Requiring negative probabilities from "the thing" researched, else that thing doesn't exist, is insufficient ground for any conclusion

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October 4, 2023

Abstract

It is demonstrated that the statistical method of the famous Aspect - Bell experiment requires negative densities and negative probabilities from "the thing" researched, else that thing doesn't exist. The thing refers here to Einstein hidden variables. This requirement in the experiment is absurd and so the results from such experiment are meaningless.

Introduction

Kolmogorov axioms

Let us start with the presentation of some relevant Kolmogorov axioms [1] that are the foundation of probability laws. A probability is a function of a set. We have three relevant axioms,

- Positivity: A probability, P, is *never* negative
- Certain event: The probability P of the universe set U is unity
- Additivity: If sets Q and R are disjoint then $P(Q \cup R) = P(Q) + P(R)$.

Here, Q and R and $Q \cup R$ are subsets of U. Sets are connected to events via an appropriate random variable.

Bell's experiment

With Bell's formula for correlation [2] Einstein hidden parameters [3, page 320, ..unvolständig..] are modeled as classical probability random variables. The value(s) of those random variables, in a universe set Λ, are

λ. The density is $\rho(\lambda) \geq 0$ for all $\lambda \in \Lambda$. The density is normalized, i.e. $\int_{\lambda\in\Lambda} \rho(\lambda)d\lambda = 1$. Bell's correlation formula

$$E(a,b) = \int_{\lambda\in\Lambda} \rho(\lambda)A(a,\lambda)B(b,\lambda)d\lambda \tag{1}$$

is therefore surely embedded in Kolmogorovian classical probabiliy. If one argues that a density somehow is associated with quantum variables, then E in (1), based on complex λ, may theoretically violate equal the quantum result. See however also [4]. In addition in the early days of quantum mechanics, Einstein already noticed the difference between classical probability, leading to Wien's law, and a kind of quantum probability leading Planck's law [5], [6]. Moreover, there is the problem of an association of a quantum (1) with an experiment, ruled by definition by classical probability laws.

Returning to a classical probability based Bell formula. In a more formal way we can state that Bell supposed that Einstein's extra parameters are somehow represented by a classical probability space (Λ, Σ, P). The Λ is the universal set, the Σ is the associated sigma-algebra [7] and P is the probability measure. The P projects a set $S \in \Sigma$ in the interval $[0,1]$. Note, $dP(\lambda) = \rho(\lambda)d\lambda$. Sometimes one also writes, $P(d\lambda)$.

In Bell's formula (1), $\boldsymbol{a}$ is the unit length setting parameter vector of Alice's instrument and $\boldsymbol{b}$ is the unit length setting parameter vector of Bob's instrument. Alice doesn't know Bob's setting and Bob doesn't know Alice's setting. We have, $A(\boldsymbol{a},\lambda) \in \{-1,1\}$ Alice's spin measurement function $B(\boldsymbol{b},\lambda) \in \{-1,1\}$ Bob's spin measurement function. There is a sufficiently large distance (see [3]) between Alice and Bob.

For photons it is sufficient to consider the angle $x = \angle(\boldsymbol{a},\boldsymbol{b})$ between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. This angle is a continuous variable in $0 \leq x < 2\pi$ and is determined in the plane orthogonal to the direction of propagation. The expectation value $E(\boldsymbol{a},\boldsymbol{b})$ in (1) then reduces to $E(x)$.

In experiment use is made of what can be called a raw product moment (rpm) correlation. This rpm correlation is of course embedded in classical probability theory.

Furthermore, an excellent example of an experiment can be found in the literature e.g. [8]. The rpm correlation for photons is,

$$R(x) = \frac{N(x,\neq) - N(x,=)}{N(x,\neq) + N(x,=)} \tag{2}$$

With, $N = N(x,\neq) + N(x,=)$ the total number of entangled photon pairs measured under angle x. Here, given $0 \leq x < 2\pi$, $N(x,\neq)$ represents the number of unequal spin measurements by Alice and Bob, i.e. $(+,-)$, $(-,+)$ and $N(x,=)$ represents the number of equal spin measurements by Alice and Bob, i.e. $(-,-)$, $(+,+)$. This enables to rewrite $R(x)$ in (2) as

$$R(x) = 1 - 2P(x,=) \tag{3}$$

Here, $P(x,=)$ equals $N(x,=)/N$. It represents the (estimate) classical probability to find "=" spin under angle $x \in [0, 2\pi)$. This is a statistical frequency of an event divided by a total, therefore a probability estimate. The N can be large if needed.

The set structure behind the probability space of the experiment is (U, Φ, P) with $U = [0, 2\pi)$ and Φ the to U associated sigma-algebra. The P is the probability measure.

