Physics in the vicinity of SAGITTARIUS A, gauss curvature values of spacetime in its vicinity

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Abstracs

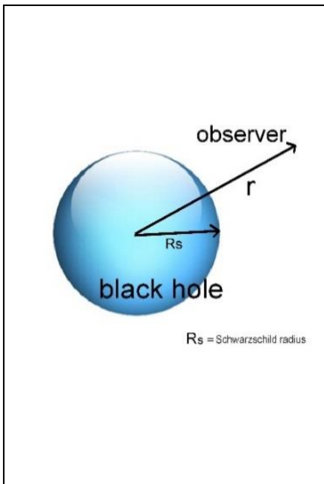
The relativistic Schwarzschild model solves Einstein's equations exactly for the assumption of a point gravitational mass and empty space in its vicinity. This model leads to a static and symmetric solution for the mathematical equation of space-time that allows its Gaussian curvature to be calculated at each point. We have calculated some curvature values and found an equation to calculate them that allows us to extend the results to a wider range of distances. Finally, we have applied these results to the real case of the black hole of our galaxy SAGITTARIUS A*, obtaining values of Gaussian curvature of the space-time in its vicinity that are valid if it is correct to apply this model.

Keywords

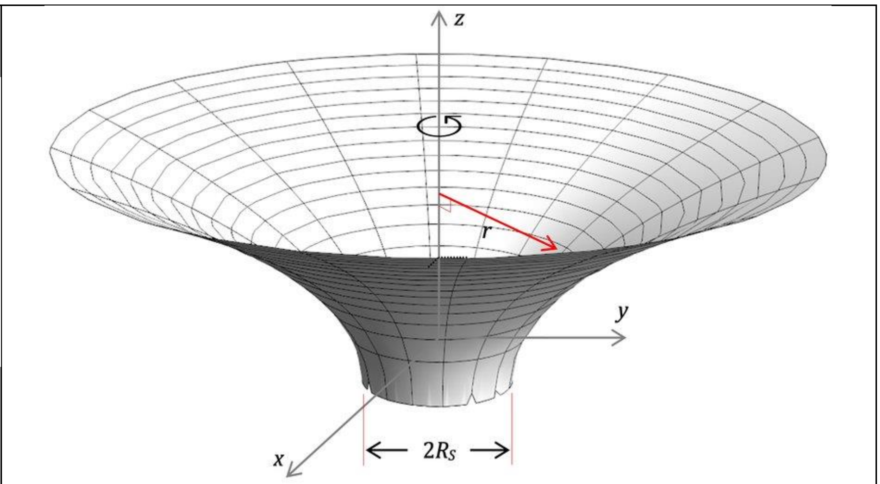
Black holes, curvature of space-time, Schwarzschild model, SAGITTARIUS A*

1.Introduction

The physical problem at hand is the calculation of the curvature of space-time caused by a spherical and static black hole at a point located at a distance "r" from the center of the black hole. This point will always be at a greater distance from the event horizon or Schwarzschild radius, “Rs” also called this radius distance from the black hole. Schwarzschild [1] solves the equations of the general theory of relativity for a point mass assumption and a surrounding empty space, establishing a metric and a space-time equation that turns out to be the Flamm paraboloid. This model leads to a solution of space-time that is stationary in time and with spherical symmetry, which simplifies the solution equation, resulting in a 2D surface, which is represented in Fig. 1.



The physical problem in Euclidean space



Space-time in the Schwarzschild metric
Flamm paraboloid

Fig. 1

2. Resolution of the mathematical problem

Flamm's paraboloid, mathematical solution to the proposed physical problem, is a surface inserted in a space R^3. Its geometry allows us to parameterize the paraboloid as a function of the observer's distance from the the point mass "r" and the azimuth angle "φ". The problem admits a mathematical treatment of differential geometry of surfaces [2], and with it we are going to calculate values of Gaussian Curvature.

