eventually acts as an unbalanced force through a distance as the arms are extended. Therefore, it becomes easier to treat the external static force as acting on the center of mass, as is done with most problems involving gravity and pseudowork. In this way, the external work that is done can easily be attributed to the kinetic energy of the center of mass. This makes the work-energy theorem easier to apply.

Here we make one final point about work and the work-energy theorem. Work has been characterized as being either external or internal to a system. For external work, "… it is important to be very clear about the choice of system, since it is *external* forces that perform the external work which appears on the left-hand side of the work-energy equation." [4] (emphasis in original) On the other hand, internal work is performed within a system in the absence of external work. For example, only internal work occurs by "flailing arms and legs" [4] as if performed by an astronaut floating in space. Internal work does not cause an increase in kinetic energy of the center of mass, contrary to what might have been inferred earlier in the literature.

We can see how an external force performs work inside a system, much like how gravity acts on masses within a system to perform work. As another example, consider a fixed magnet that performs work on a plastic ball with a piece of metal inside.

So, instead of merely interpreting external work as a force acting through a distance at the point of application on the boundary, we must consider where an external force acts to cause displacement of a system, even if within the system itself.

IV. Conclusion

This paper has looked at two examples and shown pseudowork as real work. We do not pretend to have exhausted all instances and examples of pseudowork, deformations, or paradoxical situations. Yet, the literature does indicate that there is a somewhat limited number of these specialized situations.

In this paper, we believe that it is helpful for students and teachers to see that energy within a system can be used to interact with a high inertia external body to generate an external static force. This static force is transferred within the system to act through a distance so that real work is done to change the kinetic energy of a system, per the work-energy theorem.

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