Some Speculations on Black Hole Evaporation and Quantum Gravity

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Abstract: A possible way to understand the preservation of information during black hole evaporation is explored and a new approach to quantizing General Relativity is considered. Black holes, like all physical objects, are subject to the energy-time uncertainty principle which means their spacetime characteristics must have quantum uncertainty. It is conjectured that this quantum uncertainty is what permits the escape of energy and information from black holes. It is further conjectured that spacetime and mass-energy can be quantized together (here termed metric quanta) by replacing spaces based on the real number line with a quantum information state. Eigenvalues of operators that act on the information state characterize the metric quanta and this means that differential equations do not ultimately describe nature. A formulation of General Relativity is explored that uses a quantization procedure which replaces the basis vectors for curvilinear coordinate systems with operators that act on the information state of the cosmos. This paper concludes with the possibility that these ideas can eventually be made into a workable theory.

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0. Introduction

The speculations covered in this document necessarily involve extensions to conventional ideas. However, some conventional ideas are derived and covered as a necessary part of exploring the various conjectures. This text begins with the observation that quantum uncertainty applies to all physical things, including black holes. Next, quantum uncertainty and its relation to Hawking Radiation and quantum information is postulated and calculations are given for Schwarzschild black holes. Quantum information and its relationship to spinors is explored followed by a postulated quantization of spacetime and energy that respects all the symmetries of standard physical theory. This quantized approach replaces differential equations as the basic description of the cosmos and the limitations of differential equations is explored in the context of quantum state reduction. A toy model of quantized General Relativity is presented which explores how quantum gravitational uncertainties in the metric relate to uncertainties in curvature and the energy-momentum tensor. A final discussion and summary, listing the open questions and virtues of this approach, concludes the discussion.

1. Measurements And Observations

It takes a certain amount of time to make any measurement or observation. And over that time interval, Δt, there is an intrinsic uncertainty in the energy of the measurement on the order of $\hbar/(2\Delta t)$. Furthermore, if the object under observation is large enough, light may not have enough time to travel across it in the time interval Δt. In this circumstance, the measurement may turn out to be incorrect since the object itself may have changed its size or state before the observer can see the change. This would be the case if information about such changes could not reach the observer in the time interval Δt. So, for a measurement to be valid it should be conducted over a sufficiently long time so that all the relevant information can reach the observer. Time and information are both involved in the reduction of quantum states.

Measurements, almost by definition, is information that must obtained over a certain amount of time. Because of relativity, the information may be out of date unless it can be shown that it has not been changed over the time interval that observations are being made. This time interval can be thought of as being defined by a characteristic distance. This motivates defining a certain time interval which, in this paper, is termed *the quantum characteristic time Δt.* This is the time it takes for a photon to encircle a spherical region containing an object, where the maximum width of the object is equal to the diameter of the spherical region. Nature, quantum mechanically observing itself, should need this characteristic time for state reduction and the expression of quantum information. The quantum characteristic time, which (from now on) will be referred to as the *characteristic time*, is defined to be:

$$\Delta t = \frac{\pi D}{c}$$

Where D is the diameter of a sphere enclosing an object with a maximum size D. A quantum of energy $\Delta E = \hbar c/(2\lambda)$, with wavelength $\lambda=\pi D$, can be thought of as encircling the object and that energy is the uncertainty of the energy of the object. The wavelength in this case is the characteristic distance. It is reasonable to think of the required time for a photon to encircle the object as equal to the time needed for state reduction to occur, but the characteristic time is not limited to that. The characteristic distance is what defines the wavelength and therefore uncertainty in energy. All material objects are subject to the uncertainty principle:

$$\Delta E \Delta t \approx \hbar/2 \qquad \text{Equation (1)}$$

Any object, even macroscopic objects, have an intrinsic uncertainty in their mass-energy equal to $\hbar/2$ divided by the characteristic time. For a quanta of wavelength λ state reduction can take place anywhere within and around the space containing an object of size D. Within the time associated with that, quantum fluctuations take place with the uncertainty in energy. The fluctuations can be thought of as being confined to a sphere of diameter D and, because of the finite speed of light, the fluctuations inside the sphere cannot leave the sphere in less time than the characteristic time.

2. The Characteristic Time for a Schwarzschild Black Hole

A distant observer of a black hole observes the black hole in its own tangent space. In that tangent space, the observer says a Schwarzschild black hole has a diameter of $4GM/c^2$ which is twice its Schwarzschild radius. Relative to this observer's tangent space, quantum fluctuations distort the black hole's spacetime independent of the extreme gravitational time dilation that exists within the body of the black hole itself. The equations of General Relativity show that changes in the warping of space cannot travel through space at speeds greater than the speed of light. So, changes in the warping of space at the event horizon cannot travel faster than the speed of light. Quantum fluctuations in a black hole's geometry are also restricted by these physical limitations. Distant observers of black holes should agree that the characteristic time associated with a Schwarzschild black hole should equal the circumference of the black hole divided by the speed of light:

$$\Delta t = \frac{2\pi R}{c} = \frac{4\pi GM}{c^3}$$

Where R is the Schwarzschild radius. Substituting this into equation 1 and solving for ΔE gives:

$$\Delta E = \frac{\hbar c^3}{8\pi GM} \qquad \text{Equation (2)}$$

This is the quantum uncertainty in the mass-energy of the black hole relative to distant observers. Temperature is just the average energy of a microscopic degree of freedom for a thermodynamic system (where that energy is divided by Boltzmann's constant). The temperature associated with the uncertainty of the mass-energy of the black hole is given by $\Delta E/k_B$ and is simply the Hawking Temperature:

$$T_{Hawking} = \frac{\hbar c^3}{8\pi \mathrm{GMk_B}} \qquad \text{Equation (3)}$$

3. Hawking Radiation and Information

The Hawking temperature, in this view, is the result of the fact that a black hole is a quantum mechanical object and the uncertainty in its mass-energy is nothing other than its observable quantum degree of freedom. Dividing both sides of equation 2 by c^2 gives the quantum uncertainty in the mass of the Schwarzschild black hole as calculated by distant observers:

$$\Delta M = \frac{\hbar c}{8\pi \mathrm{GM}}$$

For a Schwarzschild black hole, its radius is proportional to its mass, $R=2GM/c^2$. Since the mass of such a black hole has an uncertainty of ΔM, its Schwarzschild radius must also have a quantum gravitational uncertainty given by $\Delta R=2G\Delta M/c^2$. This means physical geometry itself must have a quantum uncertainty, that is related to Hawking radiation and information, and this leads to conjecture 1:

> **Conjecture 1**: The quantum gravitational uncertainty, in the correct quantized version of General Relativity, is responsible for Hawking radiation which results in the escape of energy and information from black holes in accordance with unitary laws.

Quantum gravitationally, the Schwarzschild black hole can be thought of as having a region of uncertainty as to where the event horizon is. This region of uncertainty can be thought of as a kind of shell, at the event horizon, with its thickness having a quantum gravitational expectation value of size $\Delta R=2G\Delta M/c^2$, relative to any distant observer. Relative to such observers, the mass in the interior of the black hole is frozen in place due to extreme gravitational time dilation [4]. One might be tempted to think of the region of uncertainty, ΔR, as a kind of atmosphere above the event horizon. But it is really a region where the event horizon has the most probable location for being found. For the purposes of this paper, this region of uncertainty will be termed the *Unruhsphere*. The relative size of the region of radial uncertainty is given by:

$$\Delta R = \frac{2G}{c^2} \Delta M = \frac{\hbar}{4\pi MC}$$

So, a Schwarzschild black hole can be depicted as in figure 1:

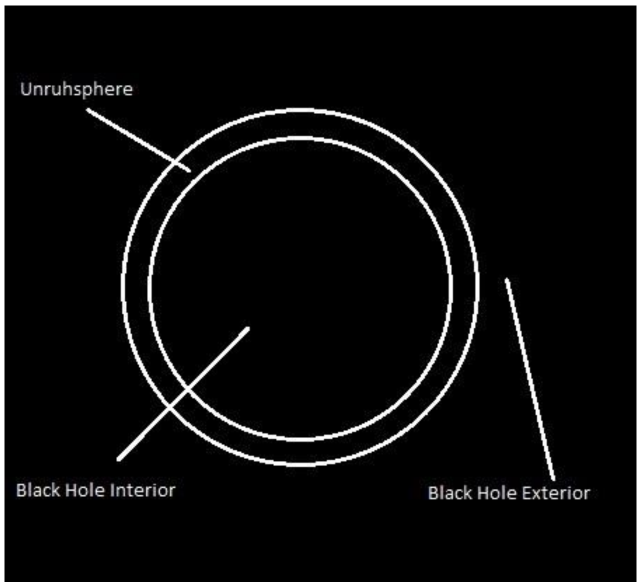

Figure 1

The Unruhsphere can be thought of as a kind of quantum overlap between the interior and exterior regions of a black hole. This quantum gravitational overlap allows energy and information “under” the event horizon to interact with quantum field fluctuations that exist “above” the event horizon. This quantum gravitational uncertainty does not exist classically. Black holes, which must be subject to constant quantum fluctuations, lack a definite Schwarzschild radius just like an electron (in an atom) does not have a definite distance from the atomic nucleus. The Unruhsphere permits energy and information, that is held in place by extreme gravitational time dilation under the event horizon, to interact and influence quantum field fluctuations that exist above the event horizon. Conjecture 1 postulates that these interactions are unitary and preserve information.

These interactions result in negative work being done on the black hole, by the quantum vacuum fluctuations above the horizon, and positive work being done on those fields by the black hole itself. So, information inside black holes escapes with, and is encoded in, the Hawking radiation. The Hawking radiation is not necessarily thermal. Relative to themselves, black holes quickly disintegrate and relative to distant observers' black holes can persist for many trillions of years. Observers falling into a black hole would immediately find themselves being torn apart by the furious quantum gravitational and quantum field activity that converts them into Hawking radiation.

