A Modified Born-Infeld Model of Electrons with Realistic Magnetic Dipole Moment

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Abstract

The original Born-Infeld model of electrons has been used to describe static electrons without magnetic dipole moment. It is not obvious how to include the magnetic field of a realistic magnetic dipole moment in the original model. This short work proposes a small modification to the original model that might allow for experimentally observed values of electric charge and magnetic dipole moment of electrons.

1 Introduction

Born and Infeld proposed a non-linear, classical field theory and described a solution that features the same electric charge and finite field energy as an electron [BIF34]. Here we write the Lagrangian $\mathscr{L}$ [BIF34, Equation 2.11] in SI units as a function of the magnetic field $\mathbf{B}$, electric field $\mathbf{E}$, speed of light c, and the parameter b of the Born-Infeld model:

$$\mathscr{L} \stackrel{\text{def}}{=} \sqrt{1 + \frac{1}{b^2}\left(\mathbf{B}^2 - \mathbf{E}^2/c^2\right) - \frac{1}{b^4}\left(\mathbf{B}\cdot\mathbf{E}/c\right)^2} - 1. \tag{1}$$

Born and Infeld motivated this Lagrangian with a "*principle of finiteness* which postulates that any satisfactory theory should avoid letting physical quantities become infinite" [BIF34]. For an example, consider electrostatic problems without magnetic fields, i.e., $\mathbf{B} = \mathbf{0}$. In this case the expression $\sqrt{1 - \mathbf{E}^2/(c\,b)^2}$ keeps the electric field finite similarly to how the expression $\sqrt{1 - \mathbf{v}^2/c^2}$ keeps velocity $\mathbf{v}$ finite in special relativity.

One shortcoming of Born and Infeld's electron model is that its magnetic dipole moment is 0, while experimentally observed electrons feature a non-vanishing magnetic dipole moment. Since Born-Infeld field theory approximates standard electromagnetism for low field energies, it is tempting to assume that one could include a magnetic dipole moment by adding its magnetic field for large distances and solving the field equations for short distances. Section 2 explains why this approach cannot succeed for the observed electric charge and magnetic dipole moment of electrons. Section 3 presents a small modification to Born-Infeld field theory that avoids the problem, while Section 4 discusses some aspects of the context of the proposed modification. Section 5 concludes this short paper.

2 The Trouble with Dipole Fields

As Born-Infeld field theory approximates standard electromagnetism for low field energies, the electromagnetic field of a static electron-like solution may be approximated for large distances by the electric field of an electric point charge and the magnetic field of a magnetic dipole moment.

Specifically, the electric field strength $|\mathbf{E}|$ at distance $r = |\mathbf{r}|$ caused by a static electron of charge q at the origin $\mathbf{r} = \mathbf{0}$ may be specified in SI units with vacuum permittivity ε_0 as

$$|\mathbf{E}| = \frac{|q|}{4\pi\varepsilon_0 r^2}. \tag{2}$$

The magnetic field strength $|\mathbf{B}|$ at distance $r = |\mathbf{r}|$ and magnetic latitude λ caused by a static magnetic dipole moment $\mathbf{m}$ may be specified in SI units with vacuum permeability μ_0 as

$$|\mathbf{B}| = \frac{\mu_0 |\mathbf{m}|}{4\pi r^3} \sqrt{1 + 3\sin^2(\lambda)}. \tag{3}$$

Note that $|\mathbf{E}|$ increases with r^{-2} for $r \to 0$, while the minimum value of $|\mathbf{B}|$ for given r increases with r^{-3}; i.e., $|\mathbf{B}|$ increases faster than $|\mathbf{E}|$ for $r \to 0$. For measured values of the constants, this means that $|\mathbf{B}|$ is orders of magnitude greater than $|\mathbf{E}|/c$ for r at the scale of femtometers, where Born-Infeld field theory is expected to deviate from standard electromagnetism.

