Calculation of Jupiter's Size and Saturn's Size by Quantum Gravity Theory with Ultimate Acceleration

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Abstract: In analogy with the ultimate speed *c*, there is an ultimate acceleration *β*, nobody's acceleration can exceed this limit *β* in a many-body system. In recent years, de Broglie matter wave has been generalized in terms of the ultimate acceleration. This paper shows that Jupiter's size and Saturn's size are the consequences of the interference of the generalized matter waves. In this calculation, Jupiter's radius is determined as 7.1491e+7m with a relative error of 5.05%; Saturn's radius is determined as 5.80057e+7m with a relative error of 3.75%. This calculation also correctly predicts the locations of Halo ring and main ring for Jupiter, ring A, B, C and D for Saturn.

1. Introduction

In analogy with the ultimate speed *c*, there is an ultimate acceleration *β*, nobody's acceleration can exceed this limit *β* in a many-body system. In recent years, de Broglie matter wave has been generalized in terms of the ultimate acceleration. Consider a particle, its relativistic matter wave is given by the path integral

$$\psi = \exp\left(\frac{i\beta}{c^3}\int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4)\right) \quad . \tag{1}$$

where u is the 4-velocity of the particle, β is the ultimate acceleration determined by experiments. The β replaces the *Planck constant* in this quantum gravity theory so that *its wavelength becomes a length on planetary-scale*. The author's early paper shows that this generalized matter wave can explain the solar quantum gravity effects correctly, such as sunspot cycle, atmospheric circulation and human lifespan [20]. The present paper shows that this quantum gravity theory with the ultimate acceleration provides a mechanism to calculate the Jupiter's size and Saturn's size correctly.

2. Extracting ultimate acceleration from a system

Similar to the Bohr model of hydrogen atom, the orbital circumference is *n* multiple of the wavelength of the planetary-scale relativistic matter wave. According to Eq. (1), consider a satellite, we have

$$\left.\begin{aligned}\frac{\beta}{c^3}\oint_L v_l dl = 2\pi n \\ v_l = \sqrt{\frac{GM}{r}}\end{aligned}\right\} \Rightarrow \quad \sqrt{r} = \frac{c^3}{\beta\sqrt{GM}} n; \quad n = 0,1,2,\dots \quad . \tag{2}$$

This orbital quantization rule only achieves a half success in the Jupiter system and Saturn system, as shown in Fig.1. The Jupiter, Metis, Adrastea, Amalthea and Thebe satisfy the quantization equation in Fig.1(a); while other outer satellites (Io, Europa, Ganymede, Callisto) fail. The Saturn, Mimas, Enceladus, Tethys and Dione satisfy the quantization equation in Fig.1(b); while other outer satellites (Rhea,Titan,Hyperion,Iapetus) fail. But, since we only study quantum gravity effects near the planets, so this orbital quantization rule is good enough as a foundational quantum theory. In Fig.1, the blue straight lines express a linear regression relation among the quantized orbits, so it gives Jupiter's β=4.013970e+10 (m/s^2) and Saturn's β=7.175115e+13 (m/s^2) by fitting the lines. The quantum numbers n=6,7,8,... were assigned to the Jupiter's satellites, the Jupiter was assigned a quantum number n=0 because it is in the **central state**. The quantum numbers n=7,8,9,... were assigned to the Saturn's satellites, the Saturn was assigned a quantum number n=0 because it is in the **central state**.

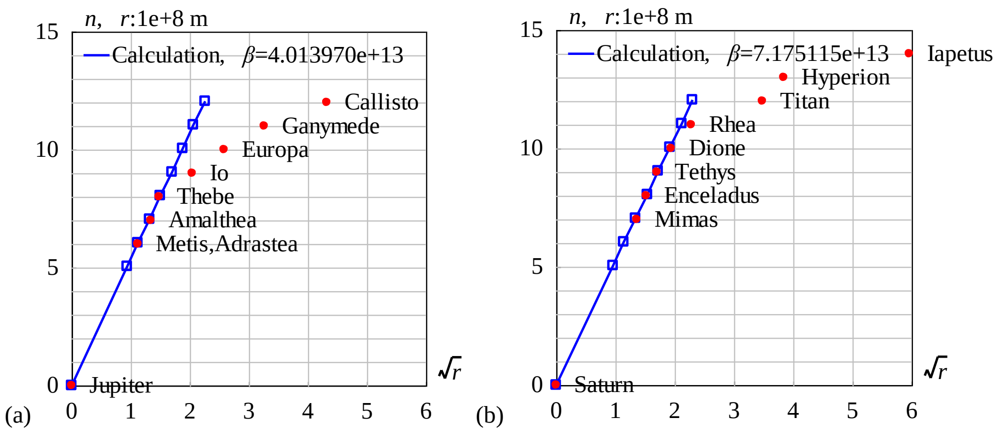

Fig.1 The inner satellites are quantized.

