THE GEOMETRIC COLLATZ CORRESPONDENCE

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ABSTRACT

The Collatz Conjecture is a math puzzle that has stumped experts and beginners for a long time. At first glance, it seems simple, but looks can be deceiving. It has become one of the most famous unsolved problems in math. One of the biggest challenges is that there's nothing quite like it in terms of comparison. This makes it hard for many to figure out where to start when trying to analyze and explore the conjecture. However, in my journey to understand this puzzle, I've found two exciting links: one connects the Collatz orbits for odd numbers with a certain type of triangle called a Primitive Pythagorean Triple, and the other ties it to another famous number called the golden ratio. On the way to explain these connections, we develop a framework for treating the Collatz Function as a process that maps integers into a space similar to computer RAM (Randomly Accessible Memory). Each orbit can be represented as a unique location in "Collatz Memory" which is specified by a tuple of three numbers: the stopping time, the page, and the offset into the page. This gives us a new way to investigate the inner structure of Collatz Orbits.

NOTE: Please excuses some of the formatting issues and lack of rigorous proofs. This paper is meant more so to share these ideas in a relatively structured form.

1 Introduction

$$f(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ 3n+1 & \text{if } n \text{ is odd} \end{cases} \tag{1}$$

The Collatz Conjecture, often dubbed the “3n + 1 conjecture”, stands as one of the most notorious unsolved problems in the realm of mathematics. Originating from the musings of Lothar Collatz in 1937, this seemingly simple problem has defied solutions and resisted all attempts at a rigorous proof, all the while captivating the imaginations of amateur and professional mathematicians alike.

The conjecture begins with any positive integer n. If n is even, it is halved ($n/2$), and if odd, it is multiplied by three and incremented by one ($3n+1$). This process is repeated, with each outcome serving as the input for the next iteration. The conjecture posits that regardless of the starting integer, the sequence will invariably arrive at the number 1, after which it will enter a perpetual loop of $4 \to 2 \to 1$.

My goal in writing this paper is not to prove the conjecture, but to start building a framework in which we can map the behaviour of Collatz orbits into some known areas of study. In fact, in most parts of this paper I’ll be assuming the conjecture is indeed true. My thinking is perhaps we can make connections to other areas of mathematics in which we might find clues to the reasons as to *why* it’s true. The areas we’ll be exploring are

- Architecture of Random Access Memory
- Diophantine Equations
- Algebraic Geometry
- Pythagorean Triples
- Harmonic Analysis
- Primes and Permutations

1.1 Outline of Concepts

1. Establish common definitions for well known and lesser known concepts.
2. Understand how we can view the Collatz Function as a mapping from integers into a "Memory Space" similar to what you would find by looking at modern computer architecture.
3. Understand how we can project this Memory Space as lines on the 2D plane.
4. Investigate how we can relate Stopping Times to solutions to Diophantine Equations
5. Investigate how we can map each orbit to an equation of a circles that appears in Memory Space.
6. Show how each circle equation can be associated with a Pythagorean Triple.
7. Show how the circle with the smallest circumference can be associated with the Golden Ratio and possibly Penrose Tilings.
8. Speculate on how we might use the ideas from this paper combined with with tools from harmonic analysis and group theory to study the distribution of the primes.

2 Common Definitions

Before we jump into the connections mentioned in the introduction, we will need to define some terms. If you are familiar with the Collatz Conjecture, you might already know these terms. Even so, revisiting them for a refresher might be beneficial.

Orbit$_n$ ($\mathbf{orbit}_n$) - The sequence of numbers you get when you follow the Collatz rules from a starting number n until you reach the number 1.

Total Stopping Time$_n$ ($\mathbf{Tstop}_n$) - The number of steps or moves needed to get to the number 1 when following the Collatz rules from a starting number n.

Stopping Time$_n$ ($\mathbf{stop}_n$) - The number of steps or moves needed to arrive at a number lower than your initial starting number n when following the Collatz rules.

3 Lesser-Discussed Definitions Explored

The definitions below cover ideas that seem less explored. I've found few formal discussions about them outside of my own research. I will mention the definitions here and expand upon them further when needed. These are not the only new concepts I will present, but these serve as a good stepping off point.

Stopping Class$_k$ ($\mathbf{Sclass}_k$) - This term gives us a way to represent stopping times as an object with properties. This will be useful when we want to compare general invariants of stopping behavior. **Stopping Class**$_k$ contains all numbers **Stopping Time** $= k$

Stopping Orbit$_n$ ($\mathbf{Sorbit}_n$) - The sequence of numbers you get when you follow the Collatz rules from a starting number n until you reach a number less than n.

Collatz Memory ($\mathbf{Smem}$) - **Collatz Memory** is analogous to random access memory in a computer. Through the Collatz Function, a positive integer can be mapped to a specific location in **Collatz Memory**. Interestingly, we can represent this memory as points on a 2D plane. Techincally you can think of **Collatz Memory** as being similar to $\mathbb{R}^2$.

Stopping Destination$_n$ ($\mathbf{Sdest}_n$) - The first number we reach in the sequence that is lower than our starting number n.

Stopping Point$_n$ ($\mathbf{Spoint}_n$) - If a Collatz Orbit has a stopping time, then it also has a stopping point. A stopping point for a number n is defined as a point (x, y) with the following properties:

$$\mathbf{x} = \mathbf{Sdest}_n \text{ - } n$$
$$\mathbf{y} = \mathbf{Sdest}_n$$

The idea of establishing a point to represent the stopping behavior of an orbit is that we can now start to talk about Collatz orbits in terms of their geometry, which will become important later.