Surface parameters (r, φ)

$0 \leq r < \infty, \;\; 0 \leq \varphi < 2\pi$

which has this parametric equation:

$x = r \cos\varphi$

$y = r \, \mathrm{sen}\varphi$

$z = 2(Rs(r - Rs))^{1/2}$

and by vector equation:

$f(x,y,z) = (r\cos\varphi, \; r\,\mathrm{sen}\varphi, \; 2(Rs(r - Rs))^{1/2})$

Determination of velocity, acceleration, and normal vectors to the surface

$\eth f/\eth\varphi = (-r\,\mathrm{sen}\varphi, r\cos\varphi, 0)$

$\eth^2 f/\eth\varphi^2 = (-r\cos\varphi, \; -r\,\mathrm{sen}\varphi, \; 0)$

$\eth f/\eth r = (\cos\varphi, \mathrm{sen}\varphi, (r/Rs - 1)^{-1/2})$

$\eth^2 f/\eth r^2 = (0, \; 0, \; (-1/(2R_s)).\,(r/Rs - 1)^{-3/2})$

$\eth f/\eth\varphi\eth r = (-\mathrm{sen}\varphi, \; \cos\varphi, \; 0)$

$n = (\eth f/\eth\varphi \times \eth f/\eth r) = (r\cos\varphi/(r/Rs - 1)^{1/2}, \; r\,\mathrm{sen}\varphi/(r/Rs - 1)^{1/2}, \; -r)$

$[n] = r\,((1/(r/Rs - 1)) + 1)^{1/2}$

$\mathbf{n} = n/[n]$

Curvature and curvature parameters

Gauss curvature $\mathbf{K = LN - M^2/EG - F^2}$

$L = \eth^2 f/\eth\varphi^2 . \, \mathbf{n}$

$E = \eth f/\eth\varphi . \, \eth f/\eth\varphi$

$N = \eth^2 f/\eth r^2 . \, \mathbf{n}$

$G = \eth f/\eth r . \, \eth f/\eth r$

$M = (\eth f/\eth\varphi\eth r) . \, \mathbf{n}$

$F = \eth f/\eth\varphi . \, \eth f/\eth r$

Completing a previous work of ours, [3], we have particularized the equations in 20 points between 1 and 1400 Schwarzschild radii, $Rs = 2GM/c^2$, calculating the corresponding curvatures as shown in the results table 1.

Thus, although in the metric there is a singularity at the point 1Rs, in the value of the Gaussian curvature for the singularity is resolved mathematically calculating a limit. We have calculated that limit for Gauss curvature.

3.- Results of curvature values

Table 1. Gaussian curvature values according to the Schwarzschild model

Distance to the center of point mass	**Value of Gauss Curvature k**	Distance to the center of point mass	**Value of Gauss Curvature k**
1Rs	**$-0{,}5000 \times Rs^{-2}$**	60Rs	**$-2{,}325.10^{-6} \times Rs^{-2}$**
1,2Rs	**$-0{,}2873 \times Rs^{-2}$**	80Rs	**$-9{,}596.10^{-7} \times Rs^{-2}$**
1,4Rs	**$-0{,}1821 \times Rs^{-2}$**	100Rs	**$-4{,}925.10^{-7} \times Rs^{-2}$**
1,6Rs	**$-0{,}1220 \times Rs^{-2}$**	200Rs	**$-5{,}963.10^{-8} \times Rs^{-2}$**
1,8 Rs	**$-0{,}0790 \times Rs^{-2}$**	400Rs	**$-4{,}800.10^{-9} \times Rs^{-2}$**
2Rs	**$-0{,}0625 \times Rs^{-2}$**	600Rs	**$-2{,}376.10^{-9} \times Rs^{-2}$**
3Rs	**$-0{,}0186 \times Rs^{-2}$**	800Rs	**$-9{,}710.10^{-10} \times Rs^{-2}$**
4Rs	**$-0{,}0078 \times Rs^{-2}$**	1000Rs	**$-5{,}059.10^{-10} \times Rs^{-2}$**
5Rs	**$-0{,}0030 \times Rs^{-2}$**	1200Rs	**$-2{,}883.10^{-10} \times Rs^{-2}$**
6Rs	**$-0{,}0023 \times Rs^{-2}$**	1400Rs	**$-1{,}810.10^{-10} \times Rs^{-2}$**