The event "$x,=$" is represented by a random variable X. A random variable connects the probability set structure with what can be found in measurement (events). Here it associates the Φ set for the angle x, from the universe, $U = [0, 2\pi)$ to the real numbers, i.e. $X : \Phi \to \mathbb{R}$. It does that in such a way that it connects a continuous variable to a set, viz. [9, page 117]. A probability is $P : \Phi \to [0, 1]$.

Therefore, a random variable X is associated to the event "=" spin under angle x and $x \in [0, 2\pi)$. Furthermore, we are allowed here to assume ideal (no loss) measurements.

The random variable X is a continuous random variable. The random variable X attains continuous values because the angle is a continuous variable. It is a mistake to think that $P(x,=)$ is the probability associated to a discrete random variable. The point of coarse graining and discreteness is dealt with in a special subsection.

Subsequently, note that for a continuous random variable, the probability in a point is zero [10]. The probability for a continuous random variable is computed [11] like e.g.

$$P(0 \leq X < x) = \int_0^x f(y)dy \tag{4}$$

In Riemannian integration, inclusion of limits doesn't make a difference in outcome.

Hypothesis

In the experiment we ask if it is possible in principle that $R(x)$ can be equal to the quantum correlation $\cos(x)$. Because, we have $\cos(x) = 1 - 2\sin^2(x/2)$, this leads to the simple testing of the hypotheses

$$\begin{aligned} H_0 &: P(0 \leq X < x) = \sin^2(x/2) \\ H_1 &: \text{The hypothesis } \ H_0 \ \text{ is false} \end{aligned} \tag{5}$$

Therefore note, the probability $P(x,=)$ is in fact $P(0 \leq X < x)$. In this way we can via the random variable X have $\sin^2(x/2)$ associated to the set structure. However, the following things immediately catches the eye.

- The function $\sin^2(x/2)$ isn't monotone on $x \in [0, 2\pi)$,

- $P(0 \le X < 2\pi) = 0$ instead of 1,
- The probability density, $f(x) = (1/2)\sin(x)$ in (4) is not positive definite for $x \in [0, 2\pi)$.

Continuity & negative probabilities

With the fact that $\sin^2(x/2)$ is not monotone on $x \in [0, 2\pi)$, negative probabilities are required to let $P(0 \le X < x)$ meet $\sin^2(x/2)$. Suppose, e.g. $S_1 = [0, \pi)$ and $S_2 = [\pi, 3\pi/2)$. Then, $S_1 \cap S_2 = \emptyset$ and when $\sin^2(x/2)$ is the probability function, $P(S_1 \cup S_2) = P([0, 3\pi/2)) = 1/2$. Because of the additivity in Kolmogorovian axioms, we also must have $P(S_1 \cup S_2) = P(S_1) + P(S_2) = 1/2$. Again, when $\sin^2(x/2)$ is the probability function it follows, $P(S_1) = 1$. But this leads, via $P(S_1) + P(S_2) = 1/2$, to $P(S_2) = -1/2$ which is outside $[0, 1]$. Hence, $\sin^2(x/2)$ cannot be a probability function for a continuous random variable. This is because negative probabilities arise from the additivity which is a basic part of probability theory [11].

Discreteness

Special attention again is given to the coarse graining and discreteness point above. Here we might use $P(x, =)$ in discrete points x. With coarse graining and/or discreteness the sum of discrete probabilities $\sin^2(x/2)$ over x in nontrivial partitioning $\mathcal{X}$, cardinality e.g. > 5 that contains $x = \pi$, always is larger than 1. The $\mathcal{X}$ is the discretisation of $U = [0, 2\pi)$. There is a discrete sigma-algebra Φ' associated. We have,

$$\sum_{x \in \mathcal{X} \cup \{\pi\}} \sin^2(x/2) > 1 \tag{6}$$

This means there are sets A with $P(A) > 1$ and $A \in \Phi'$. For $P(U')$ we also might find > 1. For comparison, the discrete Poisson probability distribution is an example of a probability distribution of a discrete random variable with a summation to unity. And please note that the Einstein hidden variables concept, [3, page 320, ..unvollständig..], wasn't rejected for a finite number of x. The claim was that for every possible $x \in [0, 2\pi)$, we have no go Einstein variables.

Conclusion

A classical probability is a function that projects a set into the interval $[0, 1]$. Bell's experiment requires a Kolmogorovian probability to be not Kolmogorovian in order to meet the quantum correlation. This remains true, even if one claims that Bell's formula is quantum mechanics. The only thing in the latter case is that there is a disconnect between experiment probability and the (supposed quantum) Bell theoretical correlation formula. Note also, we only may know the quantum world through the use of classical probability.

The points raised in the paper demonstrate that we always will find H_1 in (5), in an experiment where the rpm correlation (2) is employed. This is not because Einstein variables are impossible or that inequalities demonstrated that Einstein variables do not exist. It is because it is *not possible* in the data to find H_0 is true. Data based on negative probability doesn't exist.

We conclude that the statistics of Bell's experiment doesn't allow any sensible conclusion about go, or, no go Einstein.

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