Stopping Modulus$_k$ ($\mathbf{Smod}_k$) - As we will see, Collatz **Stopping Times** have some interesting internal structure that works similar to modular arithmetic. Each **Stopping Time k** has a maximum number of "offsets" that can be occupied (similar to a modulus). We call this maximum number of offsets the $\mathbf{StoppingModulus}_k$ where $\mathbf{k}$ is the **Stopping Time**. A **Stopping Modulus** is similar to WORD "size" when dealing with computer memory. Modern desktop computers have a WORD size of 64-bits. As we'll see later, each **Stopping Time** has a property that serves a similar purpose.

Stopping Page$_n$ ($\mathbf{Spage}_n$) - A **Stopping Page** is analogous to a page of memory in a modern computer. As we'll see, we can think of each positive integer as an argument that gets mapped to a point in the 2D plane. Points that have the same **Stopping Page**$_n$ tend to be located roughly in the same geometric area of memory.

Stopping Offset$_n$ ($\mathbf{Soffset}_n$) - A **Stopping Offset** is analogous to a memory offset. Essentially this is the distance from the lower **Stopping Page** boundary.

Stopping Signature$_n$ ($\mathbf{Ssig}_n$) - It is believed that every natural number n (excluding 1) has a finite **Stopping Time**$_n$. Many numbers n may share the same **Stopping Time** as well as the same **Stopping Page**, therefore I have created a term called **Stopping Signature**$_n$ that allows us to uniquely identify an orbit by it's location in **Collatz Memory**.The **Stopping Signature**$_n$ of an orbit can be uniquely defined by a tuple of three positive integral numbers. These properties are $(\mathbf{StoppingTime}_n, \mathbf{StoppingPage}_n, \mathbf{StoppingOffset}_n)$.

$$(\mathbf{stop}_n, \mathbf{Spage}_n, \mathbf{Soffset}_n)$$

4 Building a Geometric Intuition

In this section we will build methods to speak about Collatz orbits in terms of their geometric properties. We can do this by investigation of the stopping point $\mathbf{Spoint}_n$ of each orbit that begins with a given number n. We will focus on the **Stopping Classes** of odd numbers ≥ 3. Below is table showing the stopping times for the first 16 odd numbers ≥ 3.

n	$\mathbf{Spoint}_n$	$\mathbf{stop}_n$
3	(-1, 2)	6
5	(-1, 4)	3
7	(-2, 5)	11
9	(-2, 7)	3
11	(-1, 10)	8
13	(-3, 10)	3
15	(-5, 10)	11
17	(-4, 13)	3
19	(-8, 11)	6
21	(-5, 16)	3
23	(-3, 20)	8
25	(-6, 19)	3
27	(-4, 23)	96
29	(-7, 22)	3
31	(-8, 23)	91
33	(-8, 25)	3

Table 1: Stopping locations and stopping times for first 16 odd numbers ≥ 3

Now it's fair to ask how we determine $\mathbf{Spoint}_n$. We find this point by iteratively applying the Collatz Function.

1. Calculate the **Stopping Orbit**$_n$ for n by applying the Collatz Function until you reach a number $< n$. We call this number the **Stopping Destination** of n
2. Using the **Stopping Destination**, compute the **Stopping Point**$_n$

The **Stopping Destination**$_n$ is essentially the *end state* of applying the Collatz Function for a given positive starting integer n. To make this concrete, let's walk through an example using $\mathbf{n = 19}$. You can then apply this to any number n to compute $\mathbf{Spoint}_n$.

$$\mathbf{Sorbit}_{19} = [\ 58 \quad 29 \quad 88 \quad 44 \quad 22 \quad \mathbf{11}\ ]$$

$$\mathbf{Sdest}_{19} = 11$$

$$\mathbf{Spoint}_{19} = (\mathbf{Sdest}_{19} - 19, \mathbf{Sdest}_{19}) = (11 - 19, 11) = (-8, 11)$$

The following page shows a plot of the numbers in Table 1.