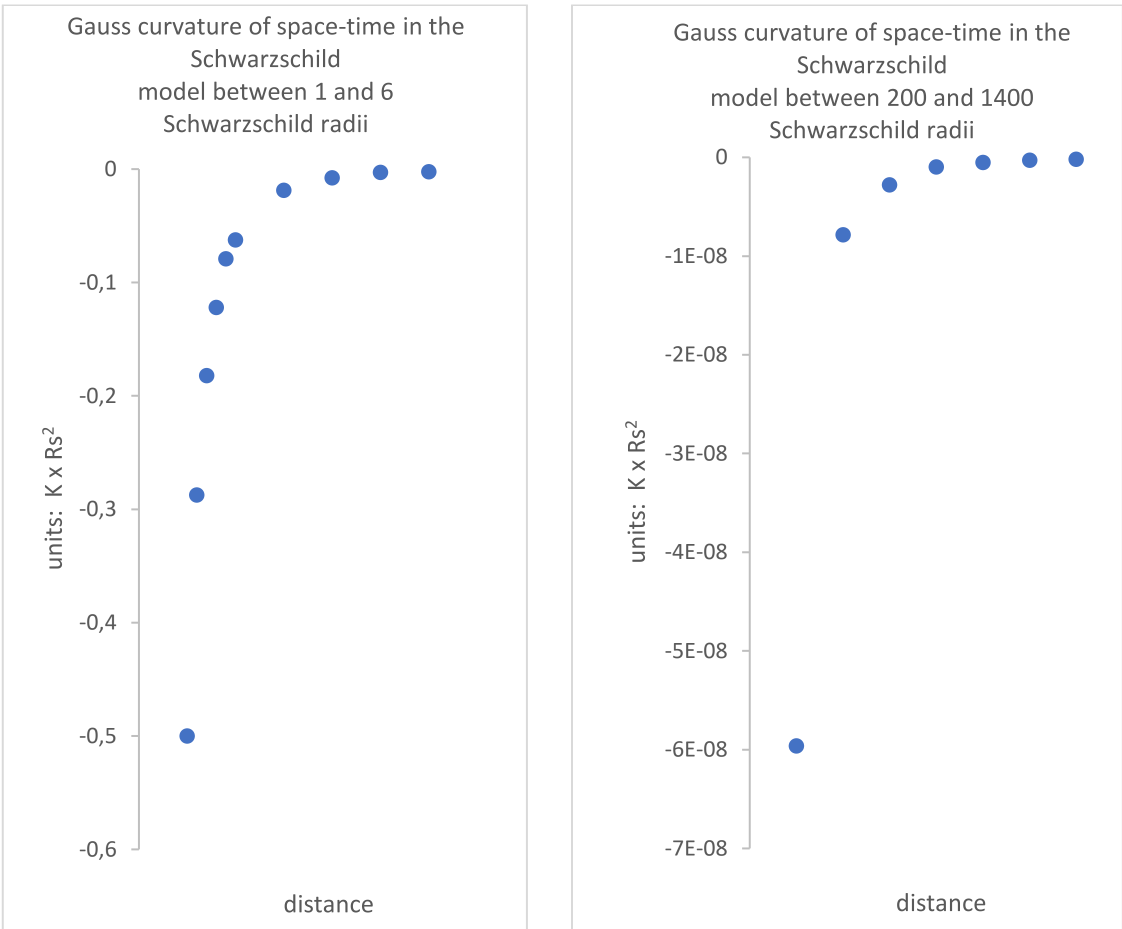


Fig. 2

4.- An equation to calculate the curvature of space-time according to the Schwarzschild model

An adjustment equation has been obtained using an Excel program by regression methods throughout this wide range of distances. The degree of quality of the fit obtained by calculating the parameter R^2 is very high, 0.9999. For this reason, it is to be expected that this equation allows interpolate the calculation of Gaussian curvature values, in this wide range of distances, with high accuracy without the need to carry out the laborious calculations that would otherwise have to be done.

Fit equation between 1 and 1400 Schwarzschild radii

Gaussian curvature $k = -0{,}5268\ (r/Rs)^{-3{,}054} \times Rs^{-2}$

Fit quality $R^2 = 0{,}9999$

Rounding decimals and according to Schwarzschild's definition of radium

$$Rs = 2GM/c^2$$

where G is the universal gravitation constant, and M is the mass of the black hole, we can express the adjustment equation we have found as the following approximate equation:

approximate equation found $\mathbf{k = -GM/c^2r^3}$ (1)

where k is the Gaussian curvature of space-time according to the Schwarzschild model.