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<Clet2020 Script>// C source code [17]
int i,j,k,n,N,N1,nP[10],Figure; double x,y,z,r,r1,rs,M,r_unit,H,beta,pD[10],D[200],S[200];
double orbit[20]={0,1.285,1.29,1.81,2.22,4.21,6.71,10.7,18.82,},a,b,A,B,r_massive,R;
double e[20]={0,0.0077,0.0063,0.0075,0.0180,0.0041,0.0094,0.0011,0.0074,0,0,0,};
int qn[20]={0,6,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20}; char str[200];
char Stars[100]={"Jupiter; ;Metis,Adrastea;Amalthea;Thebe;Io;Europa;Ganymede;Callisto;"};
main(){ Figure=1; N=9;N1=5;rs=11.209*6.378e6;M=317.8*5.97237E24;r_unit=1e8;
for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0;S[1]=0;
SetAxis(X_AXIS,0,0,6,"#if#rsr#t;0;1;2;3;4;5;6;");
SetAxis(Y_AXIS,0,0,15,"#ifn,      r#t:1e+8 m;0;5;10;15;");
DrawFrame(0x016f,1,0xafffaf); A=-0.55; r_massive=2.7; R=sqrt(r_massive); B=-0.05;
rs/=r_unit;k=1; n=1; N1=10000;
for(i=0;i<N1;i+=1) { r=35*i/N1;    r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=n) {S[k+k]=r1; S[k+k+1]=y; n+=1; if(r>rs) k+=1; }    if(y>12) break;}
Format(str,"Calculation,      #ifβ#t=%e",beta);    SetPen(2,0x0000ff);
Polyline(k,S,0.2,14,str);Plot("CARD,0,@k,XY,4,4,",S);
SetPen(2,0xff0000); Plot("OVALFILL,0,@N,XY,3,3,",D); nP[0]=TAKE;
for(i=0;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A#t
```

