**cubic ellipsoid nucleus: a summary of the research**

Ronen Yavor

This document brings together six articles that describe the development of the cubic ellipsoid nucleus theory.
It begins with the hypothesis that the structure of the nucleus determines the shape and properties of the atom and offers a model consistent with this idea.
The model does not seem to contradict the nuclear or atomic theory, but rather expands them and offers some new perspectives that could further develop these theories and other fields of physics. The theoretical and experimental data were in good agreement and the model predicts various nuclear phenomena.
We don't get complete proof of the theory, but there are several results that suggest this could be the right direction.

The achievements of the model

- The model: a visual illustration of the nucleus that connects it with the atom.
  - a justification of the structure of the periodic table: electron shells and energy levels, angular momentum, orbitals and sub-orbitals.
  - delivers the right number of protons and neutrons for the various nuclei and the correct total nuclear spin.
- The mass formula: uses a preliminary simplified theoretical mass formula adapted to the model (rather than semi-empirical).
  - calculation results agree with experimental data:
    - mass of nuclei.
    - sum of neutron and proton radius.
- The charge radius: reinforcement to the assumption that the excess neutrons are located in the nucleus envelope.
  - H and He radii are explained and calculated.
  - noble gas radii are estimated.
- Stable nuclei in dependency on the excess neutron population.
- Radioactivity: prediction of the starting points of instability in heavy and superheavy nuclei.
- Nuclear fission: explanation of the mechanism and the prediction of the expected fragments.

Attached are the research articles

**cubic ellipsoid nucleus - part 1: the model and its mass formula**

**Summary**

Ronen Yavor

**Abstract**

This paper examines the hypothesis that the structure of the nucleus determines that of the atom and its properties and attempts to construct a geometric model of the nucleus that contributes to this hypothesis.
The model proposed here suggests that the structure of the nucleus is, in general, an ellipsoid with the nucleons connected by cubic bonds and the nucleus shells correlate with those of the atom.
In accordance with the model, a preliminary simplified theoretical mass formula was created to compare it with the experimental data; the test included about 90 nuclei from $\mathrm{Ar}_{18}^{40}$ to $\mathrm{Pu}_{94}^{244}$.
The mass formula depends on two terms:

- $\mathrm{E_b}$: the total binding energy between the nucleons in the nucleus.
- $\mathrm{E_c}$: the total electric energy of the nucleus.

and has two parameters:

- $d_0$: the minimum distance between two neighboring nucleons in the cubic structure of the nucleus.
- $e_b$: the binding energy between these neighboring nucleons.

The results for the calculation parameters were:

- $d_0$ = 1.62±0.03 fm
- $e_b$= 5.72±0.03 Mev

The results for the relative errors of the mass formula calculation were:

| relative error | maximum | average | standard dev. |
|---|---|---|---|
| | 1.9% | 0.6% | 0.5% |

If we consider the nucleons for simplicity as rigid bodies, then we get a rough estimation for $d_0$ through the radii of the proton and neutron:

- $r_n \approx$ 0.80 fm, $r_p \approx$ 0.84 fm, $d_0 \approx \left(r_n + r_p\right)$ =1.64 fm
- *relative deviation of* $d_0$: $\left|\frac{d_0 - (r_n + r_p)}{(r_n + r_p)}\right| < 2\%$

These results for the mass calculation and $d_0$ strengthen the model assumptions.

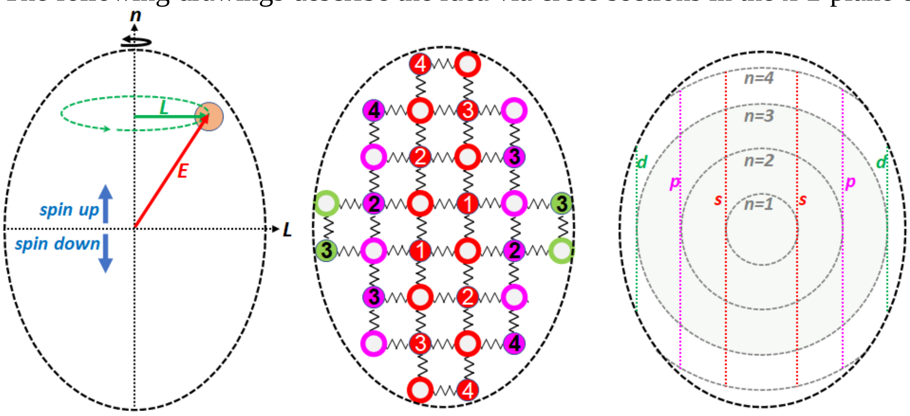
**Content**

## The model at a glance

According to the model these are the shape and properties of the nucleus:

- the nucleus has an ellipsoid shape.
- the nucleons bonds have a cubic form.
- protons are connected to neutrons (**p-n**).
- neutrons are connected mainly to protons.
- the protons are populated and organized in shells in the nucleus in a complete analogy to those of the electrons in the atom.
- the energy layers (principal quantum number **n**) grow along the **z**-axis of the nucleus in its both directions (more precisely **n** grows with its distance from the origin).
- the perpendicular distance from the **z**-axis in the **x-y**-plane reflects the angular momentum (**L**) and so the orbitals.
- the upper half of the ellipsoid is referred to as spin-up and the lower part as spin-down.
- the nucleus possibly rotates around its **z**-axis.

The following drawings describe the idea via cross sections in the **x-z**-plane of the nucleus.




***1**: a nucleon in the nucleus*   ***2**: the bonds between the nucleons*   ***3**: the energy levels of the nucleus*

1. a nucleon (**circle**) is observed inside the ellipsoid (dashed line) that encloses the nucleons and schematically defines the nucleus surface:
    - the distance from the origin represents its energy **E**.
    - the distance from the **z**-axis depicts it angular momentum **L**.
    - the nucleons in the upper half have spin up, and in the lower one spin down.
2. the bonds between the nucleons are shown for visibility as springs.
    - **protons**: full circles of the **s**, **p** and **d** sub-orbitals. **neutrons**: hollow circles.
3. the circles of equal energy states **n** in the ellipsoid.
    - the lines mark the development of the **s**, **p** and **d** sub-orbitals along the **z**-axis.
    - the **s** line crosses all **n** circles from 1 to 4 (**s1** to **s4**).
    - the **p** line begins by **n**=2 and reaches till **n**=4 (**p2** to **p4**).
    - the **d** line begins by **n**=3 and reaches the ellipsoid border, before it reaches the **n**=4 circle, and therefore there are no **d4** states at this stage (only **d3**).

## Introduction

The nucleus and the atom are governed by different forces, have a size difference of about 5 orders of magnitude and according to current physics the order of the nucleus in shells is different than that of the atom [10].
The hypothesis, that this research investigates, is that the structure of the nucleus determines the one of the atom; therefore an attempt was made to find a geometric model that could describe this and, at the same time, meet the requirements and constraints of the current theories of nuclear and atomic physics to justify this new perspective without contradictions.
The starting point was that the hypothesis holds and so, in the opposite direction, it is possible to learn and deduce from the atom about the structure of the nucleus.
Once such a model was obtained it was tested and compared with experimental data.
The methods used in this work to analyze the nucleus are essentially those of classical physics.

## Requirements

### The nucleus shape

- The structure of the nucleus shall "make sense" physically.
- The nuclear density (meaning the distance between two neighboring nucleons) is assumed to be (at least nearly) constant and the structure of the nuclear bonds is homogeneous and periodic.
- A proton is connected only to neutrons (**p-n** bond) because we assume that the **p-p** bond has a too strong electric repulsion; otherwise we could expect to observe a stable $He_2^2$ atom for instance.
- A neutron is preferably connected with protons (**p-n** bond) because it is assumed that the proton stabilizes the neutron and that the **n-n** bond alone (with no protons involved) is instable; otherwise we could expect to observe a stable **n-n** nucleus.