5. Application to the calculation of the curvatures in the vicinity of SAGITTARIUS A*

We assume that Sagittarius A* behaves like a symmetric and static black hole, that is, the Schwarzschild model is assumable for the calculation of curvatures.

Sagittarius A*, [4], [5], [6], is the supermassive black hole at the galactic center of the Milky Way. Like the nuclei of most spiral and elliptical galaxies, the Milky Way contains a black hole at its center.

The mass of SAGITTARIUS A* is estimated at 3,7 million suns if the mass of the sun [7] is $1.989.10^{30}$ Kg, the mass of Sagittarius A* is estimated at $7.16.10^{36}$ Kg, its Schwarzschild radius, Rs, thus turns out to be $10{,}61.10^9$ meters. According to these data, the values of space-time curvature in its vicinity are those expressed in Table 2.

Table 2. Values of curvatures of spacetime in the vicinity of SAGITTARIUS A*

Distance in Schwarzschild radii	Distance in meters	Gauss curvature, model Schwarzschild (m^{-2})	Gauss curvature, approximate equation found (m^{-2})
1Rs	10.10^9	$-0{,}44.10^{-20}$	$-0{,}53.10^{-20}$
1,2Rs	12.10^9	$-0{,}25.10^{-20}$	$-0{,}31.10^{-20}$
1,4Rs	14.10^9	$-0{,}16.10^{-20}$	$-0{,}19.10^{-20}$
1,6Rs	16.10^9	$-0{,}11.10^{-20}$	$-0{,}13.10^{-20}$
1,8 Rs	18.10^9	$-0{,}70.10^{-21}$	$-0{,}91.10^{-21}$
2Rs	20.10^9	$-0{,}55.10^{-21}$	$-0{,}66.10^{-21}$
3Rs	30.10^9	$-0{,}17.10^{-21}$	$-0{,}20.10^{-21}$
4Rs	40.10^9	$-0{,}69.10^{-22}$	$-0{,}83.10^{-22}$
5Rs	50.10^9	$-0{,}27.10^{-22}$	$-0{,}42.10^{-22}$

6Rs	60.10^{9}	$-0,20.10^{-22}$	$-0,24.10^{-22}$
60Rs	600.10^{9}	$-0,21.10^{-25}$	$-0,24.10^{-25}$
80Rs	800.10^{9}	$-0,85.10^{-26}$	$0,10.10^{-25}$
100Rs	1000.10^{9}	$-0,44.10^{-26}$	$-0,53.10^{-26}$
200Rs	2000.10^{9}	$-0,53.10^{-27}$	$-0,66.10^{-27}$
400Rs	4000.10^{9}	$-0,43.10^{-28}$	$-0,83.10^{-28}$
600Rs	6000.10^{9}	$-0,21.10^{-28}$	$-0,24.10^{-28}$
800Rs	8000.10^{9}	$-0,86.10^{-29}$	$-0,10.10^{-28}$
1000Rs	10000.10^{9}	$-0,45.10^{-29}$	$-0,53.10^{-29}$
1200Rs	12000.10^{9}	$-0,26.10^{-29}$	$-0,31.10^{-29}$
1400Rs	14000.10^{9}	$-0,16.10^{-29}$	$-0,19.10^{-29}$

From the results obtained we can see that our equation slightly overestimates the curvature values at all points. In any case, the order of magnitude in each of the points is of the same degree in both calculations, therefore, our simple equation does seem to be valid with a reasonable degree of accuracy in the wide range of distances between 1Rs and 1400 Rs and also, there is no objection to extending it to greater distances according to the form of Schwarzschild space-time, Flamm's paraboloid, Fig. 1.

Conclusions

First, we have calculated some values of the Gaussian curvature of spacetime according to the Schwarzschild model. As the calculations are laborious, using the results obtained, we have established an equation that allows us to interpolate and extrapolate these curvature values approximately over a greater range of distances.

SAGITTARIUS A* in some of its aspects and in a first approximation can be treated as a static and symmetric black hole and making this assumption we have applied the Schwarzschild space-time model by calculating Gaussian curvature values in its vicinity. The numerical results are presented in this work.

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