```
<Clet2020 Script>// C source code [17]
int i,j,k,n,N,N1,nP[10],Figure; double x,y,z,r,r1,rs,M,r_unit,H,beta,pD[10],D[200],S[200];
double orbit[20]={0,1.85,2.37,2.94,3.77,5.27,12.21,14.8,35.6,},a,b,A,B,r_massive,R;
double e[20]={0,0,0,0,0,0,0,0,0,0,0,0,0,0,}; int qn[20]={0,7,8,9,10,11,12,13,14,15,16,17,18,19,20};
char Stars[100]={"Saturn;Mimas;Enceladus;Tethys;Dione;Rhea;Titan;Hyperion;Iapetus;"}; char str[200];
main(){ Figure=1; N=9;N1=5; rs=9.449*6.378e6;M=95.16*5.97237E24;r_unit=1E8;
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for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0; S[1]=0;
SetAxis(X_AXIS,0,0,6,"#if#rsr#t;0;1;2;3;4;5;6;");
SetAxis(Y_AXIS,0,0,15,"#ifn,     r#t:1e+8 m;0;5;10;15;");
DrawFrame(0x016f,1,0xafffaf); A=-0.62; r_massive=4.5; R=sqrt(r_massive); B=-0.05;
k=1;    n=1; N1=10000; rs/=r_unit;
for(i=0;i<N1;i+=1) { r=40*i/N1; r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=n) {S[k+k]=r1; S[k+k+1]=y; n+=1; if(r>rs) k+=1;} if(y>12) break;}
Format(str,"Calculation,     #ifβ#t=%e",beta);    SetPen(2,0x0000ff);
Polyline(k,S,0.2,14,str);Plot("CARD,0,@k,XY,4,4,",S);
SetPen(2,0xff0000); Plot("OVALFILL,0,@N,XY,3,3,",D); nP[0]=TAKE;
for(i=0;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A#t
```

The relativistic matter wave function ψ needs a further explanation. In quantum mechanics, $|\psi|^2$ equals to the probability of finding an electron due to Max Born's explanation; in astrophysics, $|\psi|^2$ equals to the probability of finding a nucleon (proton or neutron) *averagely on an astronomic scale*, we have

$$|\psi|^2 \propto \text{nucleon-density} \propto \rho \quad . \tag{3}$$

3. Jupiter's size and Saturn's size

In interior of Jupiter or Saturn, if the coherent length of the relativistic matter wave is long enough, its head may overlap with its tail when the particle moves in a closed orbit, as shown in Fig.2. Consider a point on the equatorial plane, the overlapped wave is given by

$$\begin{aligned} &\psi = \psi(r)T(t) \\ &\psi(r) = 1 + a_1 e^{i\delta} + a_2 e^{i2\delta} + \ldots + a_{N-1} e^{i(N-1)\delta} \\ &\delta(r) = \frac{\beta}{c^3}\oint_L (v_l)dl = \frac{2\pi\beta\omega r^2}{c^3} \end{aligned} \quad . \tag{4}$$

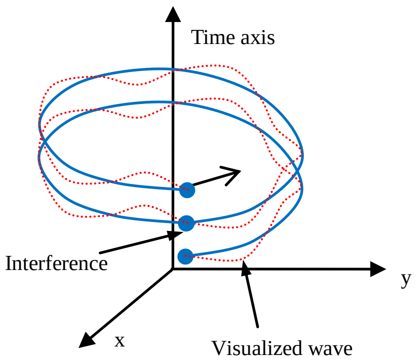


Fig.2 The head of the relativistic matter wave may overlap with its tail.

where N is the overlapping number which is determined by the coherent length of the relativistic matter wave, δ is the phase difference after one orbital motion, ω is the angular speed of the Jupiter rotation, $a_1, a_2, .., a_{N-1}$ are the amplitudes of the wavelets. The above equation is a multi-slit interference formula in optics, for a larger N it becomes the Fabry-Perot interference.

According to the study of tropic cyclones on the Pacific ocean [20] where clouds have

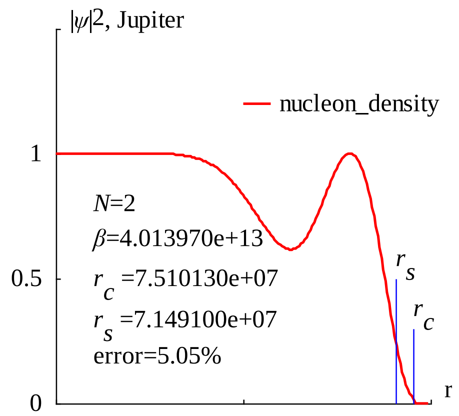
N=2, the Jupiter and Saturn also have N=2, because they are gaseous planets. Let r_s denotes the radius of planet, due to the skin effect into planetary interior, the amplitude of the first wavelet is simply assumed as

$$a_1 = \begin{cases} (\frac{r}{r_s})^5 & r \le r_s \\ 1 & r > r_s \end{cases} . \tag{5}$$

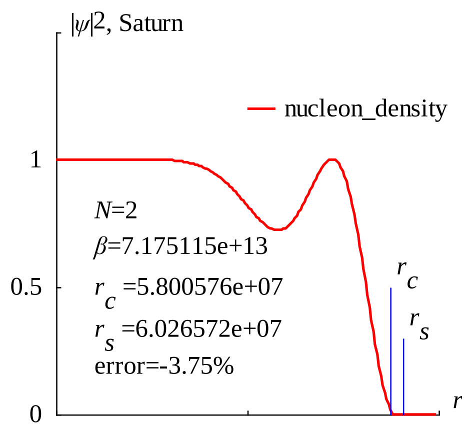
then the Jupiter's size and Saturn's size can be estimated.

Jupiter's angular speed at its equator is known as ω=2π/(9.925x3600) (s^{-1}). Its mass 317.816M_{earth} (kg), the well-known radius 11.209r_{earth} (m), the mean density 1326 (kg/m^3), the constant β=4.013970e+10 (m/s^2). According to the N=2, the matter distribution of the $|\psi|^2$ is calculated in Fig.3(a), it agrees well with the general description of Jupiter interior. The radius of Jupiter is determined as r_c =7.1491e+7 (m) with a relative error of 5.05% in Fig.3(a), which indicates that the Jupiter radius strongly depends on its rotation.

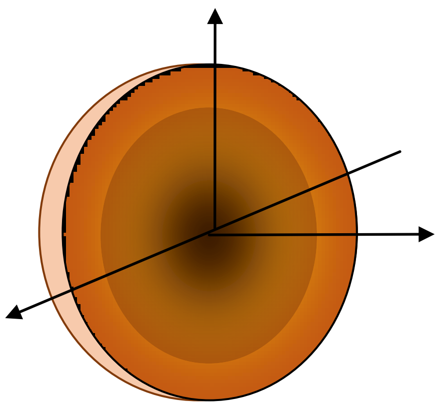
Saturn's angular speed at its equator is known as ω=2π/(10.6562x3600) (s^{-1}). Its mass 95.16M_{earth} (kg), the well-known radius 11.209r_{earth} (m), the mean density 1326 (kg/m^3), the constant β=7.175115e+13 (m/s^2). According to the N=2, the matter distribution of the $|\psi|^2$ is calculated in Fig.3(b), it agrees well with the general description of Saturn interior. The radius of Saturn is determined as r_c=5.80057e+7 (m) with a relative error of -3.75% in Fig.3(b), which indicates that the Saturn radius strongly depends on its rotation.



(a)



(b)



(c)

Fig.3 The nucleon distribution $|\psi|^2$ in the Sun is calculated. (a) Jupiter, (b) Saturn, (c)interior.

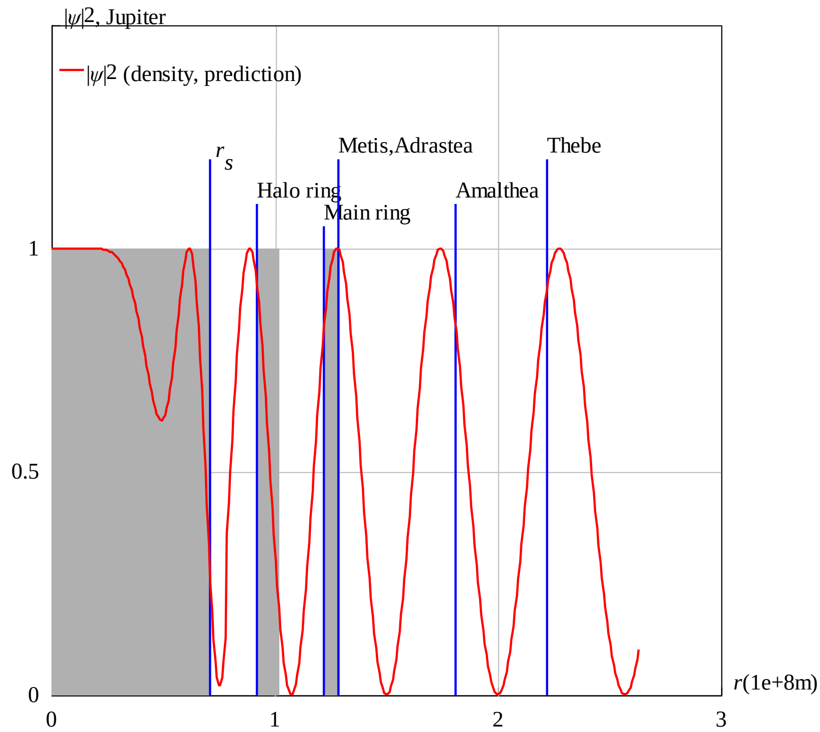
<Clet2020 Script>//C source code [17]