4. Quantifying The Escape of Information from Black Holes

The power output of Hawking radiation from a Schwarzschild black hole is given by the well-known formula $P=A\sigma T^4$ where A is the surface area of the black hole, given by $4\pi R^2$ where R is the Schwarzschild radius, σ is the Stefan-Boltzmann constant, and T is the Hawking temperature. The result is Hawking's well-known equation:

$$P=\frac{dE}{dt}=c^2\frac{dM}{dt}=\frac{\hbar c^6}{15,360\pi G^2 M^2} \qquad \text{Equation (4)}$$

The time it takes for a Schwarzschild black hole to completely evaporate, assuming no mass-energy is added to it, can be obtained by separating variables in equation 4 and integrating:

$$\frac{M_0^3}{3}=\int_0^{M_0} M^2 dM=\int_0^{t_{evap}}\frac{\hbar c^4}{15,360\pi G^2}dt=\frac{\hbar c^4}{15,360\pi G^2}t_{evap}$$

Solving for the evaporation time gives:

$$t_{evap}=\frac{5,120\pi G^2 M_0^3}{\hbar c^4}$$

Over this time interval, all the mass-energy of the black hole is carried away as Hawking radiation and all the information inside the black hole is contained in the radiation in accordance with conjecture 1. The information content of a black hole is generally considered to be determined by its entropy. Inverting equation 3 gives:

$$\frac{1}{T} = \frac{8\pi G M k_B}{\hbar c^3}$$

The entropy, S, is given by:

$$\frac{c^2 dM}{T} = \frac{dE}{T} = dS \qquad \text{Equation (5)}$$

Substituting the inverted equation 3 into equation 5 gives the maximum entropy possible for a Schwarzschild black hole:

$$\frac{4\pi G M^2 k_B}{\hbar c} = \int_0^M \frac{8\pi G M k_B}{\hbar c} dM = \int_0^{S_{max}} dS = S_{max} \qquad \text{Equation (6)}$$

This expression for the maximum entropy, S_{max}, also equals ($k_B A c^2/4G\hbar$), where A is the surface area of the event horizon $4\pi(2GM/c^2)^2$. The maximum number of qubits contained in a system with entropy S_{max} is given by n_{max}:

$$\frac{S_{max}}{k_B \ln(2)} = n_{max} \qquad \text{Equation (7)}$$

The number of microstates possible is equal to the dimension of the Hilbert space describing the system and it equals $2^{\wedge} n_{max}$. Dividing both sides of equation 6 by $k_B ln(2)$ and using equation 7 gives the maximum number of qubits that can be contained in a Schwarzschild black hole with mass M:

$$n_{max} = \frac{4\pi G M^2}{\hbar c \ln(2)} \qquad \text{Equation (8)}$$

It might be expected that the information content should be proportional to the mass (which is equal to the volume times the average density) instead of the mass squared. The fact that the information content is proportional to the square of the mass suggests that there is a lot more information present than might be naively assumed. Differentiating equation 8 with respect to time gives:

$$\frac{dn_{max}}{dt} = \frac{8\pi GM}{\hbar c \ln(2)} \frac{dM}{dt}$$

Substituting *dM/dt* from equation 4 yields the *Schwarzschild black hole baud rate*:

$$\frac{dn_{max}}{dt} = \frac{c^3}{1920 GM \ln(2)}$$

This is the maximum rate at which information leaves a Schwarzschild black hole with mass M. The information is inside the Hawking radiation. For a solar mass Schwarzschild black hole, the maximum baud rate is just under 153 qubits per second. The energy per qubit also varies inversely with the mass of the black hole. Using the product rule on the above equation and using equation 4 we get:

$$\frac{dE}{dn_{max}} = \left(\frac{dt}{dn_{max}}\right)\left(\frac{dE}{dt}\right) = \frac{\hbar c^3 \ln(2)}{8\pi GM}$$

This shows that the energy per qubit is inversely proportional to the mass of the evaporating Schwarzschild black hole.

5. The Bekenstein Bound on Information

Equation 8 gives the maximum amount of information that black holes of that type can contain. The mass of a Schwarzschild black hole can be expressed in terms of its radius, $M=Rc^2/2G$, and can be incorporated into equation 8 by replacing M^2 with $MRc^2/2G$. Doing this and letting

$E=Mc^2$ gives the Bekenstein bound on the amount of information needed to characterize an energy content, E, in a region characterized by size R:

$$n_{max} = \frac{2\pi \mathrm{RE}}{\hbar c \ln(2)} \qquad \text{Equation (9)}$$

This bound is more general and so it applies to more than just black holes. This equation gives the maximum amount of quantum information, contained in a sphere of radius R, that contains a total mass-energy content E. Viewing R and E as independent variables, the bigger R becomes the more information there can be even when E is fixed. So, more information is needed as the volume of a space increases even when the energy content does not increase. So the Bekenstein bound on information suggests that quantum information may also be describing the spacetime "continuum" in addition to its energy content. The information content is quite large. To get a feel for how large, it is useful to consider a more ordinary situation. For a mass at a constant temperature, equation 5 can be integrated to give:

$$\int_0^M \frac{c^2}{T} \mathrm{d}M = \int_0^E \frac{dE}{T} = \int_0^{S_{max}} dS$$

$$\frac{Mc^2}{T} = \frac{E}{T} = S_{max}$$

And using equation 7 gives:

$$n_{max} = \frac{Mc^2}{k_B T \ln(2)}$$

This equation gives the amount of information in a mass M at temperature T. For equation 9 to give this value, R would have to be $\hbar c/2\pi k_B T$. There is a lot of information in the cosmos and all that information is conserved. For example, a 2.04-kilogram brick at a temperature of 70 degrees Fahrenheit contains $6.5107\text{x}10^{37}$ qubits of information.

6. Quantum Information

Physical reality is quantum mechanical in nature and so, strictly speaking, there is no such thing as classical information. Quantum information, in an unreduced state, corresponds to states in a Hilbert space that are a superposition of many states. For a single qubit, an operator is needed, which acts on its Hilbert space, that defines the qubit and its relativistic quantum information that is measured and observed. The operator should be Hermitian, $H^\dagger=H$, and be parameterized in spacetime variables when an individual qubit is in an unreduced state. The approach to this problem used here follows that used in [1]. Spinors are nothing more than solutions to relativistic wave equations and they are useful in this task. A rank 1 spinor, also known as a flagpole spinor, contains four pieces of information that are used in the right-hand side of equation 10. For example, R is the magnitude of a rank 1 spinor. Equation 10 maps that spinor information to a non-null four-vector but we know that a rank 1 spinor does not correspond to a non-null four-vector. Rather, it corresponds to a null four-vector given by a different transformation (it is given by the transformation in Appendix B). Nevertheless, it is useful to consider spacetime coordinates in a spherical form with the time dimension, w, given by $w=ct$, so that it is in the same units as the spatial dimensions. Those coordinates are expressed in terms of information from a flagpole spinor:

$$\begin{aligned} w &= R\cos\alpha \\ z &= R\sin\alpha\cos\phi \\ y &= R\sin\alpha\sin\phi\sin\theta \\ x &= R\sin\alpha\sin\phi\cos\theta \end{aligned} \qquad \text{Equation (10)}$$

The most general 2x2 Hermitian matrix, using the coordinates in equation 10, is given by:

$$H = \begin{bmatrix} w+z & x-iy \\ x+iy & w-z \end{bmatrix}$$

Here the coordinates (w, x, y, z) are always real-valued. The determinant of H is:

$$det\, H = w^2 - z^2 - y^2 - x^2$$

This is the invariant spacetime interval, squared, which is the same in any inertial frame of reference and in any differential region of warped spacetime in "free-fall" coordinates. So, in another inertial frame, the operator *H'* is given by:

$$H' = \begin{bmatrix} w' + z' & x' - iy' \\ x' + iy' & w' - z' \end{bmatrix}$$

The determinant of *H'* is given by $det\ H' = w'^2 - z'^2 - y'^2 - x'^2$ which must equal the determinant of *H* due to relativistic invariance. To find the transformation that changes *H* to *H'*, the Hermitian character of these operators can be exploited by considering a 2x2 complex matrix, *L:*

$$H' = LHL^{\dagger} \qquad \text{Equation (11)}$$

Taking the conjugate transpose of this equation gives:

$$\begin{aligned} (H')^{\dagger} &= \left(LHL^{\dagger}\right)^{\dagger} \\ &= \left(L^{\dagger}\right)^{\dagger} H^{\dagger} L^{\dagger} \\ &= LHL^{\dagger} \\ &= H' \end{aligned}$$

Where $H^{\dagger} = H$ and $(L^{\dagger})^{\dagger} = L$ have been used. The primed values in *H'* must be real-valued, like they are in *H*, and this equation clearly shows that *H'* must be Hermitian too. So, setting $w' = ct'$ in the primed frame there are the following relations like equation 10:

$$\begin{aligned} w' &= R' \cos\alpha' \\ z' &= R' \sin\alpha' \cos\phi' \\ y' &= R' \sin\alpha' \sin\phi' \sin\theta' \\ x' &= R' \sin\alpha' \sin\phi' \cos\theta' \end{aligned}$$

It must be emphasized that *L* is not necessarily Hermitian. Any matrix with complex entries satisfies $(L^{\dagger})^{\dagger} = L$ and so additional information is needed to fully specify *L*. As was pointed out above, it is clear that:

$$
\begin{aligned}
det\ H' &= w'^2 - z'^2 - y'^2 - x'^2 \\
&= w^2 - z^2 - y^2 - x^2 \\
&= det\ H
\end{aligned}
$$