### Reflection of the atom properties

If the nucleus influences the atom, then it should reflect the atomic structure:

- the atomic energy levels or shells.
- the orbitals and sub-orbitals and their population sequence.
- the correct number of neutrons for each isotope.
- the total nuclear spin.
- Pauli's exclusion principle.
- Hund's rules of electronic states population may apply similarly to protons.

### Comparison with experimental data

- A preliminary simplified theoretical mass formula suitable for the model shall be constructed.

## Results

### The model

We get the following model, which is developed and explained in detail below:

- The structure of the nucleus:
    - the nucleus is in general an ellipsoid.
    - it is composed of nucleons connected in a cubic system.
    - a proton is connected to neutrons.
    - a neutron is preferably connected with protons.
    - the excess neutrons, beyond the number equal to that of the protons, are in the envelope of the ellipsoid.
- Properties:
    - the energy levels grow along the **z**-axis in both directions (more precisely with their distance from the origin). *
    - the perpendicular distance from the **z**-axis (i.e. in the **x-y**-plane) depicts the angular momentum (and so the orbitals). *
    - the upper side of the ellipsoid is arbitrarily defined as spin-up and the lower part as spin-down. *
    - the model assumes that the nucleus possibly rotates around its main axis (the **z**-axis).*
- The model achieves the following:
    - the layers of the nucleus correlate with those of the atom.
    - the model justifies the electron shells, the energy levels, the orbitals and sub-orbitals and so explains the structure of the periodic table.
    - the model delivers the right number of protons and neutrons in the nucleus and the correct nuclear spin.
    - the model doesn't contradict Pauli's exclusion principle.
    - like in the atomic physics the population sequence of the protons is possibly according to Hund's rules in the range where the electronic states follow the L-S coupling.*
- Examining the model:
    - the ellipsoid shape makes sense physically.
    - a theoretical mass formula was created and gave good results:
        - nuclei mass with an average relative error <1%.
        - combined radii of proton and neutron with a relative error <2%.

* Topics that are not essential to the first study and do not contradict the model, but help in its development and construction. They shall be developed in following studies in order to expand and establish the model.

**The Mass formula**

A preliminary simplified theoretical mass formula was developed to match the model and test its feasibility:

$m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$ experimental data from. [1]

- $m_{calc_x}$: the calculated mass of the nucleus x.
- $Z_x$: the atomic number of the nucleus x (number of protons).
- $m_p$: the mass of the proton.
- $N_x$: the number of neutrons in the nucleus x (number of nucleons $A_x$ minus $Z_x$).
- $m_n$: the mass of the neutron.
- $E_{b_x}$: the total energy of the nucleon bonds in the nucleus x.
- $E_{c_x}$: the total electric energy (between all protons) in the nucleus x.
- $c$: the speed of light.

The binding energy of the nucleus is:

$E_{b_x} = e_b \cdot n_{b_x}$

- $e_b$: the energy of a single nucleon-nucleon bond in the nucleus (assuming they are equal for all bonds in all nuclei).
- $n_{b_x}$: the number of nucleon-nucleon bonds in the nucleus x.

The electric energy of the nucleus is:

$E_{c_x} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0} \left\{ \frac{1}{2} \sum_i^{Z_x} \sum_{j \neq i}^{Z_x} \frac{1}{d_{i,j}} \right\} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0} e_{c_x}$ where $e_{c_x} := \frac{1}{2} \sum_i^{Z_x} \sum_{j \neq i}^{Z_x} \frac{1}{d_{i,j}}$

- $d_0$: the minimum distance between two neighboring nucleons in femtometer (assuming all nuclei have the same cubic structure and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices *i* and *j* measured in multiples of $d_0$: $d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$
- $e_{c_x}$: the unitless total electric energy of the nucleus (sum of the reciprocal distances).

The absolute relative error of the calculation for the nucleus is:

$rel\_err_x = \left| \frac{m_{calc_x} - m_{meas_x}}{Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}} \right| = \left| \frac{m_{calc_x} - m_{meas_x}}{mass\_defect_x} \right|$

- $m_{meas_x}$: the measured mass of the nucleus x.
- $rel\_err_x$ is represented here in percentage.
- $mass\_defect_x$: $Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}$ is the mass defect of the nucleus x.

The mass formula, in this simplified form, depends thus only on the two variables:

- $e_b$: the energy of a single nucleon-nucleon bond.
- $d_0$: the minimum distance between two neighboring nucleons.

The implementation requires two preparation calculation steps for all nuclei:

- Drawing the nucleus and counting the number of nucleon-nucleon bonds $n_{b_x}$.
- Calculating the relative total energy of the nucleus $e_{c_x}$ (sum of reciprocal distances).

## Results of the mass formula calculations

This section discusses the relative error of the mass formula calculation depending on the binding energy, $e_b$, and the distance between two neighboring nucleons, $d_0$, for 120 nuclei of the common isotopes of the elements from $Li_3$ to $Pu_{94}$ (for several elements more than one isotope was taken).
The lighter nuclei till approximately $Ar_{18}$ have larger relative errors than those of larger nuclei and are therefore shown in a different table.
The results of the mass formula calculation for 94 nuclei from $\mathrm{Ar}_{18}^{40}$ to $\mathrm{Pu}_{94}^{244}$:

| maximum | average | st. dev. | ≤ 2% * | ≤ 1% | ≤ 0.5% |
|---|---|---|---|---|---|
| 1.9% | 0.6% | 0.5% | 100% | 82% | 60% |

* the amount of nuclei with relative error smaller than or equal to 2%.

- $e_b$ = 5.72±0.03 Mev
- $d_0$ = 1.62±0.03 fm

these values are within a reasonable range [5].

If we consider the nucleons for simplicity as rigid bodies, then we get a rough estimation for $d_0$ through the radii of the proton and neutron: $r_n$ [3] *(Neutron radius)*, $r_p$ [4] *(Proton radius)*: $d_0 \approx (r_n + r_p)$.
Setting these values we get a result within a reasonable range:

- $r_n \approx 0.80$ fm, $r_p \approx 0.84$ fm, $d_0 \approx (r_n + r_p)$ =1.64 fm
- *relative deviation for* $d_0$: $\left|\frac{d_0 - (r_n+r_p)}{(r_n+r_p)}\right| = |\frac{1.62 - 1.64}{1.64}| \approx 1.3\%$

This estimation could strengthen the hypothesis of the model.

Following table shows the results of the mass formula calculation for 28 nuclei from $\mathrm{Li}_3^6$ to $\mathrm{Ar}_{18}^{36}$ at $e_b$ = 5.72MeV, $d_0$ = 1.62 fm (as found for the nuclei from $\mathrm{Ar}_{18}^{40}$):

| maximum | average | st. dev. | ≤ 3% | ≤ 2% | ≤ 1% |
|---|---|---|---|---|---|
| 7.1% | 2.4% | 1.9% | 79% | 54% | 29% |

The larger relative error of these lighter nuclei shall be analyzed by future research.

## Discussion of the results and conclusion

The theory of the cubic ellipsoid nucleus offers a different perspective that doesn't contradict current physics, but could expand its understanding and open new research direction, not only in nuclear physics, but also in atomic and possibly other fields.

The achievements of the model were:

- a tangible geometric shape for the nucleus and the connection between the structure of the nucleus and the atom.
- a reflection the structure of the periodic table in terms of the shells, the number protons and neutrons for each isotope, and qualitatively for the energy levels and orbitals; it was built that way from the beginning, but here it was shown to be possible.
- a preliminary simplified theoretical mass formula, that relates directly to the theory, rather than being semi-empirical as the common one. [11]
- the distance $d_0$ between two neighboring nucleons agrees very well with the sum of the neutron and proton radii; this strengthens the model assumption and the concept of the mass formula.
- the chemical properties of an atom are independent of its isotopes; we therefore assume that its protons have the same spatial structure for all of its isotopes; this justifies the model assumption, that the excess neutrons are located in the envelope and so leave the proton positions unchanged. we discuss this in following research.
- the model delivers the correct total nuclear spin.
- Pauli's exclusion principle holds (especially if we assume a rotation of the ellipsoid).
- The arrangement of the nucleus according to Hund does not contradict the symmetry requirement, nor does it change the number of bonds in the nucleus, but we don't discuss it further in this research.