```
int i,j,k,n,N; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=200;rs=11.209*6.378e6; r_atm =rs+rs/10; rc=r_atm; n=0; N=2;
beta=4.013970e+13;H=SPEEDC*SPEEDC*SPEEDC/beta; M=317.816*5.97237e24;
r_unit=r_atm/k; omega=2*PI/(9.925*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs; amp=pow(x,5); d=(1+amp)*(1+amp); //skin effect
delta=2*PI*omega*r*r/H; x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d; if(r>rc) z=0;
S[n]=i;S[n+1]=z; n+=2; if(rc>=r_atm && z<0.02) rc=r; }
SetAxis(X_AXIS,0,0,k,"r; ; ; ;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Jupiter;0;0.5;1; ;");
DrawFrame(FRAME_SCALE,1,0xafffaf); x=50;z=100*(rc-rs)/rs;
SetPen(2,0xff0000);Polyline(k,S,k/2,1.2," nucleon_density");SetPen(1,0xff);
x=rs/r_unit;y=0.5;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0,"#ifr#sds#t");
x=rc/r_unit;y=0.3;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0,"#ifr#sdc#t");
Format(str,"#ifN#t=%d#n#ifβ#t=%e#n#ifr#sdc#t =%e#n#ifr#sds#t =%e#nerror=%.2f%",N,beta,rc,rs,z);
TextHang(k/10,0.8,0,str);
}#v07=?>A