4.1 Plot for odd numbers less than or equal to 33

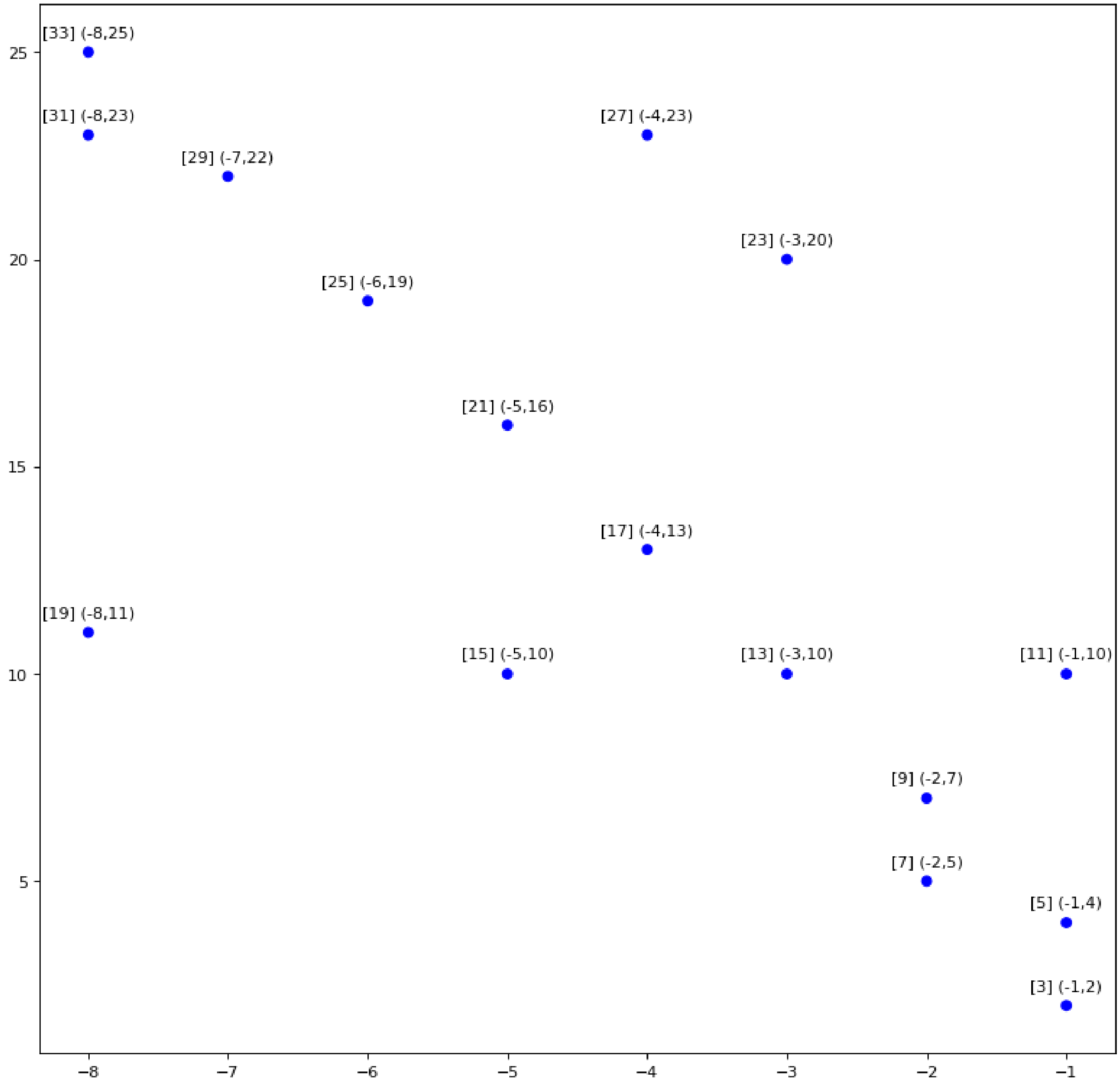

Figure 1: Plot of $\mathbf{Spoint}_n$ for odd numbers ≤ 33.

At first it doesn't seem that the points $\mathbf{Spoint}_n$ have any type of obvious pattern to them. However, if we we look at $\mathbf{Spoint}_n$ for a single stopping time $\mathbf{k}$, we do see some linearity to the points. Let's take the first 8 odd numbers where $\mathbf{stop}_n = 3$, the lowest possible stopping time for odd numbers.

4.2 Data for Orbits with Stopping Time 3

n	$\textbf{Spoint}_n$	$\textbf{stop}_n$
5	(-1, 4)	3
9	(-2, 7)	3
13	(-3, 10)	3
17	(-4, 13)	3
21	(-5, 16)	3
25	(-6, 19)	3
29	(-7, 22)	3
33	(-8, 25)	3

Table 2: Numbers where $\mathbf{stop}_n = 3$

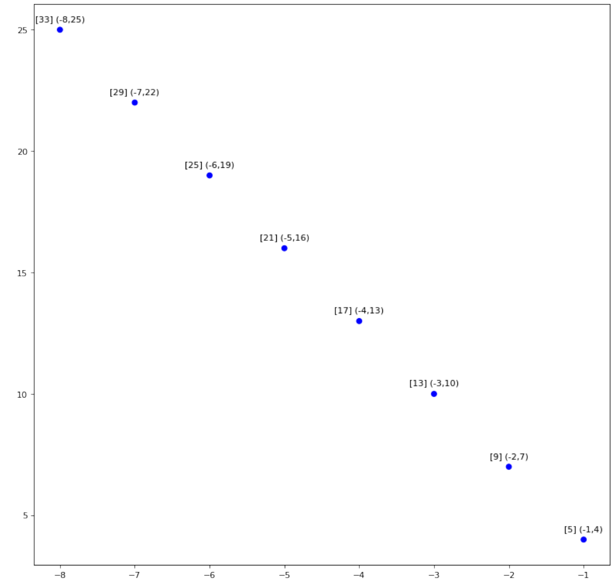

Figure 2: First 8 numbers where $\mathbf{stop}_n = 3$. Points are solutions to the equation $\mathbf{3x + y - 1 = 0}$ with the restriction $x < 0$, $y > 0$, and $|x| + |y| = n$.

We can in fact see that all of these points lie on the on the same line at locations where the coordinates are integers and satisfy the equation $\mathbf{3x + y - 1 = 0}$ $x < 0$, $y > 0$, and $|x| + |y| = n$. This is an interesting result! This may be a clue that ties each **Stopping Class**$_n$ to integer solutions of linear Diophantine equations.

Great! We're starting to see some patterns here! Let's see if these patterns continue to hold for other values of $\mathbf{stop}_n$. Below are the first 8 numbers for $\mathbf{stop}_n = 6$, the next highest allowable stopping time.