Light nuclei (below Argon and especially below Nitrogen) have a larger deviation from the mass formula calculation; the reason is assumed to be their structure that is not perfectly cubic or their density, meaning the distance between neighboring nucleons, that are slightly larger than their value by "well ordered" nuclei.
Further research shall consider this.

**Sources and references**


1. *Tables of Nuclear Data: Japan Atomic Energy Agency (JAEA)*
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3. *Neutron radius: Povh, B.; Rith, K.; Scholz, C.; Zetsche, F. (2002). Particles and Nuclei: An Introduction to the Physical Concepts. Berlin: Springer-Verlag. p. 73*
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**cubic ellipsoid nucleus - part 2: model improvement - excess neutrons**

**Summary**

Ronen Yavor

**Abstract**

In the former paper we constructed the model and found that the excess neutrons shall be found in the ellipsoid envelope.
In this work we try to find more precisely at what positions these excess neutrons are located, by expanding the drawings to more than 300 stable nuclei and optimizing their calculation while fixing the mass formula parameters at the values found in the last research [2] and at the same time requiring the correct total nuclear spin.

**Content**



**Introduction**

The results of the mass formula calculations [2] delivered two parameters:

- $d_0$: the minimum distance between two neighboring nucleons in the cubic structure of the nucleus.
- $e_b$: the binding energy between these neighboring nucleons.

and their best values were found to be:

- $d_0 = 1.62\,fm$
- $e_b = 5.72MeV$

According to the model the protons and their paired neutrons have fixed positions.
Unlike these, the excess neutrons can occupy different positions at the envelope of the ellipsoid; the neutrons have no influence on the electric energy, but they change the total binding energy and so the results of the mass formula.
At this point we assume that the results for $e_b$ and $d_0$ are correct and take the opposite direction:
we improve the mass formula calculation results by varying the positions of the excess neutrons, while keeping $e_b$ and $d_0$ fixed and ensuring the correct value of the total spin.
This way we aim to better understand were in the envelope and at what sequence the excess neutrons are populated.

**Improved calculation by iterations and spin consideration and results**

The steps of the process were as follows:

1. drawing more than 300 stable nuclei.
2. changing the drawings, if needed, to fit the spin.
3. calculating the mass formula.
4. while the parameters of the mass formula are kept fixed.
5. varying the positions of the excess neutrons.
6. better understanding the logic of the nucleus composition and returning possibly to point 2 and changing other nuclei in accordance with the knowledge gained.

The results of the first mass formula calculation for 82 stable nuclei from $Ar_{18}^{40}$ to $Pb_{82}^{208}$:

| maximum | average | st. dev. | ≤ 2% * | ≤ 1% | ≤ 0.5% |
|---|---|---|---|---|---|
| 1.9% | 0.6% | 0.5% | 100% | 78% | 60% |

* the amount of nuclei with relative error smaller than or equal to 2%.

After the improvement process the results for 296 stable nuclei from $Ar_{18}^{40}$ to $Pb_{82}^{210}$ were:

| maximum | average | st. dev. | ≤ 2% | ≤ 1% | ≤ 0.5% |
|---|---|---|---|---|---|
| 2.0% | 0.4% | 0.4% | 100% | 92% | 68% |

and even when expanding the range to 327 stable nuclei from $N_{7}^{14}$ to $Pb_{82}^{210}$ we get:

| maximum | average | st. dev. | ≤ 2% | ≤ 1% | ≤ 0.5% |
|---|---|---|---|---|---|
| 2.9% | 0.5% | 0.5% | 98% | 88% | 64% |

so we consider this as an improvement. In the next research we use these results to set the population rules for the excess neutrons of the stable nuclei.

**Sources and references**


1. *Tables of Nuclear Data: Japan Atomic Energy Agency (JAEA)*
2. *cubic ellipsoid nucleus - part 1 - the model and its mass formula*

**cubic ellipsoid nucleus - part 3: charge radius of light nuclei and noble gases**

**Summary**

Ronen Yavor

**Abstract**

In this paper we analyze the charge radius of the nucleus in the light of the cubic ellipsoid geometric model. [5]
The goal is to verify the model and its assumption that the excess neutrons are located in its envelope and expand its understanding and thus possibly also to gain new insights from it. The results match the experimental data quite well and strengthen so the model assumption. We also raise some new hypotheses as a conclusion of the calculations regarding the density of the nucleus, that might increase with the number of nucleons until it reaches a finite value in the vicinity of Argon.

**Content**

**Introduction**

The subjects we deal with in this paper are:

- The charge radius of hydrogen and helium show a large deviation of their size from the expected value; we try to explain this.
- An estimation for the charge radius of the noble gases was implemented and the results seem to support the model.
- According to the model the excess neutrons (the unpaired ones that their number exceeds the number of protons) are located on the nucleus envelope; we test this through a comparison between $\frac{\mathbf{R_c}}{\sqrt[3]{\mathbf{A}}}$ and $\frac{\mathbf{R_c}}{\sqrt[3]{\mathbf{2 \cdot Z}}}$ ; the results strengthen the model assumption. The notation is:
    - $A$: the atomic mass (the number of nucleons).
    - $Z$: the atomic number (the number of protons).
    - $R_c$: the (measured) charge radius.

We use the following data for the calculations:

- $d_0 \approx (r_n + r_p)$: the distance between two neighboring nucleons in the nucleus.
- $d_0$ = 1.62 fm [5]
- $r_n$ = 0.80 fm: the neutron radius. [3]
- $r_p$ = 0.84 fm: the proton radius. [4]

We define *single layer*: a layer with sub-orbital that appears only once in a nucleus; for instance in Neon the second layer is a *single layer*, because of the P sub-orbital. If this occurs in two different subsequent layers, we call it a *double layer*; for instance in Argon the third layer is a *double layer*, because of the P sub-orbitals.
Additional examples:
Single layers:

- layer 4 by Krypton (a single D sub-orbital)
- layer 6 by Radon (a single F)

Double layers:

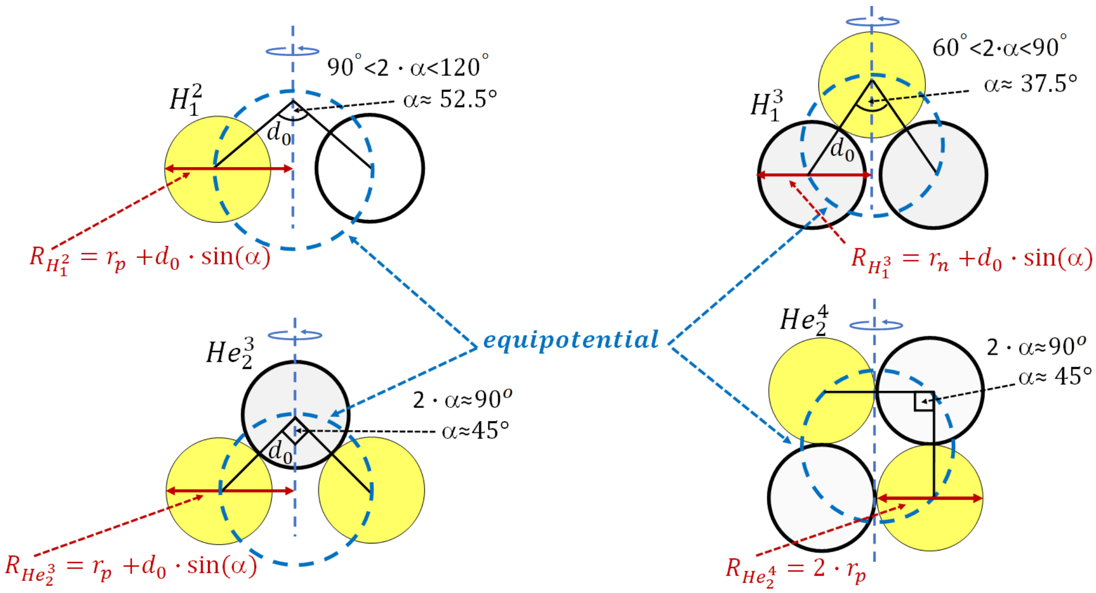
- layer 5 by Xenon (two D orbitals)
- layer 7 by Oganesson (two F orbitals)

