

Guessing that the Riemann Hypothesis is unprovable

T.Nakashima

Abstract

Riemann Hypothesis has been the unsolved conjecture for 164 years. This conjecture is the last one of conjectures without proof in "Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse" (B. Riemann). The statement is the real part of the non-trivial zero points of the Riemann Zeta function is $1/2$. Very famous and difficult this conjecture has not been solved by many mathematicians for many years. In this paper, I guess the independence (unprovability) of the Riemann Hypothesis.

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I define Möbius function $\mu(n)$ as

$$\mu(n) := \begin{cases} 1 & \text{product of even primes} \\ -1 & \text{product of odd primes} \\ 0 & \text{divisible by the square of a prime number} \end{cases}$$

The Riemann Hypothesis is that the real part of the nontrivial zero point of the ζ function is $1/2$ (Ivić[1] p44).

Theorem 1. (Ivić[1]p48, Titchmarsh[4] p370, Theorem 14.25)

$$\text{the Riemann Hypothesis} \Leftrightarrow \sum_{n=1}^m \mu(n) = O(m^{\frac{1}{2}+\epsilon})$$

Proposition 1.

$$\sum_{n=1}^m \mu(n) = O(m^{\frac{1}{2}+\epsilon})$$

I guess this proposition's unprovability. This proposition is equivalent to the Riemann Hypothesis (by theorem 1).

I define "sum of the distorted Möbius function" as $\sum_{n \leq N-1} \mu(n) + f(N)$ that is in the summation of Möbius function (up to N) add (or subtract) $f(N)$ instead of $\mu(N)$. I take $N \rightarrow \infty$. The sum of Möbius functions at infinity is set arbitrary to some extent. (I use two cases afterwards, primary case is $|\sum_{n \leq N-1} \mu(n) + f(N)| < KN^{\frac{1}{2}+\epsilon}$, $N \rightarrow \infty$ case. $f(N)$ is defined as $|\sum_{n \leq N-1} \mu(n) + f(N)| < KN^{\frac{1}{2}+\epsilon}$, secondary case is $\sum_{n \leq N-1} \mu(n) + f(N) = KN^{\frac{2}{3}}$ (is always positive), $N \rightarrow \infty$ case. $f(N)$ is defined as $f(N) := KN^{\frac{2}{3}} - \sum_{n \leq N-1} \mu(n)$.) $\lim_{N \rightarrow \infty} f(N)$ is the value free from the Möbius functions for finite n . (The case $|\sum_{n \leq N-1} \mu(n) + f(N)| > N$ is actually impossible value. I do not think this case.)

conjecture 1. *The Riemann Hypothesis is impossible to prove. In other words, the Riemann Hypothesis is an "independent proposition."*

Consider Riemann Hypothesis of Möbius function case (proposition 1). Let *ZFC* (K.Kunen[2] introduction §1) be the entire axiomatic system.

First, the Riemann Hypothesis can be disproved case, this is one possibility. Later, for finite range, there are not counter examples (to proposition 1).

I think next two models.

Model A is the case

$$\sum_{n=1}^m \mu(n) = O(m^{\frac{1}{2}+\epsilon})$$

holds for all m (containing $m = \infty$).

Model B is the case

$$\sum_{n=1}^m \mu(n) = O(m^{\frac{1}{2}+\epsilon})$$

does not hold at $m = \infty$.

There are no counter examples for proposition 1 for finite range, so Model A holds for all finite m . I think $m = N$ case, I take "sum of the distorted Möbius function" to satisfy $|\sum_{n \leq N-1} \mu(n) + f(N)| < KN^{\frac{1}{2}+\epsilon}$, $N \rightarrow \infty$. So Model A holds at infinity. Model A is consistent in *ZFC*. (Model A consists of non contradicting formulas. So that is consistent. (K.Kunen[2], Introduction §1))

I think $m = N$ case, I take "sum of the distorted Möbius function" to

satisfy $\sum_{n \leq N-1} \mu(n) + f(N) = KN^{\frac{2}{3}}$. (Temporary, I forget the Möbius function's value.) $KN^{\frac{2}{3}} = O(N^{\frac{1}{2}+\epsilon})$ does not hold, so $N \rightarrow \infty$, Model B holds at $m = \infty$. As I mentioned before, the value at infinity is the only case be free from the value of the Möbius functions for finite n . Model B is consistent in *ZFC*.

Like this condition, the Riemann Hypothesis is unprovable and is independent. (K.Kunen[2], Introduction §1)

There are two possibilities the Riemann Hypothesis can be disproved or cannot be proven.

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References

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- [2] "Set Theory An Introduction To Independence Proofs", Kunen. K. Studies in Logic and the Foundations of Mathematics, Volume 102.
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- [4] "The Theory of the Riemann Zeta function", Titchmarsh, E, C., Clarendon Press Oxford (1996)