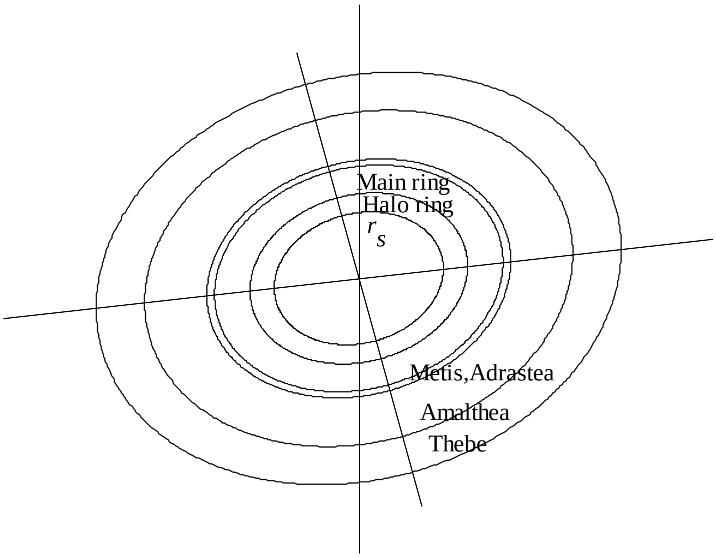
<Clet2020 Script>//C source code [17]
int i,j,k,n,N; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=200;rs=9.449*6.378e6; r_atm =rs+rs/10; rc=r_atm; n=0; N=2;
beta=7.175115e+13;H=SPEEDC*SPEEDC*SPEEDC/beta; M=95.16*5.97237e24;
r_unit=r_atm/k; omega=2*PI/(10.6562*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs; amp=pow(x,5); d=(1+amp)*(1+amp); //skin effect
delta=2*PI*omega*r*r/H; x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d; if(r>rc) z=0;
S[n]=i;S[n+1]=z; n+=2; if(rc>=r_atm && z<0.02) rc=r; }
SetAxis(X_AXIS,0,0,k,"#ifr#t; ; ; ;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Saturn;0;0.5;1; ;");
DrawFrame(FRAME_SCALE,1,0xafffaf); x=50;z=100*(rc-rs)/rs;
SetPen(2,0xff0000);Polyline(k,S,k/2,1.2," nucleon_density");SetPen(1,0xff);
x=rs/r_unit;y=0.3;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0," #ifr#sds#t");
x=rc/r_unit;y=0.5;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.1,0," #ifr#sdc#t");
Format(str,"#ifN#t=%d#n#ifβ#t=%e#n#ifr#sdc#t =%e#n#ifr#sds#t =%e#nerror=%.2f%",N,beta,rc,rs,z);
TextHang(k/10,0.8,0,str);
}#v07=?>A
```

4. Locations of satellites and rings

The calculation of near field situations has also been carried out by the same formula Eq.(4) as shown in Fig.4 and Fig.5. For Jupiter, the maxima of $|\psi(r)|^2$ point out the radii of Metis, Adrastea, Amalthea and Thebe, respectively; also approximately gives out the locations of Halo ring and Main ring; as shown in Fig.4, the overall relative error is about 5%.



(a)



(b)

Fig.4 The nucleon distribution $|\psi|^2$ is calculated in the radial direction for Jupiter.

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<Clet2020 Script>//C source code [17]
int i,j,k,n,N; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=400;rs=11.209*6.378e6; r_atm =rs+rs/10; rc=r_atm; n=0; N=2;
beta=4.013970e+13;H=SPEEDC*SPEEDC*SPEEDC/beta; M=317.816*5.97237e24;
r_unit=3e+8/k; omega=2*PI/(9.925*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs;
if(r<r_atm) {delta=2*PI*omega*r*r/H;amp=pow(x,5);}
else {delta=2*PI*sqrt(GRAVITYC*M*r)/H;amp=1;}
d=(1+amp)*(1+amp); x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d;
S[n]=i;S[n+1]=z; n+=2; if(i>350) break;}
SetAxis(X_AXIS,0,0,k,"#ifr#t(1e+8m);0;1;2;3;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Jupiter;0;0.5;1; ;");
DrawFrame(0x0133,2,0xafffaf); SetPen(2,0xff);
x=rs/r_unit;y=1.2;D[0]=0;D[1]=0;D[2]=x;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D);D[0]=D[2]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0," #ifr#sds#t ");
x=0.92e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=1.02e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]= D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Halo ring");
x=1.22e8/r_unit;y=1.05;D[0]=x;D[1]=0;D[2]=1.29e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]=D[0]; D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Main ring");
x=1.285e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Metis,Adrastea");
x=1.81e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Amalthea");
x=2.22e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Thebe");
SetPen(2,0xff0000);Polyline(n/2,S,5,1.4,"#if|ψ|#su2#t (density, prediction)"); SetPen(1,0xff);
}#v07=?>A
```

```
<Clet2020 Script>// C source code [17]
char Stars[100]={"#ifr#sds#t;Halo ring;Main ring;Metis,Adrastea;Amalthea;Thebe; ;"};
double r[10]={0.7149,0.92,1.22,1.285,1.81,2.22,0,};
double a,b,x,y,dP[10],D[100],S[2000];int i,j,k,N,nP[10];char str[200];
main(){SetViewAngle(0,50,10);a=3;N=6;
SetAxis(X_AXIS,-a,0,a,"x;");SetAxis(Y_AXIS,-a,0,a,"y;");SetAxis(Z_AXIS,-a,0,a,"z;");
DrawFrame(FRAME_LINE,2,0x00ffff); //SetPen(1,0x808000);
for(i=0;i<N;i+=1) {b=r[i];
D[0]=0;D[1]=0;D[2]=0;D[3]=0;D[4]=1;D[5]=0;D[6]=0;D[7]=0;D[8]=1;
D[9]=600;D[10]=b;D[11]=b;
Lattice(OVAL,D,S); Plot("POLYGON,0,600,XYZ,10",S[9]);
nP[0]=TAKE;nP[1]=i;TextJob(nP,Stars,str);y=r[i];x=0.2; if(i>2) y=-y; TextHang(x,y,0,str);}
a=317.816*5.97237e24; y=30;
for(i=0;i<N;i+=1) {x=r[i]*1e+8;b=sqrt(GRAVITYC*a/x);b=2*PI*x/b; b/=3600;
TextAt(100,y,"i=%d      r=%e      T=%f",i,x,b); y+=30;}
}#v07=?>A
```

For Saturn, the maxima of $|\psi(r)|^2$ point out the radii of Mimas, Enceladus, Tethys and Dione, respectively; also approximately gives out the locations of ring A,B,C and D; as shown in Fig.5, the overall relative error is about 5%.