## Results

### Hydrogen and Helium: charge radii

The charge radii of hydrogen and helium decrease as the number of their nucleons increases; we relate this to the total nuclear force that is increasing with the number of nucleons for these nuclei and so also their density.
Following drawings explain this idea:

- $\mathrm{He}_2^4$: the nucleon bonds have an angle of about $2 \cdot \alpha = 90°$ and so the charge radius is: $R_{\mathrm{He}_2^4} \approx 2 \cdot r_p$.
- $He_2^3$: the 90° angle remains unchanged, but to keep symmetry with respect to the z-axis, the nucleus it turned in 45°, resulted in: $R_{He_2^3} \approx r_p + d_0 \cdot \sin(45°)$.
- $H_1^3$: the nucleus is very similar to $He_2^3$, but the protons and neutrons are swapped, so the two neutrons are a bit nearer, due to the lack of the electric repulsion. We estimate that the angle is between that of a right-angled and an isosceles triangle. This means an angle of $60° < 2 \cdot \alpha < 90°$ ($\alpha \approx \frac{30+45}{2} = 37.5$).
  We get: $R_{H_1^3} \approx r_n + d_0 \cdot \sin(37.5°)$.
- $H_1^2$: we treat the nucleus in a similar manner to the last two nuclei, although the top nucleon is missing; we estimate the angle therefore to be slightly larger than that of $\mathrm{He}_2^4$, because of the smaller total attraction between the nucleons, so our estimation is $90° < 2 \cdot \alpha < 120°$ ($\alpha \approx \frac{45+60}{2} = 52.5$), and we get: $R_{H_1^2} \approx r_n + d_0 \cdot \sin(52.5°)$.
- **Remark**: we assume that the centers of the nucleons lie on an equipotential circle.

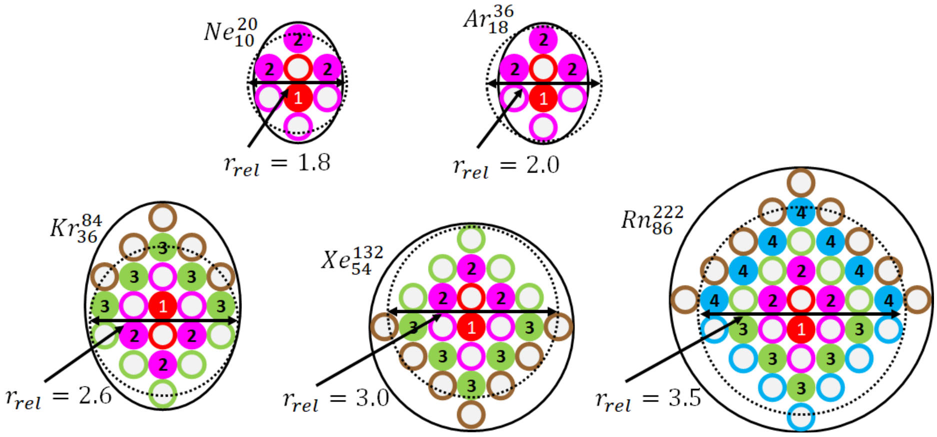



| nucleus | $R_{calc}$ | $R_{meas}$ | rel. error | formula |
|---|---|---|---|---|
| $H_1^2$ | 2.13 | 2.14 | 0.8% | $r_p + d_0 \cdot \sin(52.5°)$ |
| $H_1^3$ | 1.79 | 1.76 | 1.8% | $r_n + d_0 \cdot \sin(37.5°)$ |
| $He_2^3$ | 1.99 | 1.97 | 1.0% | $r_p + d_0 \cdot \sin(45°)$ |
| $He_2^4$ | 1.68 | 1.68 | 0.3% | $2 \cdot r_p$ |

Data of the charge radius: [2].
**Remark**: the results agree with the experimental data, but we note that this is only a rough estimation and not necessarily a proof of the model.

## Noble gases: charge radii

The noble gas nuclei of **Ne**, **Ar**, **Kr**, **Xe** and **Rn** possess according to the model complete shells, so the estimation of their charge radius is possibly easier. About Oganesson there is not enough experimental data.
The charge radius of the noble gasses is calculated in the x-y plane; a circle is drawn from the center of the nucleus till the "nearest" outside proton edge. This is a rough estimation, but it gives good results.
A double layer is slightly wider than a single layer; (the reason is not understood by the model; it could be that the excess neutron mix a bit with the proton, but other reasons are possible as well; for instance due to rotation or precession of the nucleus;) therefore, Argon is wider than Neon; Xenon is wider than Krypton and in addition there are extra neutrons in its envelope that increase the radius.




| nucleus | $R_{rel}$ | $R_{calc}$ | $R_{meas}$ | rel. error |
|---|---|---|---|---|
| $Ne_{10}^{20}$ | 1.8 | 2.92 | 3.01 | 3.0% |
| $Ar_{18}^{36}$ | 2.0 | 3.24 | 3.39 | 4.4% |
| $Kr_{36}^{86}$ | 2.6 | 4.21 | 4.19 | 0.6% |
| $Xe_{54}^{132}$ | 3.0 | 4.86 | 4.79 | 1.5% |
| $Rn_{86}^{222}$ | 3.5 | 5.67 | 5.69 | 0.4% |

Data of the charge radius [2]

- $R_{meas}$: the measured radius.
- $r_{rel}$: relative radius, the number of nucleons contained in the radius as taken from the drawing.
- $R_{calc}$: the calculated radius: $R_{calc} \approx r_{rel} \cdot d_0$.
- $d_0$: the radius of proton + the radius of the neutron. $d_0 = r_p + r_n = 1.62\ fm$ [5].

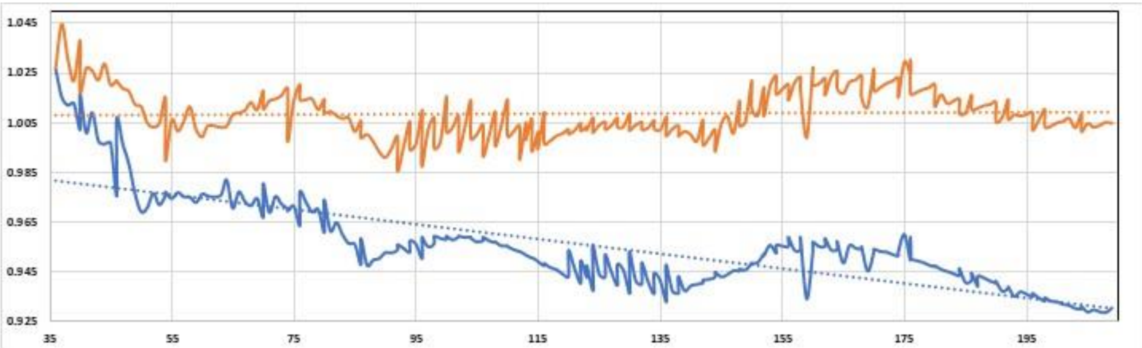
## The charge radius

According to the liquid drop model an approximately constant value would be expected for the ratio between the nucleus charge radius $\mathbf{R_c}$ and the third root of **A**, the atomic mass or the number of nucleons $\sqrt[3]{\mathbf{A}}$. We expect therefore to get $\frac{\mathbf{R_c}}{\sqrt[3]{\mathbf{A}}} \approx$ **constant**.

In reality this ratio decreases as the number of nucleons grows. This could mean a change of the nucleus density or nucleons distribution in the nucleus for larger nuclei or that the nucleus charge is concentrated toward its center by p-n pairs and the excess neutrons are located in its envelope, as the model assumes.

The number of nucleons **A** would therefore better be replaced by (twice) the number of protons **2·Z** (assuming that the number of neutrons is not smaller than that of the protons).