And since *det(H') = det(H)* taking the determinant of equation 11 gives:

$$det(H') = det(L) \quad det(H) \quad det(L^\dagger)$$

This suggests $det(L)det(L^\dagger) = 1$ which is satisfied if $det(L) = e^{ir}$ for some real number *r*. The real number, *r*, can be set to zero, so that *det(L) = 1*, because any phase factor that *det(L)* contains has no effect on the 4-vectors used in *H* and *H'*. From this, *L* is completely defined as the set of 2x2 complex matrices defined by *SL(2, C)*:

$$SL(2,\ \mathbb{C}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in \mathbb{C}^4 \ \middle|\ (ad - bc = 1) \right\}$$

As will be explained shortly, there is no loss of generality in finding the normalized eigenvectors of *H* (using equation 10) by using equation 10 in the special case where $R = 1$ and $\alpha = \pi/2$. And this gives:

$$
\begin{aligned}
z &= \cos\phi \\
y &= \sin\phi\ \sin\theta \\
x &= \sin\phi\ \cos\theta
\end{aligned}
\qquad \text{Equation (12)}
$$

And *H* now equals:

$$H = \begin{bmatrix} \cos\phi & \sin\phi\cos\theta - i\sin\phi\sin\theta \\ \sin\phi\cos\theta + i\sin\phi\sin\theta & -\cos\phi \end{bmatrix}$$

This operator has eigenvalues and normalized eigenvectors given by:

$$\lambda_1 = +1 , \quad \begin{bmatrix} \cos\left(\phi/2 \right) \\ \sin\left(\phi/2 \right) e^{i\theta} \end{bmatrix} \qquad \text{(Right Chiral Qubit)}$$

Equation (13)

$$\lambda_2 = -1 , \quad \begin{bmatrix} \sin\left(\phi/2 \right) \\ -\cos\left(\phi/2 \right) e^{i\theta} \end{bmatrix} \qquad \text{(Left Chiral Qubit)}$$

These angles range over the values [0, π] for ϕ and [0, 2π) for θ. These qubits can be expressed as linear combinations of the basis vectors $\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$:

$$| Q_R > = \cos\left(\phi/2 \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \sin\left(\phi/2 \right) e^{i\theta} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$| Q_L > = \sin\left(\phi/2 \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \cos\left(\phi/2 \right) e^{i\theta} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Just as in quantum physics, the probability that these qubits will be in one of the basis vector states, after state reduction, is equal to the co-efficient of that state times its complex conjugate. For example, the probability that $|Q_R>$ will be in the state $\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$ is equal to $\sin\left(\phi/2 \right) e^{i\theta} \cdot \sin\left(\phi/2 \right) e^{-i\theta} = \sin^2\left(\phi/2 \right)$. For the qubit $|Q_R>$, the state $\left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$ corresponds to the classical bit 0 and the state $\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$ corresponds to the classical bit 1. For the qubit $|Q_L>$, the state $\left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$ corresponds to the classical bit 1 and the state $\left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$ corresponds to the classical bit 0. If equation 10 is used instead of equation 12, in calculating the eigenvalues, then the eigenvalues are:

$$\begin{aligned} \lambda_1 &= R(\cos \alpha + \sin \alpha) \\ \lambda_2 &= R(\cos \alpha - \sin \alpha) \end{aligned} \qquad \text{Equation (14)}$$

But the normalized eigenvectors are the same as in equation 13. As mentioned previously, this means there is no loss of generality in these calculations and it also means the additional information, R and α, in equation 10 (two pieces of the flagpole spinor information) is not encoded into the qubits. Rather, that information is in the eigenvalues.

7. Relativistic Quantum Information and Spinors

Rank one spinors contain four pieces of information, the spinor size, two angles, and the flagpole angle. That information was used in the right-hand side of equation 10. Section 6 shows that this information is split: two angles are used in qubits (Equation 13) and the size and flagpole angle can be in the eigenvalues (Equation 14). This suggests that a spinor size and its flagpole angle information should be put into an operator that can act on the qubit so that a connection can be found between spinors and quantum information. The spinor size and its flagpole angle are information that must be provided by the physical system embodying the qubits. Relativity requires that the laws governing quantum information must be the same in all frames of reference. This suggests defining an operator, K, that contains the missing information in the qubit, namely size and flagpole angle, from the system containing the qubit. Assuming $|Q_R>$ and $|Q_L>$ correspond to spinors with the same parameters then the operator K has the following properties:

$$\begin{aligned} s_R &= K \mid Q_R > \\ s_L &= K \mid Q_L > \\ KK^{\dagger} &= K^{\dagger}K = R \cdot I_{2x2} \\ KK^{-1} &= K^{-1}K = I_{2x2} \end{aligned} \qquad \text{Equation (15-A)}$$

Where K is a matrix operator. If K is simply a scalar function of the spinor parameters, then:

$$\begin{aligned} s_R &= K \mid Q_R > \\ s_L &= K \mid Q_L > \\ KK^{*} &= K^{*}K = R \\ KK^{-1} &= K^{-1}K = 1 \end{aligned} \qquad \text{Equation (15-B)}$$

In these equations, S_R and S_L are right-chiral and left-chiral spinors respectively and R is the magnitude of the spinor. These equations allow for the transformations of qubits between special relativistic frames of reference. The qubits are mapped to their respective spinors, the spinors are then transformed between frames of reference in the usual way, and the operators in the new frame of reference are used to get the qubits:

$$(K')^{-1}s'_R = |Q'_R>$$
$$(K')^{-1}s'_L = |Q'_L>$$

K' is easily found using the primed values of the spinor variables (*R, ϕ, θ, α*). These equations are only needed when doing a boost on qubits, they are not necessary when only doing a rotation on qubits. Qubits have a magnitude of 1 and rotations do not change their magnitude. The rotation operator is the same for both types of qubits in equation 13. So, rotations on qubits can be done by using the unitary rotation operator:

$$U = exp\left(i\vec{\boldsymbol{\sigma}}\cdot\vec{\boldsymbol{\theta}}/2\right)$$
$$|Q'_R> = U|Q_R>$$
$$|Q'_L> = U|Q_L>$$

Where **θ** are the rotation angles and **σ** are the Pauli matrices. Since qubits have magnitude 1, i.e., $<Q|Q> = 1$, for any qubit $|Q>$:

$$\begin{aligned} <Q'|Q'> &= <Q|U^{\dagger}U|Q> \\ &= <Q|I|Q> \\ &= <Q|Q> \\ &= 1 \end{aligned}$$

So, rotations do not need all the information that boosts do. A di-qubit, which could also be called a Dirac di-bit, can be defined as follows:

$$
| Q_D > = \begin{bmatrix} | Q_R > \\ | Q_L > \end{bmatrix} = \begin{bmatrix} \cos\left(\phi/2 \right) \\ \sin\left(\phi/2 \right) e^{i\theta} \\ \sin\left(\varphi/2 \right) \\ -\cos\left(\varphi/2 \right) e^{i\beta} \end{bmatrix}
$$

With these definitions, quantities and fields in physics can be expressed in terms of qubits. For example, let K_1 and K_2 be 2x2 matrices as defined by equation 15A and define the 4x4 matrix, J, by:

$$
J = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}
$$

And with the help of the Dirac matrices in the common basis:

$$
\gamma^0 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \ \ \gamma^i = \begin{bmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{bmatrix}, \ \ \gamma^5 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}
$$

$$
\gamma^u = (\gamma^0, \gamma^i) , \quad \gamma^0 \gamma^u = (\gamma^0 \gamma^0, \ \gamma^0 \gamma^i)
$$

$$
\gamma^0 \gamma^u \gamma^5 = (\gamma^0 \gamma^0 \gamma^5, \ \gamma^0 \gamma^i \gamma^5)
$$

$$
C^{uv} = \left[\gamma^u, \ \gamma^v \right]
$$

With these definitions, two vector quantities (A^u and B^u), scalar and pseudoscalar quantities (ψ and ϕ), and an antisymmetric tensor, F^{uv}, can be formed from Dirac di-bits as follows:

$$A^u = \langle Q_D \mid J^\dagger \gamma^0 \gamma^u J \mid Q_D \rangle$$

$$B^u = \langle Q_D \mid J^\dagger \gamma^0 \gamma^u \gamma^5 J \mid Q_D \rangle$$

$$\psi = \langle Q_D \mid J^\dagger \gamma^0 J \mid Q_D \rangle$$

$$\varphi = \langle Q_D \mid J^\dagger \gamma^0 \gamma^5 J \mid Q_D \rangle$$

$$F^{uv} = \langle Q_D \mid J^\dagger \gamma^0 C^{uv} J \mid Q_D \rangle$$

A field of di-qubits can therefore correspond to scalar, pseudoscalar, vector, and tensor fields. The question arises: is it possible that quantum information is the basis for the fields that exist in nature? Can a unified theory of nature be formulated on information? It is also possible to take outer products and tensor products of qubits to obtain new quantities and define fields for them. Quantum information can be used to code up the kinds of quantities, and fields, that exist in nature, along with their operators. This raises other questions: in what ways can information-based theories of nature be formulated? Are quantum fields sets of qubits that are in some type of unreduced quantum gravitational state? Before going onto these issues, it must be noted that $|Q_R\rangle$ and $|Q_L\rangle$ are interchangeable with the following operators (e.g. $|Q_R\rangle = M_{LR}|Q_L\rangle$):

$$M_{LR} = \begin{bmatrix} 0 & -e^{-i\theta} \\ e^{i\theta} & 0 \end{bmatrix}, \qquad M_{RL} = \begin{bmatrix} 0 & e^{-i\theta} \\ -e^{i\theta} & 0 \end{bmatrix}$$

These operators are unitary, and they are inverses of each other, and they satisfy:

$$M_{LR} M_{RL} = M_{RL} M_{LR} = I$$
$$M_{LR} = M_{RL}^\dagger$$
$$M_{RL} = M_{LR}^\dagger$$

With these operators, it is possible to move chiral information into other operators. So, tensor products of qubits can have various equivalent representations. Lots of other operations can be performed on qubits. Null vectors associated with qubits are reflected through the *x* and *z* axes when their corresponding qubits are conjugated. The operator $g = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, when applied to a conjugated qubit, corresponds to a parity inversion on its associated null vector. Qubits and the operations presented so far can be generalized from flat spacetime to warped spacetime. All locations in a warped spacetime have an associated tangent space with local Minkowski geometry. For that reason, the entire formulation of spinors can be used in warped spacetime [3]. Similarly, the entire theory of quantum information formulated here can be used at events in warped spacetime. This suggests that warped “quantum” spacetime should be formalizable in terms of quantum information. It also suggests that quantum fields can be formulated in terms of quantum information.