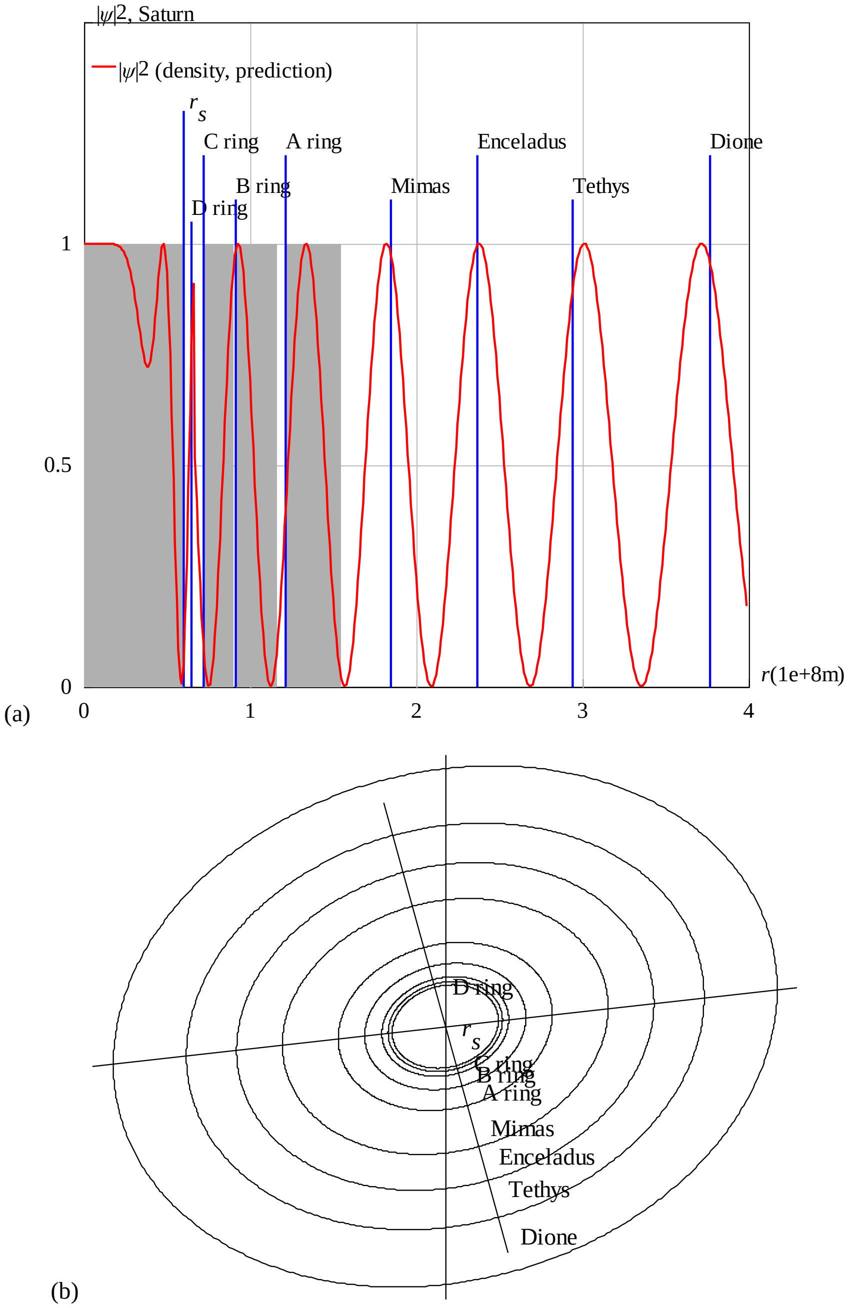

Fig.5 The nucleon distribution $|\psi|^2$ is calculated in the radial direction for Saturn.

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<Clet2020 Script>//C source code [17]
int i,j,k,n,N; double H,M,r,r_atm,r_unit,x,y,z,delta,D[10],S[1000];
double rs,rc,omega,amp,d,beta; char str[100];
main(){k=400;rs=9.449*6.378e6; r_atm =rs+rs/10; rc=r_atm; n=0; N=2;
beta=7.175115e+13;H=SPEEDC*SPEEDC*SPEEDC/beta; M=95.16*5.97237e24;
r_unit=4e+8/k; omega=2*PI/(10.6562*60*60);//angular speed
for(i=0;i<k;i+=1) {r=i*r_unit; x=r/rs;
if(r<r_atm) {delta=2*PI*omega*r*r/H;amp=pow(x,5);}
else {delta=2*PI*sqrt(GRAVITYC*M*r)/H;amp=1;}
d=(1+amp)*(1+amp); x=1+amp*cos(delta);y=amp*sin(delta); z=(x*x+y*y)/d; S[n]=i;S[n+1]=z; n+=2; }
SetAxis(X_AXIS,0,0,k,"#ifr#t(1e+8m);0;1;2;3;4;");SetAxis(Y_AXIS,0,0,1.5,"#if|ψ|#su2#t, Saturn;0;0.5;1; ;");
DrawFrame(0x0143,2,0xafffaf);    SetPen(2,0xff);
x=rs/r_unit;y=1.3;D[0]=0;D[1]=0;D[2]=x;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[0]=D[2];D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0," #ifr#sds#t ");
x=0.65e8/r_unit;y=1.05;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"D ring");
x=0.72e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=0.9e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]=D[0];D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"C ring");
x=0.915e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=1.165e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]=D[0];D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"B ring");
x=1.215e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=1.55e8/r_unit;D[3]=1;
Draw("RECT,3,XY,0,0xb0b0b0",D); D[2]=D[0];D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"A ring");
x=1.85e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Mimas");
```