4.3 Data for Orbits with Stopping Time 6

n	$\mathbf{Spoint}_n$	$\mathbf{stop}_n$
3	(-1, 2)	6
19	(-8, 11)	6
35	(-15, 20)	6
51	(-22, 29)	6
67	(-29, 38)	6
83	(-36, 47)	6
99	(-43, 56)	6
115	(-50, 65)	6

Table 3: Numbers where $\text{stop}_n = 6$

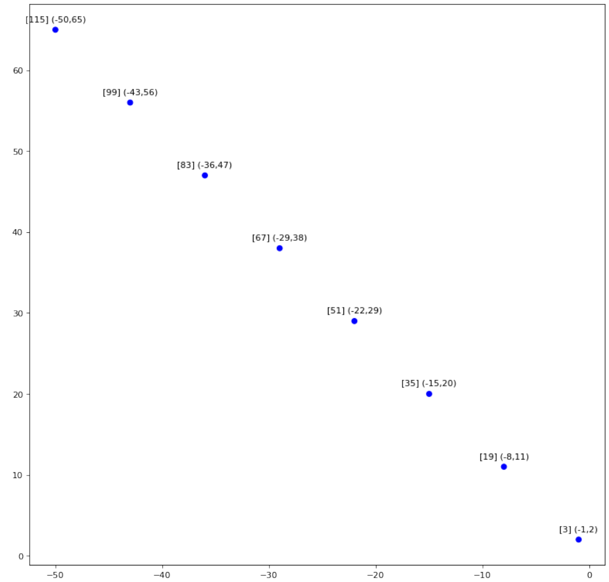

Figure 3: First 8 numbers where $\mathbf{stop}_n = 6$. Points are solutions to the equation $\mathbf{9x + 7y - 5 = 0}$ with the restrictions $x < 0$ and $y > 0$

These points seem to lie on a line as well, this time with slope -$\frac{9}{7}$. And the points seem to be the integer solutions to $\mathbf{9x + 7y - 5 = 0}$ with the restrictions $x < 0$ and $y > 0$.

4.4 Data for Orbits with Stopping Time 8

Let's take a look at one more example where $\mathbf{stop}_n = 8$. This example will serve to motivate our definition of a **Stopping Modulus** and a **Stopping Signature**. Below are the first 8 numbers where $\mathbf{stop}_n = 8$.

n	$\mathbf{Spoint}_n$	$\mathbf{stop}_n$
11	(-1, 10)	8
23	(-3, 20)	8
43	(-6, 37)	8
55	(-8, 47)	8
75	(-11, 64)	8
87	(-13, 74)	8
107	(-16, 91)	8
119	(-18, 101)	8

Table 4: Numbers where $\mathbf{stop}_n = 8$

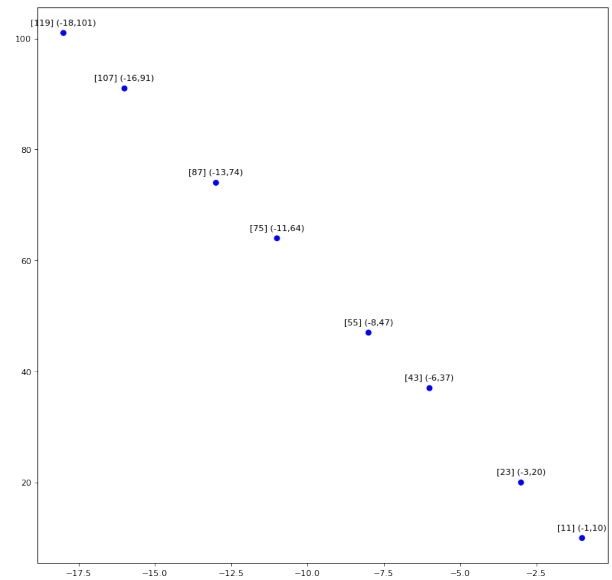

Figure 4: First 8 numbers where $\mathbf{stop}_n = 8$.

The points on the graph do *appear* to fall on the same line, but they actually lie on two separate lines. They also don't seem to be evenly spread out. The points tend to "clump" in groups of **2**. This is where our definition of **Stopping Modulus**$_k$ becomes useful. When **k = 8**, we say **Stopping Modulus**$_8$ $= 2$. This is equivalent to stating **Stopping Class**$_8$ has **Stopping Modulus 2**.

Now we can further classify numbers that belong to **Stopping Class**$_8$ by referencing them by their unique **Stopping Signatures**.

n	$\mathbf{Spoint}_n$	$\mathbf{stop}_n$	$\mathbf{Spage}_n$	$\mathbf{Soffset}_n$	$\mathbf{Ssig}_n$
11	(-1, 10)	8	0	0	(8, 0, 0)
23	(-3, 20)	8	0	1	(8, 0, 1)
43	(-6, 37)	8	1	0	(8, 1, 0)
55	(-8, 47)	8	1	1	(8, 1, 1)
75	(-11, 64)	8	2	0	(8, 2, 0)
87	(-13, 74)	8	2	1	(8, 2, 1)
107	(-16, 91)	8	3	0	(8, 3, 0)
119	(-18, 101)	8	3	1	(8, 3, 1)

Table 5: Demonstrations of **Stopping Signatures**$_8$

The **Stopping Points** of **Stopping Class**$_8$ actually lie on two separate lines that share the same slope of -$\frac{27}{5}$:

- $\mathbf{27x + 5y - 23 = 0}$ with restrictions $x < 0$, $y > 0$, and $|x| + |y| = n$ when $\mathbf{Soffset}_n$ = 0
- $\mathbf{27x + 5y - 19 = 0}$ with restrictions $x < 0$, $y > 0$, and $|x| + |y| = n$ when $\mathbf{Soffset}_n$ = 1

We've now seen how we can map each orbit to a point on a 2D plane (it's **Stopping Point**), creating a geometry that we can study. The addressing of this space resembles the modern day architecture of randomly accessible memory. I call this the **Collatz Memory** space.

We've also seen how we can start to identify properties of **Stopping Classes**. In the next section, we'll get a better sense of how the properties of **Stopping Classes** relate to each other by exploring how they map into the **Collatz Memory** Space.