8. 4-Volume Energy That is Invariant: ħ/2

Standard physical theory is currently based on the real number line. The real number line, under its usual order, is dense: between any two real numbers, no matter how close together they are, there exists an uncountably infinite number of other real numbers between them. The question arises, is this the way nature really is? Are space and time infinitely divisible? The answer to both questions is no because of the denseness of the real number line and the uncertainty principle. If time is modeled by the real number line, then at any instant of time there is no way time can `move’ to the `next’ instant in time. This is because if there were a `next' instant then the uncountable number of instants between the two would have been skipped over. And an instant in time, which lasts for no time at all, cannot exist because there is no time there. Time exists only insofar as there is a non-zero amount of proper time. Temporal extension along the time dimension is needed for existence. Similar problems hold true for space and motion in space. On the real number line, it is possible to set up supertasks which, by their very nature, are self-contradictory. By construction, a supertask forces an unending process to have an end state. It is also the real number line that makes singularities possible in physical theory: singularities in General Relativity and singularities in field theory. If the real number line corresponded to anything physical, then what could it be? Perhaps an uncountably infinite number of massless singularities (i.e., points) arranged in the standard order of the reals? If that is to be taken seriously, then what sort of physical theory gives an infinite number of massless singularities? Real numbers are usually modeled as Dedekind cuts in standard analysis and that assumes that space is infinitely divisible. According to the uncertainty principle, it is not possible to have zero uncertainty in time because that would require an infinite amount of energy, nor is it possible to have zero uncertainty in space because that would require an infinite amount of momentum. Some of these problems can be partially bypassed using relativity. From relativity, it is known that there is no flow to time so it might be conceivable that state reduction in time specifies “instants” and none of the uncountably infinite number of other “instants” between them. But other problems with the infinities remain and so that is not a satisfactory resolution. All of this suggests that the real number line, and spaces based on it, do not correspond to anything that exists in nature. Quantum physics suggests that all physical quantities are finite. Infinity is not an eigenvalue of any reasonable operator.

Removing infinities from physical theory has always been successful. From eliminating various ultra-violet catastrophes to the elimination of infinities in quantum field theory, every time infinities are removed from physical theory it has turned out to be the right course of action. This suggests that spacetime, mass-energy, and quantum fields should have some sort of quantization scheme that does not ultimately rely on the real number line but does retain all the symmetries that nature is known to have. That means the principles of locality, causality, relativity, unitary evolution, the strong equivalence principle, diffeomorphism invariance, the superposition principle, Born's rule, and so on, remain true in a new quantization scheme.

The quantization scheme in standard physics involves summing over all possible histories. The amplitudes for given events are found by summing over all possible paths and all possible interactions, in space and time, and the probability that state reduction yields a given state is determined by the complex square of the amplitudes for that state. Traditionally, it has been assumed that state reduction is truly random: that is, it is not determined by anything at all. It has been argued that this eliminates determinism. But the fact that each particle interacts with all other particles in the cosmos, prior to state reduction, suggests that the reduced state may be determined in some computational way by all the other particles in the cosmos. If that is true, then there must be an unknown physical law governing their interactions that is deterministic. Quantum physics only shows that state reduction is probabilistic, it does not rule out determinism.

What is needed is a quantization scheme that quantizes spacetime and can be related to the quantization of quantum field theory. Quantum information may be a partial way to do this. Conjecture 2 provides one possible way that energy and spacetime could be quantized together.

> **Conjecture 2**: Spacetime and energy are fused together and made up out of a finite number of quantized units (for the purposes of this paper called *metric quanta*) that contain some spacetime and some energy. Each metric quantum is a "4-volume energy" in the amount of $\hbar/2$ which is invariant in the sense of relativity. Quantum information, both in states and operators, organizes the metric quanta into the spacetime "continuum" and the mass-energy content of the cosmos, in such a way as to be experimentally consistent with quantum field theory and General Relativity.

Conjecture 2 requires some clarification. Quantum information is analogous to normalized rank 1 spinors and their operators as discussed in section 7. A metric quantum can be thought of as an invariant object where the four dimensions of spacetime contain a three-dimensional energy density (e.g., Joules per cubic meter). The energy density can be thought of as a kind of fifth dimensional quantity. The combined energy and spacetime of the metric quanta are a conjectured gravitational quantization of the spacetime continuum and its energy in the form of gravity and quantum fields. The conjecture means that the whole of physical reality, at every place and time, is made up out of an enormous number of metric quanta. Those metric quanta are governed by an enormous amount of quantum information and the eigenvalues of all their operators. It also means that quantizing General Relativity necessarily involves a new type of quantization for the quantities in quantum field theory. Standard quantum field theory has to be a low energy approximation to quantum gravity.

If the real number line can be replaced by metric quanta, quantum information and various operators, then quantum field theory needs to be reformulated so that it no longer depends on the real number line. The same thing is true with General Relativity. Reformulating General Relativity using this new approach necessarily involves abandoning the real number line of pure mathematics. It also entails that a finite amount of information quantifies physical systems rather than an infinite amount of information that is necessary for systems in classical physics. Replacing the real number line with metric quanta and quantum information also eliminates the infinities and singularities that traditionally can arise.

Since each metric quantum can be thought of as having some space, some time, and some energy, the amounts involved will not be definite until state reduction occurs and not all quantities may necessarily have definite values. In an unreduced state, a metric quantum must be in a superposition of all possible states and those states are in relation to other metric quanta. So metric quanta are not classical objects, but it might be useful to think of a metric quantum in a classical sense by the following equation (where ρ is a three-dimensional energy density):

$$\frac{\hbar}{2} = \int_{4-Region} \rho(x, y, z, t) \; dxdydzdt \qquad \text{Equation (16)}$$

Equation 16 should not be taken too seriously. Regions of spacetime are themselves made up of metric quanta and it is quantum information that provides the structure. And the region of integration itself has quantum uncertainties. Because nature is fundamentally quantum mechanical, the classical imagery involved in equation 16 is deceptive. Due to quantum uncertainty, the three-dimensional energy density, ρ, may only be in a partially reduced state when state reduction occurs for some of its properties. For example, if the integral takes the form:

$$\frac{\hbar}{2} = mc^2 t$$

Then there can still exist an uncertainty between m and t. And if one of them is specified then so is the other but the function ρ may still not have a specific form. Conjecture 2 permits space, time, and energy as different expressions of metric quanta. Conjecture 2 also permits quantum information, and various operators on that information, to structure the metric quanta in such a way as to make the physical world. Spacetime and its energy content is quantized in units of *ħ/2*. This quantization scheme can retain the symmetries of physical theory based on the real number line. For example, when a metric quantum describes a massive particle, relativistic mass and time dilation preserve its invariance: $m't' = (m\gamma)\left(t/\gamma\right) = mt = \hbar/2c^2$ where γ is $1/\sqrt{1-v^2/c^2}$. Although if such a particle is considered to have a relatively long world-line then that world-line's existence would need many metric quanta.

Metric quanta offer new possibilities as to what quanta are. Traditionally, the quanta have been thought of as objects. The kinds of objects typically considered are infinitely small points, in which case the quanta are some types of naked singularities. The quanta have been postulated to be vibrations of strings that are infinitely thin, as in the string-theory paradigm. Both of those approaches bring infinity into physical theory: infinitely small points or infinitely thin strings. Other possibilities are that the quanta are tiny little black holes or tiny worm holes but infinities are still involved in constructions like that. The very notion of an object seems to be a classical notion. It is not clear how objects like these could do the complicated summing over histories that physical quanta do; no mechanism is given as to how that could work.

Metric quanta suggests that the basic quanta of nature are not objects at all. Rather, they are just energy confined to a region of space, where the confinement is possibly due to quantum gravitation. For example, in the equation $\hbar/2 = mc^2t$, setting $x = ct$ fixes the mass of a particle when x is fixed, and then the equation becomes $\hbar/2 = mcx$. Of course, this requires that there exists some yet to be discovered theory that localizes the energy to a spatial region x. The obvious challenge is to try and formulate these ideas into a coherent theory. If that can be done, then a new picture of the cosmos can emerge. With metric quanta, it can be conjectured that when an individual "particle" undergoes a "sum over all histories" it can be thought that its associated metric quanta "cover" the whole of the "spacetime continuum" so that ρ in equation 16 has an extremely small average value (that is until state reduction occurs). Equivalently, by the energy-time uncertainty principle, the metric quantum for a "particle" in an unreduced state can be thought of as encompassing all the energy in the cosmos when the time uncertainty is small enough. This is what is to be expected from equation 16 since equation 16 is a kind of uncertainty principle.

From conjecture 2, the cosmos can be thought of as a large, but finite, number of metric quanta that relate to each other in unreduced states and are connected in a manner that is consistent with quantum field theory and General Relativity. The number of metric quanta is finite because the information content and energy content must be finite in accordance with the Bekenstein bound (explained below). Since the number of metric quanta is finite, the total amount of energy that exists must be finite, and conserved, and there cannot be an infinite amount of spacetime. The cosmos must be positively curved on the largest scales. A positively curved universe is closed, and its mathematics can be formulated so that the total energy is set equal to zero [2]. This means that energy lost by the gravitational field exactly equals energy gained in the form of non-gravitational energy. The finite number of metric quanta imposes an additional constraint: it is not possible to have an infinite amount of gravitational energy since the total amount of energy that exists must be finite. So, even though General Relativity allows for the possibility of an infinite amount of gravitational energy for a closed universe, conjecture 2 requires that boundary conditions are imposed on the field equations to prevent that. One way to do this is to define the gravitational energy-momentum tensor to be equal to the Einstein curvature tensor multiplied by the appropriate constant and then impose cutoffs on the volume integral of that gravitational energy-momentum tensor. According to conjecture 2, these boundary conditions must be a consequence of the correct quantized form of General Relativity.

