Zero Times Zero Equals Nonzero

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Abstract

The current Multiplication and Division Properties of Zero are flawed and illogical. This paper illustrates why and presents logical solutions that resolve the issue of dividing by zero.

Content

The current Multiplication Property of Zero states that any number times 0 is 0. I believe this to be true for all nonzero numbers, but logically not true for 0.

If we think of multiplication as the sum of groups (i.e. 3 times 5 is the sum of 5 groups of 3), then 0 times 0 can be thought of as the sum of 0 groups of 0. The sum of 0 groups of any number or object logically cannot be that same number or object (i.e. the sum of 0 groups of 1 cannot be 1, the sum of 0 groups of apples cannot be apples, etc.).

Therefore, 0 groups of 0 cannot be 0.

If we can agree that zero and nonzero are binary sets, and if we can agree that 0 groups of any nonzero number must be 0 (i.e. 0 groups of 2 must be 0), then logically the reverse must also be true, where 0 groups of 0 must be nonzero. **Therefore, we can say that 0 times 0 = nonzero.**

Another more abstract way to think about this is music. If notes represent nonzero numbers and rests represent 0 (both being binary groups that represent quantities of something and nothing), then a musical piece with 0 rests must consist of all notes and no rests (i.e. 0 groups of rests must be all notes).

If we were to apply the music analogy to the current multiplication property of zero (which currently states that 0 groups of 0 is 0), then when asked to play a musical piece with 0 groups of rests, a musician would play nothing but rests (i.e. no notes), which obviously does not satisfy the initial request or condition and would be considered completely illogical.

This logic can be applied to any binary analogy. For example, flipping a coin. If heads represents nonzero and tails represents 0, they 0 flips of tails cannot be tails. Rather, it must be any amount of heads other than 0 (i.e. nonzero).

Therefore, if we assume 0 times 0 = nonzero to be true, then we can also arrive at the following:

Multiplication

1. nonzero*0 = 0 (Current Multiplication Property)
2. 0*nonzero = 0 (Commutative Law of Line 1)
3. 0∗0 = nonzero (Proposed Multiplication Property)
4. 0∗0∗0 = a
   a. We can rewrite as (0∗0)*0 = a
   b. If we substitute (0∗0) from Line 3 we have: (nonzero)*0 = a
   c. a = 0
   d. Therefore, 0∗0∗0 = 0
   e. This would be true for all 0 to an odd power (i.e. 0^odd number)
5. 0∗0∗0∗0=b
   a. We can rewrite as (0∗0)*(0∗0) = b
   b. If we substitute (0∗0) again from Line 3 we have: (nonzero)*(nonzero) = b
   c. b = nonzero
   d. Therefore, 0∗0∗0∗0 = nonzero
   e. This would be true for all 0 to an even power (i.e. 0^even number)

Division

1. 0/nonzero = 0 (Current Division Property)
2. nonzero/0 = c
   a. Through cross-multiplication we have: 0*c = nonzero
   b. Because Line 3 under Multiplication states that 0∗0 = nonzero, c must be 0
   c. Therefore, c = 0
   d. nonzero/0 = 0
   e. Logically, this makes sense because since no groups (or 0 groups) of zero can ever produce 7 (or nonzero), we can say that 0 groups of 0 produces 7 (or nonzero), or that 0 doesn’t go into 7 any amount of times (i.e. 0 times).
3. 0/0 = d

a. Through cross-multiplication we have: 0*d = 0
b. Because Line 3 under Multiplication states that 0∗0 = nonzero, d must be nonzero.
c. Therefore, d = nonzero
d. 0/0 = nonzero
e. Logically, this makes sense because any groups of 0 other than 0 groups (i.e. any nonzero groups of 0) equals 0.

In closing, we can arrive at the following:

1. 0∗0 = 0/0 (both equal nonzero)
2. 0/nonzero = nonzero/0 (both equal 0)
3. 0∗0 and 0/0 both equal 0^even power (all equal nonzero)
4. 0/nonzero and nonzero/0 both equal 0^odd power (all equal 0)