5 Exploring The Collatz Memory Space

We've already seen how we can map positive integers $n \geq 3$ from **Stopping Class**$_k$ to **Stopping Points** which are located on the 2D plane. One valid question we might ask is "Does every positive integer get mapped into **Collatz Memory**? I've written python code that tests the first 1,000,000 odd numbers $n > 1$, and every number does map to a unique **Stopping Point**.

Every even number also maps to a **Stopping Point**. Since every integer has **Stopping Time** $= 1$, then the following holds true for all even numbers.

Lemma 1. *For all even numbers,* $\mathbf{StoppingPoint}_n$ = $(-\frac{\mathbf{n}}{\mathbf{2}}, \frac{\mathbf{n}}{\mathbf{2}})$

Moreso, if the Collatz Conjecture is true, I assume the following to be true.

Conjecture 1. *Every number* $\mathbf{n > 1}$ *maps to a unique* $\mathbf{StoppingPoint}$

5.1 Infinite Stopping Classes

In section 4 we explored three $\mathbf{StoppingClasses}$: 3, 6, and 8. One might ask the question "How many unique $\mathbf{StoppingClasses}$ exist?" Since there seem to be an infinite amount of $\mathbf{StoppingTimes}$, this leads to my next set of conjectures. I'm not sure if they are obvious, which is why they're left as conjectures.

Conjecture 2. *There are an infinite number of unique* $\mathbf{StoppingClasses}$*.*

Conjecture 3. $\mathbf{StoppingClass}_1$, $\mathbf{StoppingClass}_3$*, and* $\mathbf{StoppingClass}_6$ *are the only* $\mathbf{StoppingClasses}$ *with* $\mathbf{StoppingModulus} = 1$

Conjecture 4. *For every* $\mathbf{k \geq 8}$*,* $\mathbf{StoppingClass}_k$ *has a unique* $\mathbf{StoppingModulus} \geq 2$*.*

Conjecture 5. *For every* $\mathbf{j \geq 8}$*, if* $\mathbf{k > j}$ *then* $\mathbf{StoppingModulus}_k > \mathbf{StoppingModulus}_j$.

5.2 Parameterizing Stopping Classes

Empirical evidence seems to indicate that for all positive integers $j > 1$ having $\mathbf{StoppingTime}_k$, all $StoppingPoints$ in $\mathbf{StoppingClass}_k$ fall on lines that have the same slope. Below you'll find a table of information on the first 10 **Stopping Classes**, including the slopes of the lines passing through all points in $\mathbf{StoppingClass}_k$

$\mathbf{stop_k}$	$\mathbf{Smod}_k$	Slope
1	1	-1
3	1	-3
6	1	$-\frac{9}{7}$
8	2	$-\frac{27}{5}$
11	3	$-\frac{81}{47}$
13	7	$-\frac{243}{13}$
16	12	$-\frac{729}{295}$
19	30	$-\frac{2187}{1909}$
21	85	$-\frac{6561}{1631}$

Table 6: Slopes of lines passing through $\mathbf{StoppingPoints}$ belonging to $\mathbf{StoppingClass}_k$

There are few a interesting observations to point out from this table.

1. The $\mathbf{stop_k}$ column is a well known sequence: A122437. Sticking with our RAM architecture analogy, it would seem that stopping tim
2. The $\mathbf{Smod_k}$ column is also a well known sequence: A100982.
3. The numbers appearing in the **slope** column are a combination of well known sequences.
   - The numerators are simply powers of 3 A000244.
   - The denominators seem to be the difference between and the next larger or equal power of 3^n and 2. from this series A063003
   - All terms in the sequence appear to be negative.

It is pretty well known that the sequences in items 1 and 2 are related to the Collatz Conjecture. However, I have not come across any literature stating direct connections between the sequence appearing in item 3. This seems to imply there is some relationship connecting **Stopping Classes** and the gaps between the powers of 2 and 3 to the Collatz Conjecutre.

It's fairly obvious to see how powers of 2 effect the Collatz Conjecture. When a power of two turns up within an orbit, the orbit falls directly to 1 in $log_2(n)$ steps. But powers of 3 show no obvious pattern as far as I've explored.

5.3 Descretizing The Slope

Table 6 shows the sequence made from the slopes for the first 10 $\mathbf{StoppingClasses}$. There are well known formulas for the two sequences, but none that are precise. To avoid any heuristic arguments, we want to avoid having to use any rounding functions like *floor*(n) or *ceil*(n). It turns out, there is a way to compute this sequence discretely in the following manner.

1. Let $l =$ the number of digits in the base 2 representation of 3^n.
2. Then $a_n = 3^n - 2^l$

Remarkably, this generates the sequence of denominators of the slopes. It's a clue that there may be a deep connection between the geometry described by $\mathbf{StoppingPoints}$. This could perhaps open the Collatz Conjecture up to being studied by the beautiful field of algebraic geometry.

In the next section, we'll explore the upper and lower bounds of this slopes of lines through **Stopping Classes**

5.4 Finding an Upper Bound for the value of slopes representing Stopping Classes

According to **Lemma 1**, we can pretty easily see that all stopping points for even numbers must fall on the line $\mathbf{y} = -\mathbf{x}$. In the figure below, you'll see the **Stopping Points** for 2, 4, 6, 8, and 10, and the line **y = -x**.