Removing infinities from physical theory eliminates any possibility of singularities, ultra-violet catastrophes, non-renormalizable quantities, and so on. In equation 16, if ρ is assumed to be an average density then it is impossible for a metric quantum to encompass an infinite amount of spacetime. Zero times an infinite quantity is nonsensical. There are functions, ρ, that satisfy equation 16 if the region of integration is infinite but that does not apply here. The removal of infinities from physical theory also requires that there must be a "big crunch" at some time in the future. Since the amount of spacetime is finite, there cannot be any new time after the big crunch nor any time before the big bang. So, the negative pressure in dark energy must cease at some time in the future just as any inflationary like expansion in the very early universe had to cease. Quantum gravity suggests cosmic inflation and the current dark energy inflation cannot be eternal (e.g., see [12], [13], and [14]) and that is necessary when removing infinities.

Letting the total time between "big bang" and "big crunch" be T_{total}, and the total energy that exists is E_{total}, (relative to the co-moving CMB rest frame) then the total number of metric quanta that make up the cosmos is $(E_{total}T_{total})/(\hbar/2) = n_{metric\_quanta}$. That can be related to the total amount of information that exists using equation 9. In equation 9, assuming R is a constant distance that characterizes the size of spacetime, substituting the characteristic time $2\pi R/c=T_{total}$ gives n_{qubits}:

$$n_{qubits} = \frac{2\pi \mathrm{RE}}{\hbar c \ln(2)} = \frac{T_{total} E_{total}}{\hbar \ln(2)} = \frac{N_{metric\ quanta}}{2\ln(2)}$$

Applying the Bekenstein bound to the entire cosmos implies that the number of metric quanta that exists is approximately equal to $2ln(2)$ times the total information content of the universe (the number of qubits). The amount of quantum information calculated here is quite a bit more than the quantum information that can be estimated for the observable universe. However, qubits associated with gravitational energy are also included and the universe is larger than just its observable part. A consequence of Conjecture 2 is that the size of the cosmos is determined by the amount of quantum information that exists.

9. Do Differential Equations Ultimately Describe Nature?

Contemporary physics understands the evolution of the universe in terms of differential equations. This means that the initial or boundary conditions contain the information content of the universe and physical laws evolve those initial or boundary conditions over time according to the differential equations that are said to describe nature. The equations are evolved forward in time based on the initial or boundary conditions and, apart from state reduction, the evolution is clearly deterministic. For this approach to describe nature, the activity taking place at any place and time must receive its instructions on how to behave from the past. This approach works extremely well in most situations. But are differential equations fundamentally the right approach? In quantum mechanics, reduced states cannot be calculated from the differential equations. So, state reduction offers a counter example to the traditional approach. As mentioned previously, abandoning the real number line, and spaces based on it, suggests that the universe and its physical laws should not fundamentally be based on differential equations. However, differential equations still have problems with boundary and initial conditions coming from the past. For example, in the situation visualized in figure 2, a gravitational body receives its instructions on how to behave from a future event, relative to its own time and place.

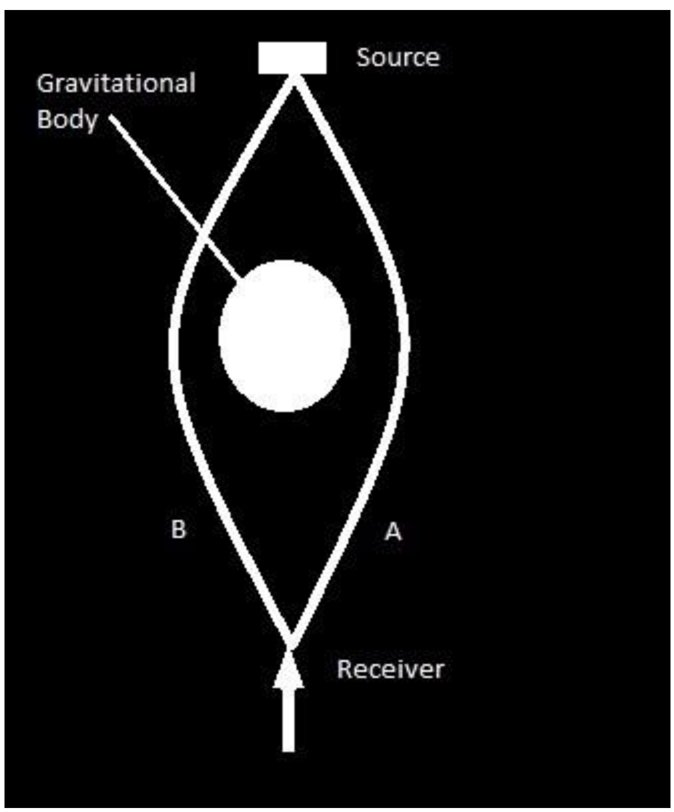

Figure 2

The gravitational body in figure 2 can be a planet, a star, or even a galaxy. The source in figure 2 is a light source that emits a photon and, quantum mechanically, half of the wave function for the photon resides along path A and the other half of its wave function resides along path B. In both paths, the photon's course is bent around the gravitational body due to the warping of space. The receiver can be an astronomer on Earth who receives the photon emitted from the source. Suppose the distance from the source to the gravitational body is one billion light years and the distance from the gravitational body to the receiver is also one billion light years. Then it takes about two billion years for the photon to travel from the source to the receiver. It is assumed that the gravitational body has no horizontal motion at all relative to the receiver. The receiver's instruments can be configured to detect either the wave-like behavior of the photon or the particle-like behavior of the photon.

If the receiver's instruments are configured to detect the wave-like behavior of the photon then, after receiving the photon, the horizontal motion of the gravitational body, relative to the receiver, should remain unchanged. And, since its horizonal speed was zero to begin with, it should be zero after the photon arrives. But if the receiver's instruments are configured to detect the particle-like behavior of the photon then the state of the system undergoes state reduction forcing the photon to have traversed a particular path. In the case it traversed path A then the horizontal momentum of the photon would be changed, during its journey, from traveling to the right to traveling to the left. By conservation of momentum, the change in the photon's momentum must be compensated for by a change in the momentum of the gravitational body. The gravitational body must now have horizonal momentum causing it to travel to the right relative to the receiver. In the case state reduction caused the photon to have traversed path B then the horizonal momentum of the photon would have changed from traveling to the left to traveling to the right and, by conservation of momentum, the gravitational body would have to start traveling to the left relative to the receiver. In the case of state reduction, the gravitational body gets its moving instructions from a billion years in its own future. Whether or not the gravitational body starts moving to the left or to the right depends on whether the receiver, in this case an astronomer on Earth, chooses to configure the receiving instrument to measure either the particle-like or wave-like nature of the receiving photon.

In this example, the gravitational body's behavior is not entirely determined by initial or boundary conditions in its past. Its behavior cannot be entirely calculated using differential equations with only past initial and boundary conditions. A new paradigm is needed here that replaces differential equations, and spaces based on the real number line. Conjecture 2 postulates that a new, yet to be fully developed theory, can be formulated where quantum information and metric quanta replace differential equations in describing the cosmos. A possible approach to doing that is explored in the next section.

10. A Toy Model of Quantized General Relativity

The solutions of General Relativity are either a metric tensor, given an energy-momentum tensor, or an energy-momentum tensor, given a matric tensor, or both an energy-momentum tensor and a metric tensor if only partial information of either tensor is given. A given solution can be specified entirely by a metric tensor which corresponds to a given energy-momentum tensor. Since energy, momentum, and their states have an intrinsic quantum uncertainty (and since the energy-momentum tensor is proportional to the Einstein curvature tensor) it necessarily follows that spacetime curvature should have a quantum uncertainty. And, therefore, the metric tensor should have quantum uncertainty since its curvature does. In general, every symmetric metric tensor corresponds to a specified curvilinear coordinate transformation. Associated with a curvilinear coordinate transformation is a set of basis vectors, defined at each location on the manifold, and the dot product of any two basis vectors defines a component of the metric tensor. In this section, these basis vectors are promoted to operators acting on a quantum information state. In section 7, it was shown that information states, with appropriate operators, could define two vector quantities (A^u and B^u), scalar and pseudoscalar quantities (ψ and ϕ), and an antisymmetric tensor, F^{uv}, and so it is natural to think that fields, including gravitational fields, could also be described as operators acting on an information state.