Therefore, $\mathbf{B}^2 - \mathbf{E}^2/c^2$ is positive and $\sqrt{1 + (\mathbf{B}^2 - \mathbf{E}^2/c^2)/b^2} > 1$ at the scale of femtometers instead of approaching 0 for $r \to 0$. This prevents the Born-Infeld Lagrangian from keeping field strengths finite. Thus, while Born-Infeld field theory keeps field strengths finite for a static charged particle-like solution without magnetic dipole moment ($\mathbf{B} = \mathbf{0}$), it cannot provide an electron-like solution of finite energy featuring realistic electric charge and magnetic dipole moment.

Born and Schrödinger tried to include a realistic magnetic dipole moment by adapting the parameter b [BS35], but apparently they ignored the problematic effect of the magnetic dipole field.

Fortunately, a small modification to the Lagrangian might avoid this problem as described in the next section.

3 An "Easy" Fix

In order to solve the issue discussed in Section 2, a modified Lagrangian $\tilde{\mathscr{L}}$ is proposed with

$$\tilde{\mathscr{L}} \stackrel{\text{def}}{=} \sqrt{1 - \frac{1}{b^2}\left(\mathbf{B}^2 - \mathbf{E}^2/c^2\right)} - 1. \tag{4}$$

Compared to the original Lagrangian $\mathscr{L}$, the b^{-2} term is multiplied with -1, and the b^{-4} term is neglected as its sign has to be determined by future work.

With this change, a static electron-like solution with realistic electric charge and magnetic dipole moment might exist because the term $\sqrt{1 - \left(\mathbf{B}^2 - \mathbf{E}^2/c^2\right)/b^2}$ approaches 0 for $r \to 0$ as $|\mathbf{B}|$ increases faster than $|\mathbf{E}|$. Therefore, this term might keep magnetic field strength as well as electric field strength finite. Preliminary numeric experiments supported this hypothesis.

4 Discussion

While the original Lagrangian $\mathscr{L}$ cannot describe electron-like solutions with realistic electric charge and magnetic dipole moment, the modified Lagrangian $\tilde{\mathscr{L}}$ cannot describe electron-like solutions without magnetic dipole moment. However, the latter is consistent with the Standard Model of particle physics, which does not include charged leptons without magnetic dipole moment.

Furthermore, experimentally observed magnetic dipole moments of charged leptons decrease with increasing proper mass while the absolute value of the electric charge of all charged leptons is identical to the elementary charge. If muon and tau are considered heavier and therefore smaller excited states of an electron-like solution [Dir62] and the magnetic field strength is the primary reason for finite field strengths, it is plausible that muon and tau feature a smaller magnetic dipole moment than the electron, because smaller distances would result in greater magnetic field strength for the same magnetic dipole moment, which is not plausible if the magnetic field strength of all charged lepton-like solutions is limited by the same parameter b. On the other hand, the electric field strength of these solutions is not limited in the same way.

The original motivation for this work, however, were numeric experiments with a different modification of the Born-Infeld Lagrangian [Kra23]. In these numeric experiments, electrons are rotating waves that are pulled onto a circular orbit by a centripetal force. When a realistic magnetic dipole moment was included, solutions to the field equations could only be computed with the modification proposed here. For the resulting solutions, the Lorentz force due to the electric and magnetic field around the peak of the rotating wave could be estimated. It turned out that the Lorentz force due to the electric field is about an order of magnitude too weak to be identified as the mentioned centripetal force, while the Lorentz force due to the magnetic field is of the correct order of magnitude. This is another reason to believe that the magnetic field strength plays a more important role in models related to Born-Infeld field theory than previously assumed.

5 Conclusion

This work proposes a new modification to the Born-Infeld Lagrangian that might lead to a model of electrons with realistic electric charge and magnetic dipole moment. While preliminary numeric experiments support this hypothesis, more numeric experiments are necessary to confirm it.

References

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A Revisions

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