The following graph shows this by comparing between $\frac{\mathbf{R_c}}{\sqrt[3]{\mathbf{A}}}$ and $\frac{\mathbf{R_c}}{\sqrt[3]{\mathbf{2 \cdot Z}}}$ for nuclei from $Ar_{18}$ up to $Cm_{96}$ vs. **Z** (for nuclei smaller than $Ar_{18}$ the number of protons and neutrons is quite equal so there is no major difference between the two).




*Graph: a comparison between* $\frac{R_{measured}}{A^{1/3}}$ *and* $\frac{R_{measured}}{(2 \cdot Z)^{1/3}}$ (*raw data from* [2]**)***.*

*Dotted lines: linear fit.*

The following table summarizes the calculations of the above data.

| | $\frac{R_{measured}}{A^{1/3}}$ | $\frac{R_{measured}}{(2Z)^{1/3}}$ |
|---|---|---|
| max | 1.027 | 1.045 |
| min | 0.928 | 0.985 |
| Δ = max-min | 0.098 | 0.059 |
| average | 0.955 | 1.009 |
| standard dev. | 0.018 | 0.010 |

The result for $(2 \cdot Z)^{\frac{1}{3}}$ fits the data better than the one for $A^{\frac{1}{3}}$. The reason is that the protons and neutrons build a core with an equal number of both, and the excess neutrons are located in the envelope of the nucleus, and so don't influence its charge distribution.

Data of the charge radius [2].

## Discussion of the results and conclusion

The results strengthen the model, yet no decisive proof was delivered. The conclusions are:

- The charge radius results for hydrogen and helium are well explained by the model.
- The results for the noble gas nuclei also strengthen the assumption of the model.
- The excess neutrons of the nuclei, beyond the number equal to that of the protons, seem to be located in its envelope, as the model predicts.

## Sources and references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4. *Proton radius: Yong-Hui Lin, Hans-Werner Hammer and Ulf-G. Meißner: New insights into the nucleon's electromagnetic structure; Physical Review Letters, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.128.052002*
5. *cubic ellipsoid nucleus - part 1 - the model and its mass formula*

**cubic ellipsoid nucleus - part 4: nuclear stability and excess neutrons - closed sub-orbitals**

**Summary**

Ronen Yavor

**Abstract**

This paper analyzes the sources of nuclear stability according to model [5] in dependency on the number of excess neutrons (neutrons beyond the number equal to that of the protons). Although the research included all nuclei up to Lead (Pb), the focus here is on nuclei with closed sub-orbitals, because this simplifies the analysis; further research will expand the discussion to nuclei with even number of protons and then to all nuclei.

According to the model the nuclei are built up of layers and each of them has a population range of excess neutrons, in which the nucleus is stable.

If the number of these excess neutrons is below the minimum for this range then there is lack of neutrons that stabilize the proton bonds and a proton emission occurs. [4]
Above the maximum number of neutrons for the range (neutrons excess) a neutron emission occurs due to lack of protons that stabilize the neutrons. [3]

**Remark**: these statements regarding the mechanisms that cause proton or neutron instability and their emission are not part of this work.

The population range of the excess neutrons depends on the number of layers of the nucleus and the protons population of each specific layer.
This is the focus of this work.

The results of this research are guidelines for the population of stable nuclei with excess neutrons in a similar manner to Hund's rules in atomic physics (for the electron population of light atoms).

**Content**

## Introduction

In this research we want to deepen the understanding of the model and expand it by investigating the nucleus stability in dependency on the number of excess neutrons in it.
The number of neutrons has two limits:

- lower limit: below it a lack of neutrons leads to proton emission. [4]
- upper limit: beyond which the excess of neutrons leads to neutron emission. [3]

Elements larger than Lead (Pb) don't have stable nuclei at all and therefore other mechanisms shall be considered in addition to the lack or excess of neutrons; these will be discussed in following research.
We observe here mainly nuclei of full sub-orbitals (s, p, d, f) in order to simplify the discussion and make the idea clearer. Further works shall expand the discussion to nuclei with even number of protons and then to all nuclei.
According to the model the core of the nucleus is built from an equal number of protons and neutron that are connected in cubic p-n bonds.
Till approximately the end of the third row of the periodic system, meaning around Z=18 or 20 protons, additional neutrons are not crucial for the stabilization of the nucleus; the excess neutrons help by the stabilization of nuclei with odd number of protons or appear also in several isotopes, but there are also stable nuclei with no excess neutrons.
From the fourth row excess neutrons are required for stability in general and we assume here that this is due to the electric forces that act on the surface protons.
The electric force is analyzed in this work and we can see a correlation between the number of protons on which a force above certain level acts and the number of neutrons required for the nucleus stability.
How the excess neutrons stabilize the protons we don't try to solve at this stage, but detailed tables are offered here for the number of excess neutrons along the population process of every sub-orbital.

## Population rules for the excess neutrons of stable nuclei

### The force on the surface protons of the nucleus ellipsoid

We first calculate the relative electric force on each proton in the envelope.
The electric force on the proton $\boldsymbol{j}$ in the $\boldsymbol{x}$ direction is:

$$F_{j_x} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0^2} \left\{ \sum_{i \neq j}^{Z} \frac{(x_i - x_j)}{{d_{i,j}}^3} \right\} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0^2} f_{j_x}$$

the unitless relative electric force in the $\boldsymbol{x}$ direction is defined as: $f_{j_x} \coloneqq \sum_{i \neq j}^{Z} \frac{(x_i - x_j)}{{d_{i,j}}^3}$

where:

- $d_0$: the minimum distance between two neighboring nucleons in femtometer (assuming all nuclei have the same cubic structure and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices *i* and *j* measured in multiples of $d_0$: $d_{i,j} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2 + \left(z_j - z_i\right)^2}$
- Z: atomic number (number of protons).

Similarly we get the relative forces in the $\boldsymbol{y}$ and $\boldsymbol{z}$ direction: $f_{j_y}$ and $f_{j_z}$.

The total relative force on the proton $\boldsymbol{j}$ in the envelope is: $f_j = \sqrt{{f_{x,j}}^2 + {f_{y,j}}^2 + {f_{z,j}}^2}$ (absolute value).

### The range of excess neutrons

In order to calculate the number of excess neutrons in the envelope we define:

- $Z_{env}$ : the number of neutrons in the envelope (the excess neutrons).
- $n_{min} = \sum_{i}^{Z_{env}} \begin{cases} 1: & f_{max} \leq f_i \\ 0 \end{cases}$ : minimum number of excess neutrons.
- $n_{opt} = \sum_{i}^{Z_{env}} \begin{cases} 1: & f_{opt} \leq f_i \\ 0 \end{cases}$ : intermediate or optimum number of excess neutrons.
- $n_{max} = \sum_{i}^{Z_{env}} \begin{cases} 1: & f_{min} \leq f_i \\ 0 \end{cases}$ : minimum number of excess neutrons.

The relative forces $f_{min}, f_{opt}, f_{max}$ are found by trial, while learning the model. We note to not confuse: $n_{min}$ is calculated via $f_{max}$ whereas $n_{max}$ via $f_{min}$.
We roughly estimate these limits here with:

- $f_{min} = 0.295$
- $f_{opt} = 0.310$
- $f_{max} = 0.325$

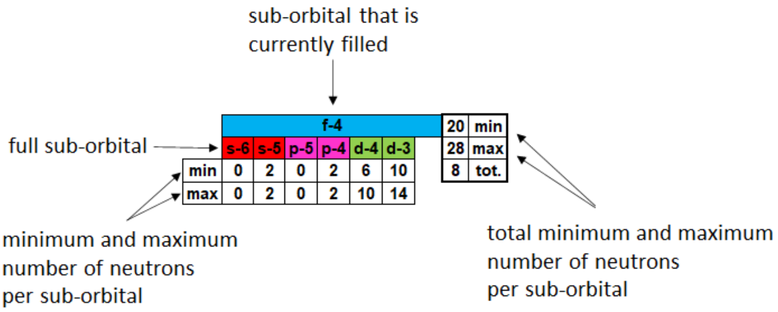
This is only a support tool in the development of the population rules. It helps us estimating the upper and lower limits of the excess neutrons for stable nuclei, but it is not exact and in order to determine the precise locations additional rules shall be considered.