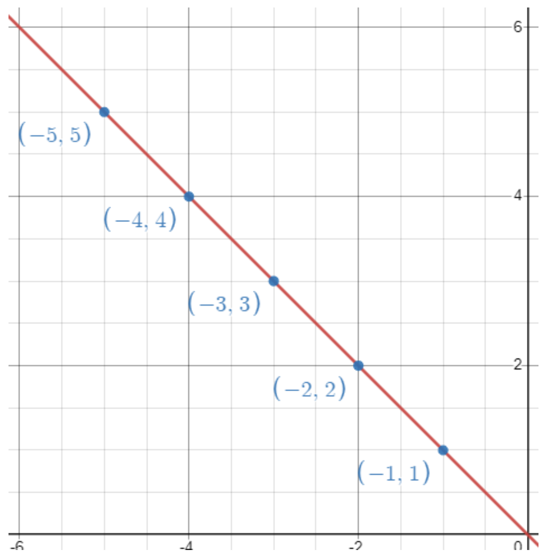

Figure 5: **Stopping Points** for 2, 4, 6, 8, and 10

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In fact, the slope of the line intersecting the **Stopping Points** of even numbers must have the maximum slope allowed for any line representing $\mathbf{StoppingClass}_k$. Remember, these points are found by computing the $\mathbf{StoppingDestination}_n$ if it exists, which must be lower than n by definition. The only way to reach a number lower than n (as per the rules of the Collatz Function), is to divide by two. Since you can only divide by two when you encounter an even number, this means $\mathbf{StoppingDestination}_n$ can only be reached after a "divide by two" operation. This leads to the following observation:

Lemma 2. *The maximum slope for a line representing* $\mathbf{StoppingClass}_k$ *is -1 and belongs to the line passing through the **Stopping Points** of the even numbers.*

5.5 Finding a Lower Bound for the value of slopes representing Stopping Classes

As n increases, intuitively you can think of the the lines as both getting steeper (having a higher negative magnitude value for slope) and moving higher up the y-axis (increasing the value of the y intercept). It's almost like there's a translation of slide + rotate occurring. See Figure 6 for the first 3 **Stopping Point** Lines.

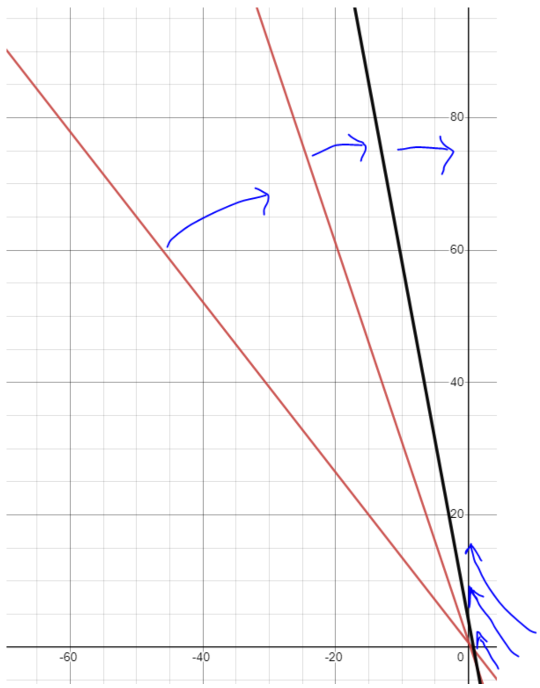

Figure 6: Plot of first 3 $\mathbf{StoppingPoints}$ lines.

If **Conjecture 2** turns out to be true and there are indeed an infinite number of **Stopping Classes**, then it may follow that there are an infinite number of lines that describe their **Stopping Points**! As n grows, we should expect these lines to have decreasing slopes, but higher y-intercepts. As n increases, we're getting closer and closer to mapping $\mathbf{y} = -\mathbf{x}$ onto $\mathbf{x} = \mathbf{0}$ via a rotation about the origin. Perhaps we can employ some tools from calculus or topology to make some definitive statements about the convergence of these two lines.

In the next section we will turn to investigating the **Stopping Points** and see an interesting way we can map each number n to a unique circle on the 2D plane. This is where an unlikely number shows up - the Golden Ratio.

6 Mapping Orbits to Circles

Table 1 shows $\mathbf{StoppingPoints}$ of first 16 odd numbers $\geq n$. I found an interesting property that seems to hold for all $\mathbf{StoppingPoint}_n$ where $n \geq 3$. These points all appear to lie on their own unique circles located in **Stopping Memory**.

Each circle $\mathbf{StoppingCircle}_n$ seems to have the following properties.

1. It is centered at point $(\mathbf{x}, \mathbf{y})$ where $|\mathbf{x}| + |\mathbf{y}| = \frac{n}{2}$
2. It intersects 4 other points.
   (0, 0)

   $(\mathbf{SpointX}_n, \mathbf{SpointY}_n)$

   (0, $\mathbf{SpointY}_n$)

   ($\mathbf{SpointX}_n$, 0)

3. If you order the sequence of y-intercepts of $\mathbf{StoppingClass}_n$ by n, you seem to get this sequence: A122437, which is the sequence built from following the trajectory of $2n + 1$ in the $3n + 1$ problem.
4. The radius of each circle where n is odd seems to be directly related to the hypotenuse of primitive triangles. A008846. For $\mathbf{StoppingCircle}_n$ with radius r, it appears that there is a corresponding primitive Pythagorean triple $a^2 + b^2 = c^2$ where $r^2 = c$. See the table below for the first 8.

$\mathbf{n}$	$\mathbf{r^2}$	Index In Ordered List	a	b	c
3	$\frac{5}{4}$	1	3	4	5
5	$\frac{17}{4}$	3	8	15	17
7	$\frac{29}{4}$	5	20	21	29
9	$\frac{53}{4}$	9	28	45	53
11	$\frac{101}{4}$	19	20	99	101
13	$\frac{109}{4}$	20	60	91	109
15	$\frac{125}{4}$	23	44	117	125
17	$\frac{185}{4}$	26	57	176	185

Table 7: Slopes of lines passing through $\mathbf{StoppingPoints}$ belonging to $\mathbf{StoppingClass}_3$

5. The circles existing in $\mathbf{StoppingClass}_k$ seem to have a strong relationship amongst each other. To illustrate, see figure 7 which shows the first 8 circles in $\mathbf{StoppingClass}_3$.

