

Can be for a number k , $(2[k]m)+1$ always prime for all number m ?

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0- Abstract:

This paper is about hyperoperators. In this paper I ask myself and the mathematical community if there is possible that a k -ation of the number 2 will be always a number prime for any number m if we add the number one to the result.

1- Introduction:

We will use hyperoperators tools to enunciate this conjecture. Lets refresh some concepts. The addition is “ n copies of 1 added to a combined by succession.”. The multiplication is “ n copies of a combined by addition.”. The exponentiation is “ n copies of a combined by multiplication.”. The tetration is “ n copies of a combined by exponentiation, right-to-left.”.[1] The pentation is “ n copies of a combined by tetration, right-to-left.” and so on.

Going to the “easy” counterexamples we have that in:

- a) Addition: $(2+3)+1=(2[1]3)+1=6$ which is not prime
- b) Multiplication: $(2 \times 4)+1=(2[2]4)+1=9$ which is not prime.
- c) Exponentiation: $(2^3)+1=(2[3]3)+1=9$ which is not prime
- d) Tetration: $(2 \uparrow 2 \uparrow 2 \uparrow 2 \uparrow 2)+1=(2[4]5)+1$ which is not prime, is a number with 19729 digits [2].

Going with more hard examples we have pentation, the first cases are:

- e) Pentation: $(2[5]1)+1=2+1=3$ which is prime.
 $(2[5]2)+1=4+1=5$ which is prime.
 $(2[5]3)+1=65536+1=65537$ which is prime.

But, the problem begins with $(2[5]4)+1$ which is huge number, one of that number that I can not calculate even understand at all.

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2: Conjecture:

As the title said the conjecture is the following:

“Can be for a number k , $(2^{[k]m})+1$ always a primer number for all number m ?”

I do not know if there exists a counterexample in pentation or not, if there is one we should look for 6-ation. Anyway the conjecture goes further and can allow any k -ation.

3: Conclusions:

Since old times mathematicians have been looking for a sequence of number or a determinate formulae that always give us prime numbers. That is a fair trying, Fermat or Mersenne will be in that way, even Eiseinstein. Sometimes is hard to proof it, and in the most of cases we have ended obtaining a disproof of the conjectures, if someone asks to me if I believe if this conjecture is true or false I would say that is false, but I do not have enough mathematical tools to disproof it.

4: References:

[1] <https://en.wikipedia.org/wiki/Pentation> & <https://en.wikipedia.org/wiki/Tetration>

[2] $(2^{2^{\dots^{2^2}}})$ (n times) + 1. Serie. <https://oeis.org/A007516>