**The population of sub-orbitals of stable nuclei**

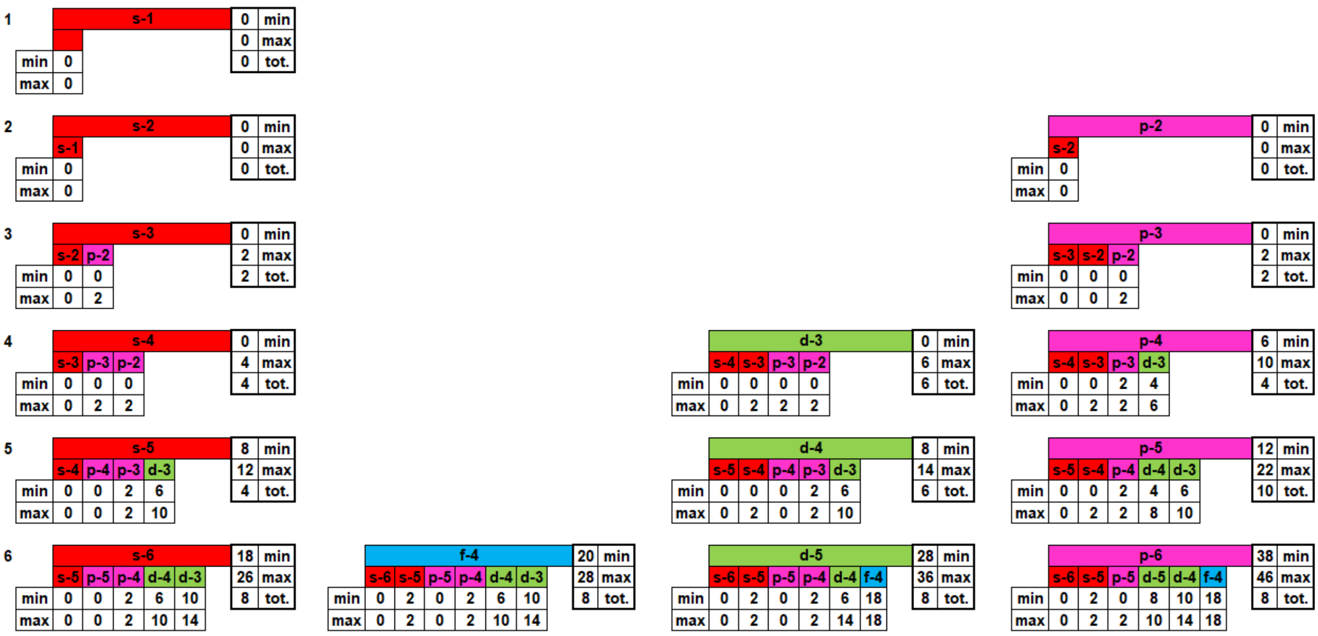
In a former research of this series we improved the model and found the locations of the excess neutrons for many stable nuclei [6]. Here we use this knowledge.

**Remark**: according to the model all excess neutrons are located in the envelope, but not all neutrons that are located in the envelope are excess neutrons. [5]

The following illustrations show the population process of the excess neutrons along the sub-orbitals of the periodic table.
The outermost sub-orbital of the layer that is currently in filling process is the upper one in the drawing and is marked with its name and color (in the example below: row/layer: 6, sub-orbital: **f-4**); below it we draw the population of the outermost sub-orbitals of every layer of the ellipsoid with its minimum and maximum number of excess neutrons.
On its right the total minimum and maximum number of excess neutrons is shown.



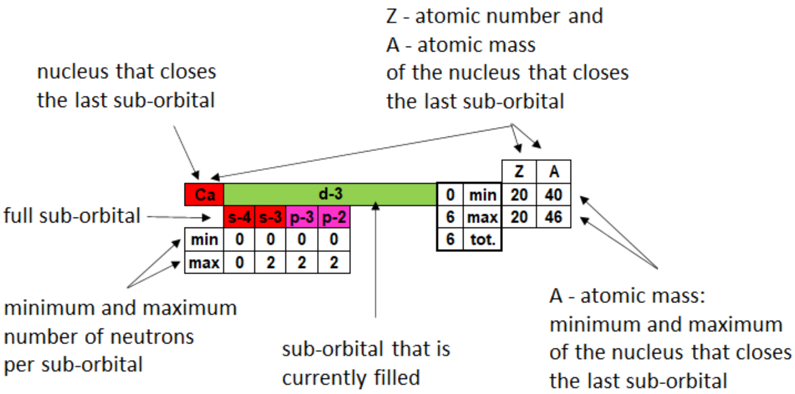

*Explanation on how to read the table below.*



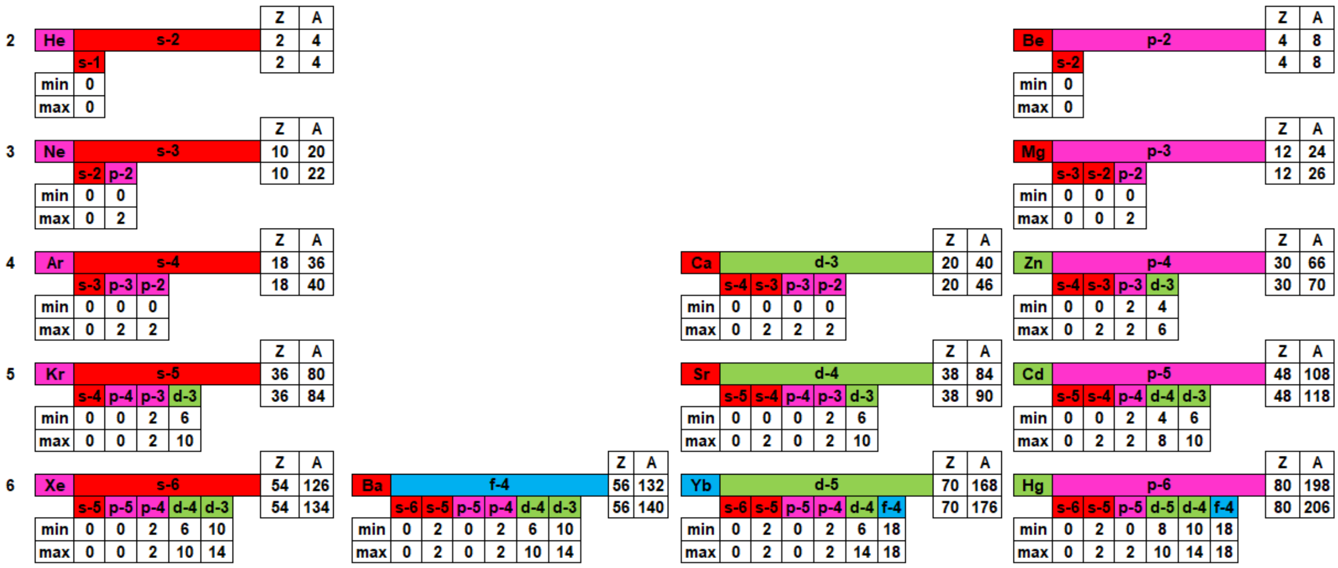

*The number of excess neutrons per sub-orbital along the process of filling in the elements of the periodic table.*
***s*** *sub-orbital,* ***p*** *sub-orbital,* ***d*** *sub-orbital,* ***f*** *sub-orbital.*

## The sub-orbitals of stable isotopes

The following drawings are similar to those of the last section, only the nucleus of the element that closed the last sequence (the sub-orbital of the last layer that was filled) is shown on the left and on the right the minimum and maximum values of its atomic mass A are calculated.