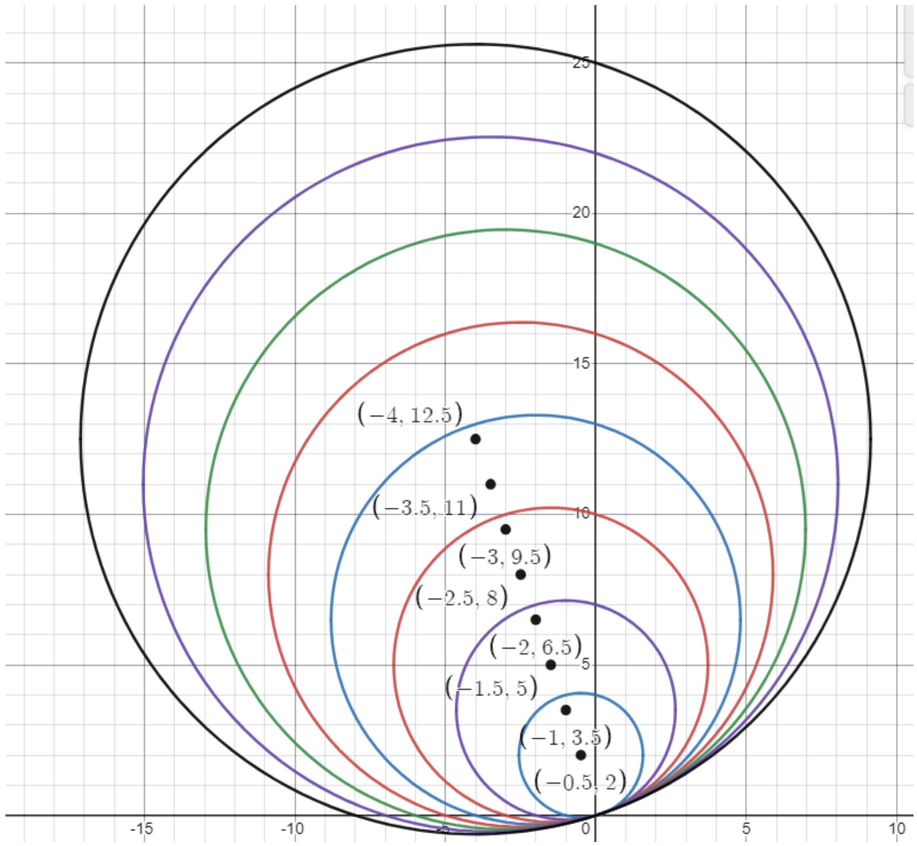

Figure 7: Equations of the first 8 **Stopping Circles** of **Stopping Class**$_3$ listed in table 8.

n	Equation
5	$(x + .5)^2 + (y - 2)^2 = 4.25$
9	$(x + 1)^2 + (y - 3.5)^2 = 13.25$
13	$(x + 1.5)^2 + (y - 5)^2 = 27.25$
17	$(x + 2)^2 + (y - 6.5)^2 = 46.25$
21	$(x + 2.5)^2 + (y - 8)^2 = 70.25$
25	$(x + 3)^2 + (y - 9.5)^2 = 99.25$
29	$(x + 3.5)^2 + (y - 11)^2 = 133.25$
33	$(x + 5)^2 + (y - 12.5)^2 = 172.25$

Table 8: Equations of the circles in figure 7

6. If you parameterize the x coordinate of $\textbf{StoppingCircle}_3$, which is the smallest $\textbf{StoppingCircle}$ and first odd prime value with a **Stopping Time**, you end up with the equation:

$$x = -.5 + \sqrt{\frac{5}{4}} \times \cos(\theta)$$

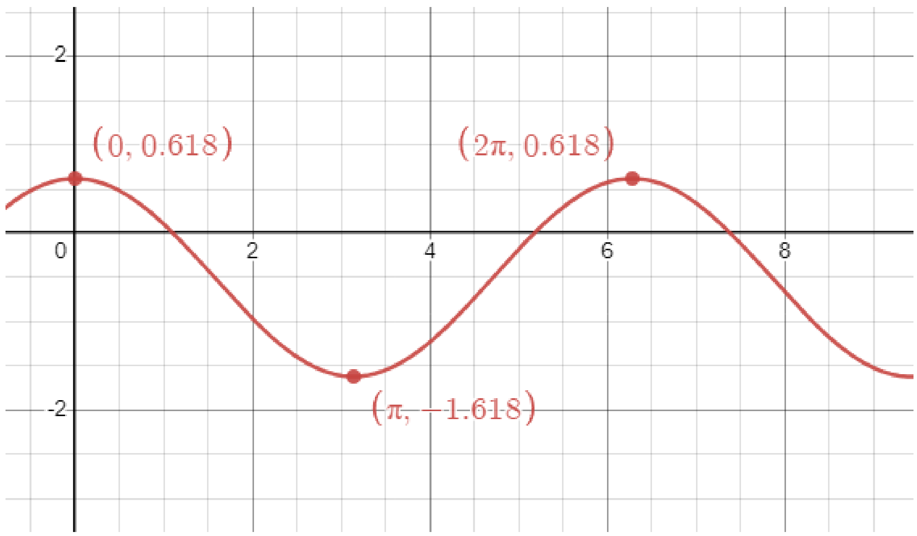

Figure 8: Parameterization of **Stopping Circle**$_3$ listed in table 8.