The law of conservation of quantum information states that the total amount of quantum information that exists is conserved. The total amount of quantum information that exists must be immense and, since infinities should be removed from physical theory, the total amount of quantum information should be finite as explained in section 9. In this toy model, the quantum information state is a pure state, but toy models could be created that include mixed states. So, in this model, the total quantum information that exists is defined to be the state $|\Psi>$ as follows:

$$|\psi> = |Q_1> \otimes |Q_2> \otimes |Q_3> \otimes \ldots \otimes |Q_n> \qquad \text{Equation (17)}$$

Here, n, is the total number of qubits that exist. Obviously, n must be an extremely large number, and nobody knows why it is so big. The bigger n is the bigger $|\Psi>$ is and the bigger $|\Psi>$ is the more operators there can be that act on that information state. And, therefore, there are more observables in the universe. The fact that the universe is large is because there is an enormous amount of quantum information that exists. And, based on the reasoning in section 9, the reverse is true. The qubits, denoted by $|Q_k>$, can be any of the types of qubits described in sections 6 and 7, appropriately normalized as needed. Changing the type of qubit used only results in changes in the operators to be discussed next. If the qubits are the kind specified in equation 13 then $<\Psi|\Psi>=1$ should hold true. Observables are eigenvalues of operators acting on the state $|\Psi>$. As discussed in section 8, the real number line needs to be removed from physical theory and replaced with something else. In this toy model, the relativistic spaces that use the real number line are replaced by the quantum information vector in equation 17. Something similar needs to be done with the other fields in nature but that will not be explored here. In this kind of approach, the whole of physical theory can be formulated without the real number line. This model beings with the manifold for the curvilinear coordinate transformation. The manifold basis vectors are:

$$\vec{i}_0 = (1, 0, 0, 0)$$
$$\vec{i}_1 = (0, 1, 0, 0)$$
$$\vec{i}_2 = (0, 0, 1, 0)$$
$$\vec{i}_3 = (0, 0, 0, 1)$$

The index zero corresponds to the time dimension, and the indices 1, 2, and 3 correspond to *x*, *y*, and *z* dimensions respectively. In all these calculations, the summation convention applies to subscripts and superscripts and the sum is from zero to three for both Greek indices and non-Greek indices. The basis vectors for the curvilinear coordinate system are given by operators, ***g***, acting on $|\Psi>$ returning a basis state, $|\Psi_K>$, and their covariant form is given by equation 18.

$$|\psi_k> \ = g_k^l \,|\psi> \vec{i}_l \qquad \text{Equation (18)}$$

These basis vectors for the curvilinear coordinate system define the metric for the entire cosmos: from “big bang” to “big crunch”. Therefore, equation 18 gives the entire history of the cosmos. The operators, ***g***, are Hermitian. Taking the conjugate transpose of an equation like equation 18 gives:

$$<\psi_m| \ = \ <\psi| \, g_m^n \, \vec{i}_n \qquad \text{Equation (19)}$$

Here $\left(g_m^n\right)^\dagger = \left(g_m^n\right)$ because the operator is Hermitian. The inner product of Equation 18 and 19 gives the metric as a linear function of the Kronecker delta:

$$
\begin{aligned}
<\psi_m \mid \psi_k> &= <\psi \mid g^n_m\, g^l_k \mid \psi> \vec{i}_l \cdot \vec{i}_n \\
&= <\psi \mid g^n_m\, g^l_k \mid \psi> \delta_{nl} \\
&= g_{mk}
\end{aligned}
\qquad \text{Equation (20)}
$$

Switching the order of the inner product between the two equations gives:

$$
\begin{aligned}
<\psi_k \mid \psi_m> &= <\psi \mid g^l_k\, g^n_m \mid \psi> \vec{i}_l \cdot \vec{i}_n \\
&= <\psi \mid g^l_k\, g^n_m \mid \psi> \delta_{nl} \\
&= g_{km}
\end{aligned}
\qquad \text{Equation (21)}
$$

In general, the operators in these two different orders will not commute. This means that there must be quantum uncertainty in the metric, and it must be the difference between equation 20 and 21 which can be expressed in terms of a commutator:

$$
\Delta\, g_{mk} = g_{mk} - g_{km} = <\psi \mid \left[g^n_m\, ,\ g^l_k \right] \delta_{nl} \mid \psi> \qquad \text{Equation (22)}
$$

There must be an enormous number of operators, ***g***, to describe the whole of the quantized spacetime "continuum". The exact form of these operators will depend on how the quantum information is organized in equation 17 and so the operators themselves have yet to be specified. For example, the operators could use positional encoding. The information should have a representation like that. There must also be contravariant operators and they will be distinguished from their covariant counterparts using the tilde symbol.

$$
\mid \psi^\alpha> \;=\; \tilde{g}^{\,\alpha}_{\beta} \mid \psi> \vec{i}^{\,\beta} \qquad \text{Equation (23)}
$$

Because the reciprocal basis vectors are orthogonal, we want $<\Psi^\alpha|\Psi_K>=\delta^\alpha{}_K$. Imposing this condition means that the operators must satisfy:

$$
\begin{aligned}
<\psi^{\alpha} \mid \psi_{k}> &= <\psi \mid \tilde{g}^{\alpha}_{\beta}\left(\vec{i}_{l} \cdot \vec{i}^{\beta}\right) g^{l}_{k} \mid \psi> \\
&= <\psi \mid \tilde{g}^{\alpha}_{\beta}\left(\delta^{\beta}_{l}\right) g^{l}_{k} \mid \psi> \\
&= <\psi \mid \tilde{g}^{\alpha}_{\beta} g^{\beta}_{k} \mid \psi> \\
&= \delta^{\alpha}_{k}
\end{aligned}
$$

Assuming *<Ψ|Ψ>=1*, the operators must satisfy:

$$\tilde{g}^{\alpha}_{\beta} \cdot g^{\beta}_{k} = \delta^{\alpha}_{k}$$

The contravariant form of the metric can be found by the same procedure used in equations 19 through 21 but using equations like equation 23 instead of equations 20 and 21. The result is:

$$<\psi^{\delta} \mid \psi^{\alpha}> = g^{\delta\alpha}$$

Just like its covariant counterpart, the metric expressed in its contravariant form has a quantum uncertainty and its equation is like equation 22 except the contravariant operators are used. In ordinary Riemannian geometry, the metric can change basis vectors to their reciprocal counterparts and vice versa. For example:

$$\vec{g}^{\alpha} = \vec{g}_{\delta} g^{\delta\alpha} \qquad \text{Equation (24)}$$

This imposes another condition on the operators. Define the following vectors:

$$
\begin{aligned}
\mid \psi^{\alpha}> &= \tilde{g}^{\alpha}_{\beta} \mid \psi> \vec{i}^{\beta} \\
\mid \psi_{\delta}> &= g^{u}_{\delta} \mid \psi> \vec{i}_{u}
\end{aligned}
$$

Requiring these vectors satisfy equation 24 means:

$$\tilde{g}^{\alpha}_{\theta} \mid \psi> \vec{i}^{\theta} = g^{u}_{\delta} \mid \psi> \vec{i}_{u} <\psi \mid \tilde{g}^{\delta}_{\gamma} \tilde{g}^{\alpha}_{\beta} \delta^{\gamma\beta} \mid \psi>$$

Taking the dog product of both sides of this equation with $\vec{i}_m$ gives:

$$\tilde{g}^{\alpha}_{m} \mid \psi> = g^{u}_{\delta} \mid \psi> <\psi \mid \tilde{g}^{\delta}_{\gamma}\tilde{g}^{\alpha}_{\beta}\delta_{um}\delta^{\gamma\beta} \mid \psi>$$

And equating operators on both sides gives:

$$\tilde{g}^{\alpha}_{m} = g^{u}_{\delta} \mid \psi> <\psi \mid \tilde{g}^{\delta}_{\gamma}\tilde{g}^{\alpha}_{\beta}\delta_{um}\delta^{\gamma\beta}$$

This means that the operators are constrained by identities that involve all the information in the cosmos. This is not surprising since quantum gravity is expected to behave like quantum field theory where quantum gravity obeys Feynman like path integrals. The contravariant and covariant form of the metric are inverses of each other. Imposing this constraint gives yet another relationship between the operators:

$$\begin{aligned}\delta^{\delta}_{m} &= g^{\delta\alpha}g_{\alpha m} \\ &= <\psi \mid \tilde{g}^{\delta}_{\gamma}\delta^{l\beta}\tilde{g}^{\alpha}_{\beta} \mid \psi> <\psi \mid g^{l}_{\alpha}g^{n}_{m}\delta_{\ln} \mid \psi>\end{aligned}$$

Assuming *<Ψ|Ψ>=1*, the operators must satisfy:

$$\delta^{\delta}_{m} = \tilde{g}^{\delta}_{\gamma}\delta^{l\beta}\tilde{g}^{\alpha}_{\beta} \mid \psi> <\psi \mid g^{l}_{\alpha}g^{n}_{m}\delta_{\ln}$$

The quantum uncertainty in the metric, defined in equation 22, must be relatively small since the classical form of General Relativity works so well in describing nature. But the question arises, how do uncertainties in the metric correlate with uncertainties in curvature and energy states? What is needed is to put the uncertainties in the metric through the machinery of the field equations for gravity and see what uncertainties they require for the energy-momentum tensor. In this toy model, metric uncertainties will be assumed to have the form:

$$\Delta g_{mk} = \frac{\hbar}{E_m T_k}\left(1 - \delta_{mk}\right) \qquad \text{Equation (25)}$$

In equation 25, on the right-hand side there is no implied sum over the indices *m* and *k*. When the Kronecker delta is equal to 1, there are no uncertainties because the diagonal components are subtracted from themselves according to equation 22. That does not mean they cannot have quantum uncertainty in relation to other observables. The denominator in the term $\hbar/(E_m T_k)$ contains an energy, not necessarily related to characteristic time, and a characteristic time that will, in general, depend on the specific situation and keep the metric uncertainty relatively small. The operators corresponding to quantum uncertainties of Christoffel symbols of the first kind will be zero since the terms in equation 25 are constants.

$$\Delta\left[ij,k\right] = \frac{1}{2}\left[\frac{\partial}{\partial x^i}\left(\Delta g_{jk}\right) + \frac{\partial}{\partial x^j}\left(\Delta g_{ik}\right) - \frac{\partial}{\partial x^k}\left(\Delta g_{ij}\right)\right] = 0$$

Quantum uncertainty in Christoffel symbols of the second kind are given by:

$$\Delta\, \Gamma^m_{no} = (\Delta\, g^{m\alpha})\left[no,\alpha\right]$$

Differentiating the identity $g_{mn}g^{no}=\delta_n^{\,o}$ gives the useful relations:

$$\delta g^{m\alpha} = -\, g^{mv} g^{\alpha t} \delta g_{vt}$$
$$\Delta\, g^{m\alpha} = -\, g^{mv} g^{\alpha t} \Delta\, g_{vt}$$

In these and subsequent calculations differentials and small differences are treated as interchangeable. All of this allows the expression for uncertainty in Christoffel symbols of the second kind to be expressed in terms of uncertainty in the covariant form of the metric:

$$\Delta\, \Gamma^m_{no} = \left(-\, g^{mv} g^{\alpha t} \Delta\, g_{vt}\right)\left[no,\alpha\right] \qquad \text{Equation (26)}$$