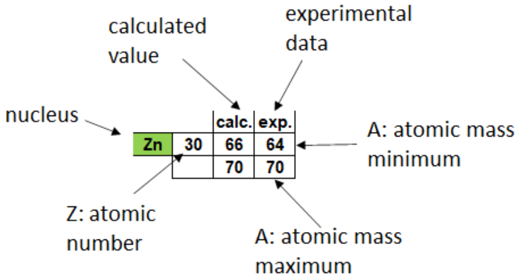

*Explanation on how to read the table below.*




*The number of excess neutrons per sub-orbital along the process of filling in the elements of the periodic table with the nuclei of the elements that closed the last sequence (the sub-orbital of the last layer that was filled).*
***s*** *sub-orbital,* ***p*** *sub-orbital,* ***d*** *sub-orbital,* ***f*** *sub-orbital.*

## The number of stable isotopes for nuclei with full sub-orbitals

The range of stable isotopes with closed sub-orbital was calculated in the last section.
Here we compare it with the experimental data.
The result verifies the population process and strengthen the model.




*Explanation on how to read the table below.*

| | Nucleus | Z | calc. (min) | exp. (min) | calc. (max) | exp. (max) |
|---|---|---|---|---|---|---|
| 2 | Be | 4 | 8 | 9 | 8 | 9 |
| | Ne | 10 | 20 | 20 | 22 | 22 |
| 3 | Mg | 12 | 24 | 24 | 26 | 26 |
| | Ar | 18 | 36 | 36 | 40 | 40 |
| 4 | Ca | 20 | 40 | 40 | 46 | 46 |
| | Zn | 30 | 66 | 64 | 70 | 70 |
| | Kr | 36 | 78 | 80 | 86 | 86 |
| 5 | Sr | 38 | 84 | 86 | 90 | 88 |
| | Cd | 48 | 108 | 106 | 118 | 114 |
| | Xe | 54 | 126 | 124 | 134 | 134 |
| 6 | Ba | 56 | 132 | 130 | 140 | 138 |
| | Yb | 70 | 168 | 168 | 176 | 176 |
| | Hg | 80 | 198 | 196 | 206 | 204 |

*A comparison between the calculated and experimental data for the upper and lower limits of stable isotopes of the nuclei of the elements with closed sub-orbitals.*
***s*** *sub-orbital,* ***p*** *sub-orbital,* ***d*** *sub-orbital,* ***f*** *sub-orbital.*

## Discussion of the results and conclusion

Analyzing the stable nuclei according to the model, leads to the creation of a set of rules for the population of the excess neutrons (the neutrons beyond the number equal to that of the protons) that are located in the envelope of the nucleus.
The rules were gained by an iterative process.
We can see the pattern of the rules through the tables that describe it.
The calculations of the electrical forces on the surface protons help us to justify the rules, but this is still not a general description of the process and its justification.
To summon up, we describe in this paper the rules and explain them, but we don't deliver a clear proof for why this is done exactly so.
The main assumptions are that it has to do with:

- lower limit for excess neutrons that are required for the stability of protons on the envelope of the nucleus; below this limit proton emission [4] occurs.
- upper limit for excess neutrons; beyond it there are not enough protons anymore to stabilize them and neutron emission [3] occurs.

These mechanisms shall be further developed in another research.

## Sources and references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3. *Neutron emission* - *(Wikipedia)*
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6. *cubic ellipsoid nucleus* - *part 2* - *model improvement* - *excess neutrons*

**cubic ellipsoid nucleus - part 5: instability of heavy elements**

**Summary**

Ronen Yavor

**Abstract**

This paper examines the hypothesis that the stability of heavy nuclei, beyond Lead ($Pb_{82}$) depends on the energy balance of a proton inside it, meaning the difference between the binding energy and the electric energy of the center proton.
The value of the maximum electric energy of the center proton is calculated for all heavy nuclei and compared with the number of nuclear bonds, that is required to keep the proton stable and so the nucleus as a whole.

The research results are the following:

- from about Lead ($Pb_{82}$) and till about Dubnium ($Db_{105}$) six nuclear bonds are required to keep the center protons stable; with some certain probability one nuclear bond could break for a certain amount of time, leading to instability and radioactivity occurs. The half-life (or the probability that the radioactive decay occurs) depends on various factors, such as the general form of the nucleus, the number of excess neutrons it possesses and its symmetry.
- beyond Dubnium ($Db_{105}$) more than six nuclear bonds are required to keep the center protons stable and therefore these nuclei are constantly unstable and have as a result a short half-life.

The explanation of these results with the help of illustration and calculation is another reinforcement for the model.

**Content**

## Introduction

Our former paper in this series dealt with the instability of nuclei due to the lack or excess of neutrons. [13]
In this research we discuss the instability of heavy nuclei that is divided in two:

- radioactivity, which is the subject of this research.
- nuclear fission, which will be discussed in the following research.

The model assumption is that the basic mechanism for both phenomena is the same, but for fission additional requirements must be fulfilled.

## The radioactivity hypothesis

The mechanism that is assumed, according to the model, to determine instability of heavy nuclei, beyond Lead (Pb), is the electric energy, that overcomes the binding energy (of the strong nuclear force) between the nucleons.
The instability is assumes to occur in the middle of the ellipsoid, where the electric energy reaches its maximum value; when we get to the nuclear fission, this idea will be further discussed and strengthen.

The calculations provide a rough prediction of the nucleus stability. We find that for nuclei larger than about Lead ($Pb_{82}$) and till about Rutherfordium ($Rf_{104}$) six nuclear bonds are required to keep the center protons stable.

The model hypothesis is that due to movements or fluctuations within the nucleus there is a certain probability that these six bonds are temporarily reduced to five bonds every certain timespan; as a result the center proton becomes unstable, possibly ending with a radioactive emission; after several radioactive steps of this type the nucleus is transformed to ($Pb_{82}$) where five bonds are sufficient to keep the center protons stable and radioactivity ends.
We explain this from another angle; the center proton has six bonds also for nuclei smaller than Pb (due to the nucleus geometry) and because only five bonds at most are required (for nuclei smaller than Pb) there is a redundancy and so even if one bond is missing for a short while, it doesn't lead to radioactivity; the probability for a simultaneous lack of two bonds is too low and so these nuclei are practically stable.

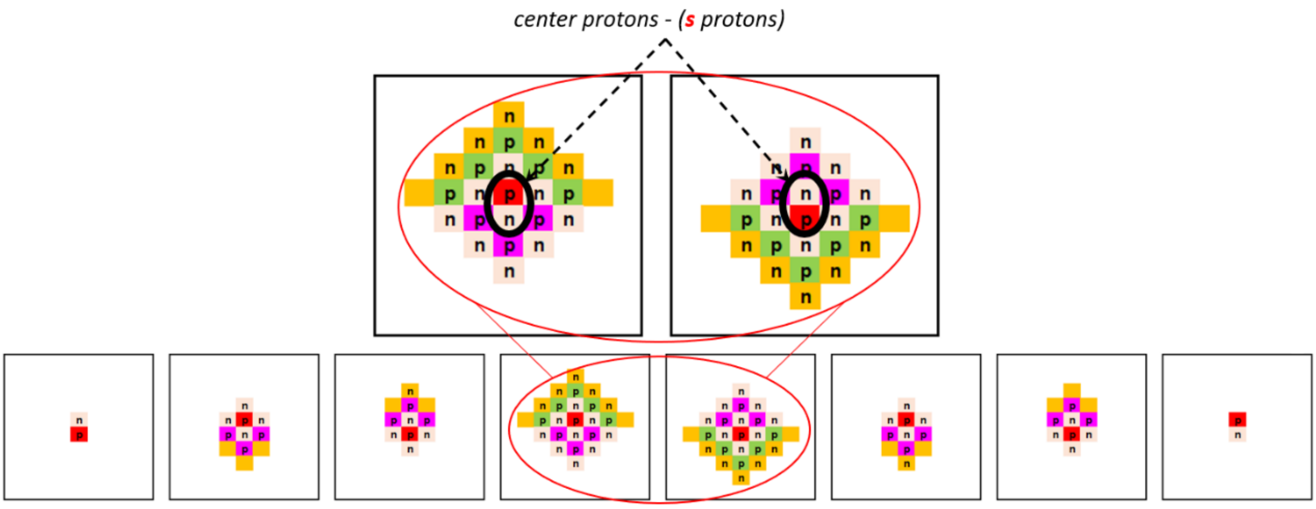
For nuclei beyond Rutherfordium ($Rf_{104}$) even six bonds are not enough to keep the center protons stable, meaning that they are inherently unstable, and therefore these nuclei have a short half-life.