There are a few remarkable facts about this function related to the Golden Ratio $\phi \approx 1.618...$.

(a) The amplitude of the sin wave produce by this function is equal to $(2 \times \phi) - 1$

(b) The sin wave produced by this function oscillates between between a maximum value of $\phi - 1$ and a minimum value of $-\phi$.

This is such a beautiful connection between two of the most popularly known transcendental numbers. This link seems like it would be a very good explanation for some of the behaviors we see in the Collatz Function. I have my suspicions that we may be able to connect the Collatz Function to Penrose Tilings. Collatz Orbits seem to traverse the 2D plane without repeating, similarly to the way Penrose Tilings cover the plane without repeating any patterns.

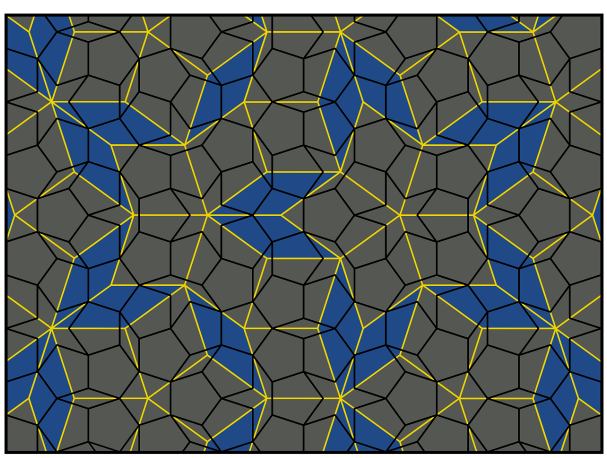

Figure 9: Example of the pentagonal Penrose tiling (P1)

7 Conclusion and Follow Up Questions / Topics

In this paper I've tried to show how we might be able to examine the Collatz Conjecture from a geometric perspective. Though most of the ideas in this paper aren't proven, I've empirically tested them up to the first 1 million integers. I can't help but think all of the pieces to either prove or disprove the Collatz Conjectre (and maybe even others) are related to the geometry described by the constructions laid out in this paper. In closing, I will pose a series of speculative questions about some of the ideas I've presented. I look forward to diving into these ideas further!

1. Could we better describe **Collatz Memory** and the relationship between **Stopping Points** if we use the complex plane rather than a Cartesian plane? The slide and rotation action we see in the slops of **Stopping Lines** makes me think this could be modeled better by complex numbers.
2. Each $\mathbf{StoppingPoint}_n$ appears at a single location in **Collatz Memory**. This memory space seems to be addressed almost *exactly* like memory in the RAM of modern computers. Could we apply the patterns we see in Collatz to lay out programs more efficiently in memory in an effort to avoid fragmentation?
3. As n increases, the slopes of $\mathbf{StoppingLines}_3$ seem to go off to negative infinity. However, they do so in a pattern intricately described by the Collatz orbits of the integers. Could we use tools from Calculus to study the behavior we see here?
4. Could the connection between ϕ and $\mathbf{StoppingCircle}_3$ indicate there is some kind of relation to the harmonic series, but closely related to ϕ? The Fibonacci sequence comes to mind.
5. I find the connection to solutions of linear Diophantine equations to be incredibly interesting. Are there tools from group theory that we can use to study the symmetry of **Stopping Classes**?
6. I have a hunch that the **Stopping Circles** may be related to the Riemann Zeta Function. Could some of the ideas in this paper help us get closer to shedding light on the Riemann Hypothesis? Is there a way in some sense that we could combine sum of all **Stopping Circles** in a way similar to the Harmonic Series? If we did, would we see the same behavior we see in the Riemann Zeta Function on the domain of complex numbers?

8 Inspiration and Information

I won't do an official list of sources at this time, but just in case some of the ideas do hold water, I would like to call out some people whose work I enjoy and who undoubtedly introduced me to new ways of thinking about science. The concepts in this paper came from combining a bunch of ideas from different areas of mathematics, so I'm sure I took inspiration from all of these folks at one point or another. I'm a huge supporter of anyone who is creating educational content that gets people excited about how the world works.

I know many are missing from this list, but here they are in no particular order.

- **Edward Frenkel** - I've listened to numerous lectures of his, and his book "Love and Math" really got me interested in the Langland's Program. The way he speaks of the beauty of mathematics really resonates with me.
- **John Conway** - I became fascinated with Conway's Game of Life in my teens. The book he coauthored called "The Symmetries of Things" is also one of my favorites.
- **Jeffrey Lagarias** - Though I don't understand all of the material in his book "The Ultimate Challenge: The 3x+1 Problem", it has been the best resource to learn about attempts at tackling this problem.
- **Robert Penrose** - His ideas about tilings gave me the motivation to look at the Collatz Conjecture as if it was a physical space.
- **Derek Muller** - I've watched so many videos on Derek's channel called Veritasium. I actually used his video to introduce the 3x+1 problem to my family!
- **Jade Holmes** - I've been watching her channels for years and always get excited when a new video is released. She's one of the first educational content creaters I started watching on YouTube.
- **Andreas Holmstrom** - He runs a small educational community called Peak Math where he leads an exploration into the mysteries of L functions. The content he produces is amazing, and the community he's put together is a lot of fun to be a part of.
- **Brady Haran** - Numberphile is probably my most watched channel. I can't tell you how many hours of content I've watched off of that channel.
- **Grant Sanderson** - 3Blue1Brown is my go to channel for if I want to try to intuitively understand a topic. Grant's way of producing content in a visual way has helped me grasp so many concepts that would've been out of reach with text alone.