Uncertainties in Einstein curvature need to be expressed in terms of the uncertainties in the metric and the Christoffell symbols found so far. An arbitrary change in the Einstein tensor is given by:

$$G_{kl} = R_{kl} - \frac{1}{2} g_{kl} R$$

$$\delta G_{kl} = \delta R_{kl} - \frac{1}{2} g_{kl} \delta R - \frac{1}{2} R \delta g_{kl} \qquad \text{Equation (27)}$$

$$\delta G_{kl} = \delta\left(R_{ikl}{}^{i} \right) - \frac{1}{2} g_{kl} \delta\left(g^{ab} R_{ab} \right) - \frac{1}{2} R \delta g_{kl}$$

The Ricci tensor variation is:

$$R_{ikl}{}^{i} = \Gamma^{i}_{l\,i,\,k} - \Gamma^{i}_{l\,k,\,i} + \Gamma^{i}_{s\,k} \Gamma^{s}_{l\,i} - \Gamma^{i}_{s\,i} \Gamma^{s}_{l\,k}$$

$$\delta R_{ikl}{}^{i} = \delta \Gamma^{i}_{l\,i,\,k} - \delta \Gamma^{i}_{l\,k,\,i} + \Gamma^{s}_{l\,i} \delta \Gamma^{i}_{s\,k} + \qquad \text{Equation (28)}$$

$$+ \Gamma^{i}_{s\,k} \delta \Gamma^{s}_{l\,i} - \Gamma^{s}_{l\,k} \delta \Gamma^{i}_{s\,i} - \Gamma^{i}_{s\,i} \delta \Gamma^{s}_{l\,k}$$

The Ricci scalar variation is:

$$\delta R = R_{i\alpha\beta}{}^{i} \delta g^{\alpha\beta} + g^{\alpha\beta} \delta R_{i\alpha\beta}{}^{i} \qquad \text{Equation (29)}$$

Equation 27 can be expressed as:

$$\delta G_{kl} = \delta\left(R_{ikl}{}^{i} \right) - \frac{1}{2} g_{kl} R_{i\alpha\beta}{}^{i} \delta g^{\alpha\beta} \qquad \text{Equation (30)}$$

$$- \frac{1}{2} g_{kl} g^{\alpha\beta} \delta\left(R_{i\alpha\beta}{}^{i} \right) - \frac{1}{2} R \delta g_{kl}$$

Equation 28 with the indices used in equation 29 are:

$$R_{i\alpha\beta}{}^{i} = \Gamma^{i}_{\beta\, i,\, \alpha} - \Gamma^{i}_{\beta\, \alpha,\, i} + \Gamma^{i}_{s\,\alpha}\Gamma^{s}_{\beta\, i} - \Gamma^{i}_{s\, i}\Gamma^{s}_{\beta\,\alpha}$$

$$\begin{aligned} \delta R_{i\alpha\beta}{}^{i} &= \delta\Gamma^{i}_{\beta\, i,\, \alpha} - \delta\Gamma^{i}_{\beta\,\alpha,\, i} + \Gamma^{s}_{\beta\, i}\delta\Gamma^{i}_{s\,\alpha} + \\ &\quad + \Gamma^{i}_{s\,\alpha}\delta\Gamma^{s}_{\beta\, i} - \Gamma^{s}_{\beta\,\alpha}\delta\Gamma^{i}_{s\, i} - \Gamma^{i}_{s\, i}\delta\Gamma^{s}_{\beta\,\alpha} \end{aligned} \qquad \text{Equation (31)}$$

Using equation 25, and the four equations that come after it, substitution gives:

$$\Delta\, g^{m\alpha} = -\, g^{mv} g^{\alpha t}\frac{\hbar}{E_v T_t}\left(1 - \delta_{vt}\right) \qquad \text{Equation (32)}$$

Combining this with equation 26 gives:

$$\begin{aligned} \Delta\,\Gamma^{m}_{no} &= (-g^{mv}g^{\alpha t})\,[no, \alpha]\frac{\hbar}{E_v T_t}\left(1 - \delta_{vt}\right) \\ &= g^{mv}\Gamma^{t}_{n\, o}\frac{\hbar}{E_v T_t}\left(\delta_{vt} - 1\right) \end{aligned} \qquad \text{Equation (33)}$$

A helpful term can be defined. Define:

$$\frac{\hbar}{E_v T_t}\left(\delta_{vt} - 1\right) = c_{vt} \qquad \text{Equation (34)}$$

Taking a partial derivative of equation 33, and using equation 34, gives:

$$\Delta\,\Gamma^{m}_{n\, o\, ,k} = \left[g^{mv}{}_{,k}\Gamma^{t}_{n\, o} + g^{mv}\Gamma^{t}_{n\, o,k}\right]c_{vt} \qquad \text{Equation (35)}$$

Although partial differentiation in equation 35 should be replaced with an operator, in this approximation simple partial differentiation has been used. Using this equation, and equation 33, the following relations can be found:

$$\delta\Gamma^{i}_{l\,i\,,k} = \left[g^{iv}{}_{,k}\Gamma^{t}_{l\,i} + g^{iv}\Gamma^{t}_{l\,i,k}\right] c_{vt}$$
$$\delta\Gamma^{i}_{l\,k\,,i} = \left[g^{iv}{}_{,i}\Gamma^{t}_{l\,k} + g^{iv}\Gamma^{t}_{l\,k,i}\right] c_{vt}$$
$$\delta\Gamma^{i}_{s\,k} = g^{iv}\Gamma^{t}_{s\,k} c_{vt}$$
$$\delta\Gamma^{s}_{l\,i} = g^{sv}\Gamma^{t}_{l\,i} c_{vt} \qquad \text{Equation (36)}$$
$$\delta\Gamma^{i}_{s\,i} = g^{iv}\Gamma^{t}_{s\,i} c_{vt}$$
$$\delta\Gamma^{s}_{l\,k} = g^{sv}\Gamma^{t}_{l\,k} c_{vt}$$

Substituting these into equation 28 gives:

$$\begin{aligned}\delta R_{ikl}{}^{i} = &\left[g^{iv}{}_{,k}\Gamma^{t}_{l\,i} + g^{iv}\Gamma^{t}_{l\,i,k} - g^{iv}{}_{,i}\Gamma^{t}_{l\,k} - g^{iv}\Gamma^{t}_{l\,k,i}\right] c_{vt} \\ &+ \left[\Gamma^{s}_{l\,i} g^{iv}\Gamma^{t}_{s\,k} + \Gamma^{i}_{s\,k} g^{sv}\Gamma^{t}_{l\,i}\right] c_{vt} + \\ &- \left[\Gamma^{s}_{l\,k} g^{iv}\Gamma^{t}_{s\,i} + \Gamma^{i}_{s\,i} g^{sv}\Gamma^{t}_{l\,k}\right] c_{vt}\end{aligned} \qquad \text{Equation (37)}$$

Doing the same thing for equation 31 that was done for equation 28 gives:

$$\begin{aligned}\delta R_{i\alpha\beta}{}^{i} = &\left[g^{iv}{}_{,\alpha}\Gamma^{t}_{\beta\,i} + g^{iv}\Gamma^{t}_{\beta\,i,\alpha} - g^{iv}{}_{,i}\Gamma^{t}_{\beta\,\alpha} - g^{iv}\Gamma^{t}_{\beta\,\alpha,i}\right] c_{vt} \\ &+ \left[\Gamma^{s}_{\beta\,i} g^{iv}\Gamma^{t}_{s\,\alpha} + \Gamma^{i}_{s\,\alpha} g^{sv}\Gamma^{t}_{\beta\,i}\right] c_{vt} + \\ &- \left[\Gamma^{s}_{\beta\,\alpha} g^{iv}\Gamma^{t}_{s\,i} + \Gamma^{i}_{s\,i} g^{sv}\Gamma^{t}_{\beta\,\alpha}\right] c_{vt}\end{aligned} \qquad \text{Equation (38)}$$

Equation 38 can be written more succinctly by introducing a new term $\boldsymbol{\omega}$:

$$\delta R_{i\alpha\beta}{}^{i} = \omega^{ivt}_{i\beta\alpha}\, c_{vt}$$

Equation 30 needs to be expressed in terms of equations 37 and 38 and lowering or raising the metric as needed. To this end, it is useful to recall that:

$$\delta g^{\alpha\beta} = -g^{\beta v} g^{\alpha t} \delta g_{vt}$$

$$= g^{\beta v} g^{\alpha t} \frac{\hbar}{E_v T_v} \left(\delta_{vt} - 1 \right) \qquad \text{Equation (39)}$$

$$= g^{\beta v} g^{\alpha t} c_{vt}$$

So, the third right-most term on the right-hand side of equation 30 becomes:

$$-\frac{1}{2} g_{kl} R_{i\alpha\beta}{}^{i} g^{\beta v} g^{\alpha t} c_{vt} \qquad \text{Equation (40)}$$

The second right-most term on the right-hand side of equation 30, with the help of equation 38, becomes:

$$-\frac{1}{2} g_{kl} g^{\alpha\beta} \omega^{ivt}_{i\beta\alpha} c_{vt} \qquad \text{Equation (41)}$$

Where the $\boldsymbol{\omega}$ indexed term contains the terms from equation 38. The first right-most term on the right-hand side of equation 30 becomes:

$$-\frac{1}{2} R \delta g_{kl} = \frac{1}{2} R c_{kl} \qquad \text{Equation (42)}$$

Equation 37, expressed in terms of $\boldsymbol{\omega}$, becomes:

$$\delta R_{ikl}{}^{i} = \omega^{ivt}_{kli} \cdot c_{vt} \qquad \text{Equation (43)}$$