```
x=2.37e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Enceladus");
x=2.94e8/r_unit;y=1.1;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Tethys");
x=3.77e8/r_unit;y=1.2;D[0]=x;D[1]=0;D[2]=x;D[3]=y; Polyline(2,D);TextHang(x,y+0.03,0,"Dione");
SetPen(2,0xff0000); Polyline(n/2,S,5,1.4,"#if|ψ|#su2#t (density, prediction)");
}#v07=?>A
```

```
<Clet2020 Script>// C source code [17]
char Stars[100]={"#ifr#sds#t;D ring;C ring;B ring;A ring;Mimas;Enceladus;Tethys;Dione; ;"};
double r[10]={0.603,0.65,0.72,0.915,1.215,1.85,2.37,2.94,3.77,0,};
double a,b,x,y,dP[10],D[100],S[2000];int i,j,k,N,nP[10];char str[200];
main(){SetViewAngle(0,50,10);a=4;N=9;
SetAxis(X_AXIS,-a,0,a,"x;");SetAxis(Y_AXIS,-a,0,a,"y;");SetAxis(Z_AXIS,-a,0,a,"z;");
DrawFrame(FRAME_LINE,2,0x00ffff); //SetPen(1,0x808000);
for(i=0;i<N;i+=1) {b=r[i]; D[0]=0;D[1]=0;D[2]=0;D[3]=0;D[4]=1;D[5]=0;D[6]=0;D[7]=0;D[8]=1;
D[9]=600;D[10]=b;D[11]=b;
Lattice(OVAL,D,S); Plot("POLYGON,0,600,XYZ,10",S[9]);
nP[0]=TAKE;nP[1]=i;TextJob(nP,Stars,str);y=r[i];x=0.2; if(i>1) y=-y; if(i==0) y=0;
TextHang(x,y,0,str);}
a=95.16*5.97237e24; y=30;
for(i=0;i<N;i+=1) {x=r[i]*1e+8;b=sqrt(GRAVITYC*a/x);b=2*PI*x/b; b/=3600;
TextAt(100,y,"i=%d      r=%e      T=%f",i,x,b); y+=30;}
}#v07=?>A
```

5. Improvement by the band theory

For far field situations and for improving the precision, we should invoke the band theory that is a sophisticated knowledge in semiconductors and superconductors for electronic quantum mechanics. For example, in Jupiter, the first band consist of Metis, Adrastea, Amalthea and Thebe; the second band consist of Io, Europa, Ganymede and Callisto. As shown in Fig.6. There also exists **forbidden gap** in the bands, for example, Ceres is considered as locating within the forbidden gap in the solar system with other eight planets [23]. By the way, the band theory supports the dark matter concept [20][23].

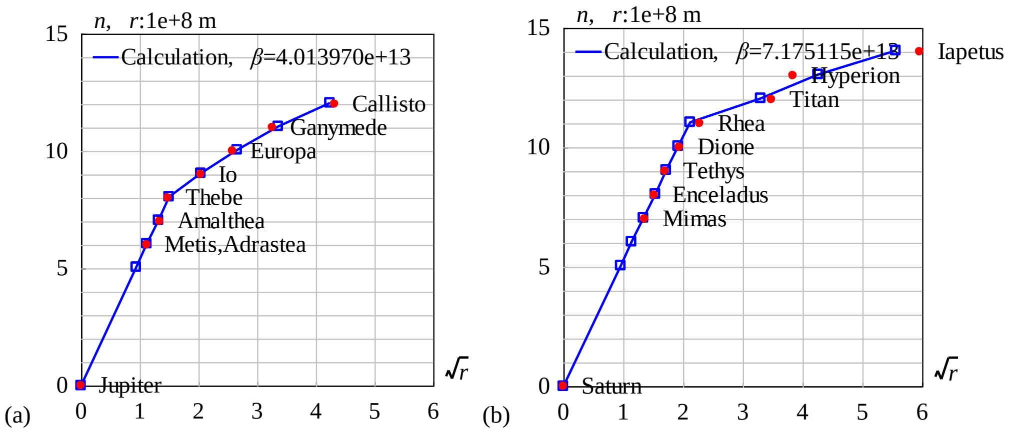

Fig.6 The satellites are quantized by the band theory [23].