All the variations in the curvature terms, equations 40 through 43, can be collected and be substituted into equation 30 with the indices of equation 40 being raised by the metrics. The result is:

$$\delta G_{kl} = \left[\omega^{ivt}_{kli} - \frac{1}{2} g_{kl} R_i^{\ tvi} - \frac{1}{2} g_{kl} g^{\alpha\beta} \omega^{ivt}_{i\alpha\beta}\right] c_{vt} + \frac{R}{2} c_{lk} \qquad \text{Equation (44)}$$

The bracketed term in equation 44 can be labeled as $\tilde{K}$ and substituting $-\delta g_{kl} = c_{kl}$ and letting the differentials become differences gives:

$$-\Delta G_{kl} = \left[\tilde{K}^{vt}_{kl}\right] \Delta g_{vt} + \frac{R}{2} \Delta g_{kl} = -k_0 \Delta T_{kl} \qquad \text{Equation (45)}$$

In this toy model of quantized General Relativity, equation 45 says how quantum gravitational variations in the metric relate to quantum variations in the energy momentum tensor and those variations are *linear* in metric variations at any spacetime location. The toy model used in this section is very simple. The variations are assumed to be "directionless" starting with equation 27. More complicated toy models could be produced if the variations were not "directionless" but varied along a particular co-ordinate direction defined by the basis vectors given by equation 18. Equation 45 says that the quantum uncertainties in energy and momentum of quantum fields, for example, must produce quantum uncertainties in the metric, i.e., spacetime. So, the quantizing of quantum fields, and their energy states, must be part of the quantization of the gravitational field. The canonical quantization procedure in quantum field theory must be a low energy approximation to a different and more general quantization scheme that includes gravity.

11. Conjectured Properties of Quantum Gravitation and Unification

In the previous sections, it has been conjectured that the physical cosmos consists of an immense number of metric quanta governed by eigenvalues of operators that operate on the total information state of the universe (given by $|\Psi\rangle$). It is assumed that the operators involve the constant *ħ/2* so that their eigenvalues specify how space, time, energy, and fields are configured through the whole of cosmic history. However, the speculations in this paper are obviously incomplete. Questions arise within the paradigm outlined here. How do nature's quantum fields relate to the information state and what are their operators? The quantization of spacetime used quantum information to define a "quantized" curvilinear coordinate system but the exact form of those operators is unknown. Spaces based on the real number line, and the real number line itself, have largely been removed from the theory. The state, $|\Psi\rangle$, only partially eliminates the use of the real number line. An obvious problem is that the real number line was substantially used in sections 6 and 7 in defining quantum information. To rectify this problem, the qubits defining the information state, $|\Psi\rangle$, should be reformulated in such a way that each qubit's state is a function of all the other qubit states (instead of being defined as functions of two real angles ϕ and θ) and that eliminates the manifold and the metric that lives on it. An obvious problem and open question in this approach is specifying the mathematical relationship between all the qubits; it is not at all clear how to do that. How that mathematical relationship between the qubits should be formulated isn't known but it makes sense that the state of each qubit is a function of all the others. Formulating the Feynman path integral in terms of quantum information necessarily involves summing over all information amplitudes.

12. Summary

The open issues of this paper, outlined in section 11, show that this approach to quantum gravity is incomplete. Nevertheless, the basic principles presented here offer solutions to issues commonly encountered when trying to extend standard physical theory. By removing the real number line (and infinity) from physical theory, the problem of singularities is avoided. Metric quanta, governed by information and eigenvalues of operators, permit a kind of quantization of General Relativity and that approach should work for the other fields in nature. Even though the real number line, spacetimes based on it, and mathematics based on those spaces is eliminated, integration, differentiation and differential equations should still be useful for calculating the behavior of large systems within the generalized quantization that has been presented here. This is because, in the theory outlined here, the `granularity' of physical systems will be much finer grained than the Planck quantities and by many orders of magnitude. Each instance of state reduction involves all the information and energy that exists everywhere else. The basic ideas in this paper about quantum gravity can be summarized as follows: operators, that contain quantum information, and are functions of $\hbar/2$, act on the total information state of the cosmos. Their eigenvalues specify the characteristics of metric quanta and the number of metric quanta approximately equals $2ln(2)$ times the number of qubits. As proof of principle, a toy model of quantized General Relativity was given but toy models for other quantum fields could also have been given. None of these models rely on spaces based on the real number line. The basic ideas in this paper about black hole evaporation can be summarized as follows: quantum gravitational uncertainties permit the escape of energy and information during black hole evaporation. The ideas in this paper provide the building blocks for a more complete and fully formulated theory.

Appendix A

The Unruh Temperature

The uncertainty principle gives the minimum uncertainty of the energy of a physical system. That energy is given by:

$$\Delta E = \frac{\hbar}{2\Delta t}$$

In a relativistic inertial frame, this uncertainty can be thought of as permitting the existence of virtual quanta over the time interval Δt. The smaller Δt is, the bigger ΔE can be. Overall, these virtual quanta do not give a quantum temperature to the space in the inertial frame. However, this is not necessarily true in a non-inertial frame of reference. A constantly accelerating observer in Minkowski spacetime views that spacetime in Rindler coordinates. The Minkowski spacetime, parameterized in terms of Rindler coordinates with the acceleration taking place along the *x*-axis, has the line element:

$$ds^2 = \left(1 + \frac{ax}{c^2}\right)^2 dw^2 - dx^2 - dy^2 - dz^2$$

In the non-inertial frame of reference, the "Rindler frame", virtual quanta that exist over some time interval $\Delta t'$ will fall down the potential ax/c^2 gaining more energy on average than they would if there was no acceleration present. Therefore, the uncertainty principle will behave differently than it does in an inertial frame, relative to observers in the Rindler frame. Because the virtual quanta gain more energy, on average, from the ax/c^2 potential, they must exist for less time than they would if that potential was absent. So, in the Rindler frame, the uncertainty principle is:

$$\frac{\hbar}{2\Delta t'}\left(1 + \frac{a\Delta x}{c^2}\right) = \Delta E'$$

This is because $ds/dw=(1+a\Delta x/c^2)$ is the average percentage increase in energy of the virtual quanta falling an average distance of Δx down the potential. Multiplying out the above equation gives:

$$\frac{\hbar}{2\Delta t'} + \frac{\hbar}{2\Delta t'}\left(\frac{a\Delta x}{c^2}\right) = \Delta E'$$

Defining $\Delta E''=\hbar/2\Delta t'$ to be the energy available over time $\Delta t'$ if no acceleration were present then the above equation becomes:

$$\delta(\Delta E') = \Delta E' - \Delta E'' = \frac{\hbar}{2\Delta t'}\left(\frac{a\Delta x}{c^2}\right) \qquad \text{Equation (A1)}$$

The term $\delta(\Delta E')$ describes the difference between the uncertainties of the energies between quantum fluctuations in a Rindler frame of reference and quantum fluctuations in an inertial frame of reference. The uncertainty in time, $\Delta t'$, must equal the characteristic time associated with the size of the system, Δx, that contains the extra energy $\delta(\Delta E')$. The characteristic time for Δx is $\pi\Delta x/c$ and this must equal $\Delta t'$:

$$\Delta t' = \frac{\pi\Delta x}{c}$$

Substituting this equation into equation A1 gives:

$$\delta(\Delta E') = \frac{\hbar a}{2\pi c}$$

Because of conservation of energy, $\delta(\Delta E')$ is only a virtual energy that must be returned to the quantum vacuum after time $\Delta t'$. But, because of the acceleration, the quantum vacuum acquires a quantum temperature in the amount $\delta(\Delta E')/k_B$:

$$T_{Unruh} = \frac{\hbar a}{2\pi c k_B} \qquad \text{Equation (A2)}$$

This is the well-known Unruh temperature. The temperature is proportional to the acceleration. Setting the acceleration to $a=GM/R^2$, with $R=2GM/C^2$, into equation A2 gives the Hawking temperature for a Schwarzschild black hole:

$$T_{Hawking} = \frac{\hbar c^3}{8\pi GMk_B}$$

This can be interpreted as the temperature that an observer, far away from the black hole, observers the black hole to have with acceleration *a* at its event horizon according to the distant observer.

Appendix B

Rank 1 Spinor Transformations and their corresponding null vectors.

A spinor, $|s>$, corresponds to a null vector with an associated flagpole angle, α, given by the below transformation, where $w=ct$ and σ_i are the Pauli matrices:

$$\begin{aligned} w &= <s \mid s> \\ x &= <s \mid \sigma_x \mid s> \\ y &= <s \mid \sigma_y \mid s> \\ z &= <s \mid \sigma_z \mid s> \\ e^{-i\alpha} &= \frac{\mid s>^T \sigma_x \mid s>}{\sqrt{x^2+y^2}} \end{aligned} \qquad \text{Equation (B1)}$$

Note that the last relation in equation B1, giving the flagpole angle α, involves only taking the transpose of the spinor, not its conjugate transpose, and the radical in the denominator is always positive. So, given a spinor $|s>$, it is straightforward to calculate the flagpole (x, y, z, α) and w always equals $\sqrt{x^2+y^2+z^2}$. The spinor can be reconstructed from its flagpole, (x, y, z, α), by letting w equal the positive value of $\sqrt{x^2+y^2+z^2}$ in equation B2:

$$| s> = \left(e^{-i\alpha/2} \right) \begin{bmatrix} \sqrt{\frac{w+z}{2}} \sqrt{\frac{x-iy}{\sqrt{x^2+y^2}}} \\ \sqrt{\frac{w-z}{2}} \sqrt{\frac{x+iy}{\sqrt{x^2+y^2}}} \end{bmatrix} \qquad \text{Equation (B2)}$$

The usual technique for taking square roots of complex numbers, DeMoivre's formula, yields two solutions for k=0 and k=1. Using k=0 in equation B2 gives the spinor $|s>$ and using k=1 gives the spinor $-|s>$. So, technically speaking, a flagpole spinor needs a sign bit in addition to the numbers (x, y, z, α).

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