```
<Clet2020 Script>// C source code [17]
int i,j,k,n,N,N1,nP[10],Figure; double x,y,z,r,r1,rs,M,r_unit,H,beta,pD[10],D[200],S[200];
double orbit[20]={0,1.285,1.29,1.81,2.22,4.21,6.71,10.7,18.82,},a,b,A,B,r_massive,R;
double e[20]={0,0.0077,0.0063,0.0075,0.0180,0.0041,0.0094,0.0011,0.0074,0,0,0,};
int qn[20]={0,6,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20}; char str[200];
char Stars[100]={"Jupiter; ;Metis,Adrastea;Amalthea;Thebe;Io;Europa;Ganymede;Callisto;"};
main(){ Figure=2; N=9;N1=5;rs=11.209*6.378e6;M=317.8*5.97237E24;r_unit=1e8;
for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0;S[1]=0;
SetAxis(X_AXIS,0,0,6,"#if#rsr#t;0;1;2;3;4;5;6;"); SetAxis(Y_AXIS,0,0,15,"#ifn,      r#t:1e+8 m;0;5;10;15;");
DrawFrame(0x016f,1,0xafffaf); A=-0.55; r_massive=2.7; R=sqrt(r_massive); B=-0.05;
rs/=r_unit;k=1; n=1; N1=10000;
for(i=0;i<N1;i+=1) { r=35*i/N1;    r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=n) {S[k+k]=r1; S[k+k+1]=y; n+=1; if(r>rs) k+=1; }    if(y>12) break;}
Format(str,"Calculation,     #ifβ#t=%e",beta);   SetPen(2,0x0000ff);
Polyline(k,S,0.2,14,str);Plot("CARD,0,@k,XY,4,4,",S);
SetPen(2,0xff0000); Plot("OVALFILL,0,@N,XY,3,3,",D); nP[0]=TAKE;
for(i=0;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A#t
```

```
<Clet2020 Script>// C source code [17]
int i,j,k,n,N,N1,nP[10],Figure; double x,y,z,r,r1,rs,M,r_unit,H,beta,pD[10],D[200],S[200];
double orbit[20]={0,1.85,2.37,2.94,3.77,5.27,12.21,14.8,35.6,},a,b,A,B,r_massive,R;
double e[20]={0,0,0,0,0,0,0,0,0,0,0,0,0,0,}; int qn[20]={0,7,8,9,10,11,12,13,14,15,16,17,18,19,20};
char Stars[100]={"Saturn;Mimas;Enceladus;Tethys;Dione;Rhea;Titan;Hyperion;Iapetus;"}; char str[200];
main(){ Figure=2; N=9;N1=5; rs=9.449*6.378e6;M=95.16*5.97237E24;r_unit=1E8;
for(i=0;i<N;i+=1) {y=e[i]; x=orbit[i]*(1+sqrt(1-y*y))/2; D[i+i]=sqrt(x); D[i+i+1]=qn[i]; }
nP[0]=REGRESSION; nP[1]=N1; DataJob(nP,D,pD); a=pD[0]; b=pD[1];
beta=b*SPEEDC*SPEEDC*SPEEDC/sqrt(GRAVITYC*M*r_unit); S[0]=0; S[1]=0;
SetAxis(X_AXIS,0,0,6,"#if#rsr#t;0;1;2;3;4;5;6;");SetAxis(Y_AXIS,0,0,15,"#ifn,     r#t:1e+8 m;0;5;10;15;");
DrawFrame(0x016f,1,0xafffaf); A=-0.62; r_massive=4.5; R=sqrt(r_massive); B=-0.05;
k=1;    n=1; N1=10000; rs/=r_unit;
for(i=0;i<N1;i+=1) { r=40*i/N1; r1=sqrt(r); y=a+b*r1;
if(Figure==2 && r1>R)    y+=A*b*(r1-R)+B*b*R*r/r_massive;
if(y>=n) {S[k+k]=r1; S[k+k+1]=y; n+=1; if(r>rs) k+=1;} if(y>15) break;}
Format(str,"Calculation,     #ifβ#t=%e",beta);    SetPen(2,0x0000ff);
Polyline(k,S,0.2,14,str);Plot("CARD,0,@k,XY,4,4,",S);
SetPen(2,0xff0000); Plot("OVALFILL,0,@N,XY,3,3,",D); nP[0]=TAKE;
for(i=0;i<N;i+=1) { nP[1]=i;TextJob(nP,Stars,str); TextHang(D[i+i]+0.3,D[i+i+1],0,str);}
}#v07=?>A#t
```

6. Conclusions

In analogy with the ultimate speed c, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β in a many-body system. In recent years, de Broglie matter wave has been generalized in terms of the ultimate acceleration. This paper shows that Jupiter's size and Saturn's size are the consequences of the interference of the generalized matter waves. In this calculation, Jupiter's radius is determined as 7.1491e+7m with a relative error of 5.05%; Saturn's radius is determined as 5.80057e+7m with a relative error of 3.75%. This calculation also correctly predicts the locations of Halo ring and main ring for Jupiter, ring A, B, C and D for Saturn.

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