## The research

### The radioactivity of heavy nuclei

We make the following definition:
**The two centers (or center protons) of the nucleus**: the two **s** sub-orbital protons in the middle of the nucleus; there are two central layers in the middle of the nucleus; one at the positive side of the **z**-axis (spin-up side) and the other at its negative (spin-down side); therefore there are also two centers (or center protons).

As an example the centers of the $Kr_{36}^{84}$ nucleus are shown.




*The two centers of the $Kr_{36}^{84}$ nucleus; the nucleus and zoom in on its center layers.*

The center protons have the largest electric energy in the nucleus.
Next we calculate this energy for the heavy nuclei and compare it with the number of bonds (of the strong nuclear force) that are required in order to stabilize these protons (to compensate the electric energy).

**Maximum electric energy as a function of the number of nuclear bonds**

The binding energy of the proton $\boldsymbol{x}$ in the nucleus is: $E_{\boldsymbol{b}_x} = e_{\boldsymbol{b}} \cdot n_{\boldsymbol{b}_x}$ [13] where:

- $n_{\boldsymbol{b}_x}$ is the number of nucleon-nucleon bonds of the proton $\boldsymbol{x}$ in the nucleus.
- $e_{\boldsymbol{b}} = 5.72\ MeV$: the energy of a single nucleon-nucleon bond in the nucleus (assuming they are equal for all bonds in all nuclei).

The electric energy of the proton $\boldsymbol{x}$ in the nucleus is:

$$E_{\boldsymbol{c}_x} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0}\left\{\sum_{j\neq i}^{Z_x}\frac{1}{d_{i,j}}\right\} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0}\,e_{\boldsymbol{c}_x} \quad \text{with} \quad e_{\boldsymbol{c}_x} := \sum_{j\neq i}^{Z_x}\frac{1}{d_{i,j}}$$

- $d_0 = 1.62\ fm$: the minimum distance between two neighboring nucleons in femtometer (assuming all nuclei have the same structure of cubic bonds and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices *i* and *j* measured in multiples of $d_0$: $d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$
- $e_{\boldsymbol{c}_x}$: the unitless relative electric energy of the proton $\boldsymbol{x}$ in the nucleus (sum of the reciprocal distances).

We analyze the maximum electric energy that a proton can have in dependency on its number of bonds: $\left(E_{\boldsymbol{b}_{proton_x}} - E_{\boldsymbol{c}_{proton_x}}\right) \geq 0$ or $\left(e_{\boldsymbol{b}} \cdot n_{\boldsymbol{b}_{proton_x}} - \frac{e^2}{4\pi\epsilon_0}\frac{1}{d_0}\,e_{\boldsymbol{c}_{proton_x}}\right) \geq 0$

and get the following table for this equation:

| $n_{\boldsymbol{b}_{proton\_x}}$ | $e_{\boldsymbol{c}_{proton\_x}}$ max. value | $E_b$ [Joule] | $E_c$ [Joule] | $E_b$-$E_c$ [Joule] |
|---|---|---|---|---|
| 1 | 6.43 | 9.2E-13 | 9.2E-13 | 0.00 |
| 2 | 12.87 | 1.8E-12 | 1.8E-12 | 0.00 |
| 3 | 19.30 | 2.7E-12 | 2.7E-12 | 0.00 |
| 4 | 25.73 | 3.7E-12 | 3.7E-12 | 0.00 |
| 5 | 32.16 | 4.6E-12 | 4.6E-12 | 0.00 |
| 6 | 38.60 | 5.5E-12 | 5.5E-12 | 0.00 |

*the maximum relative electric energy as a function of the number of nuclear bonds*

This means that a proton with a single nuclear bond can sustain, at most, a relative electric energy of 6.43; a proton with two bonds, 12.87 and so on; a proton with 5 nuclear bonds can hold at most a relative electric energy of 38.60.

**Results**

**maximum electric field of the heavy nuclei**

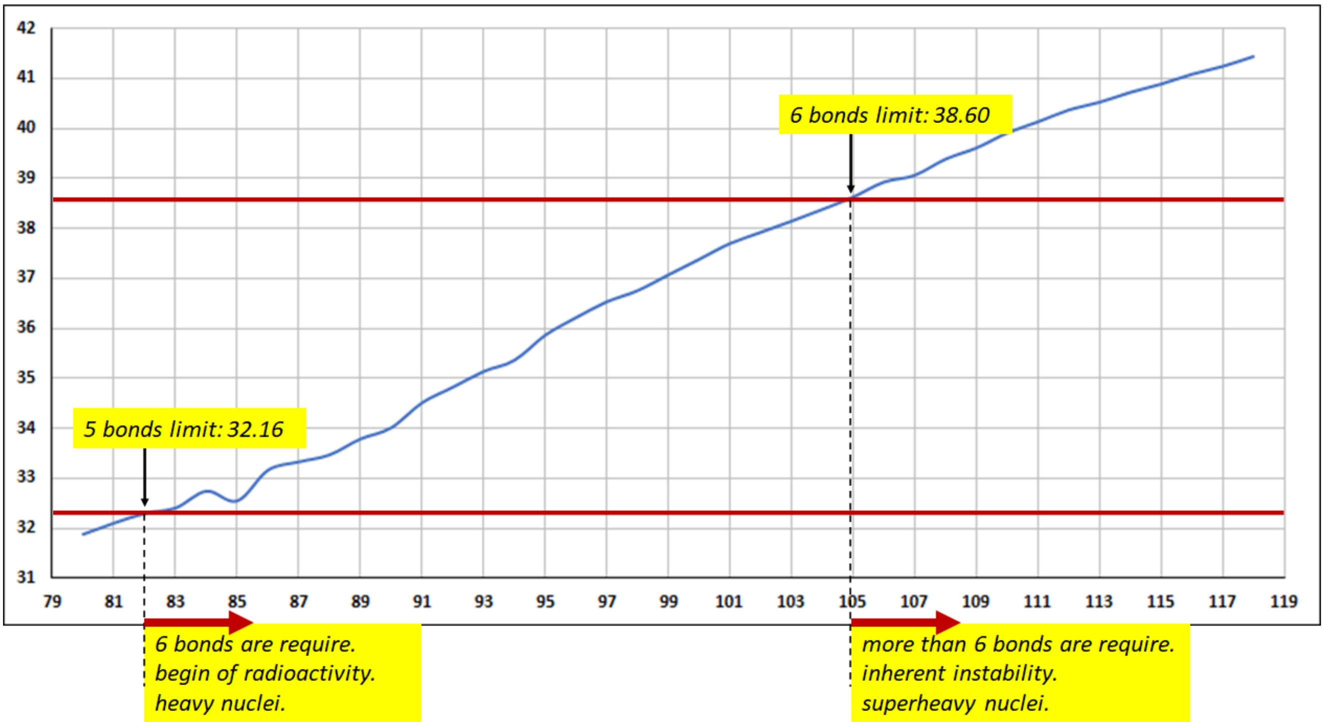
The following table shows the results from the last section. data from [1].

| nucleus | Z | max. half-life | bonds | max. $e_{c_p}$ | | |
|---|---|---|---|---|---|---|
| **Os** | 76 | stable | 5 | 30.61 | | |
| **Ir** | 77 | stable | 5 | 30.99 | | |
| **Pt** | 78 | stable | 5 | 31.30 | | |
| **Au** | 79 | stable | 5 | 31.62 | 1.7% | deviation |
| **Hg** | 80 | stable | 5 | 31.87 | 0.9% | from the limit |
| **Tl** | 81 | stable | 5 | 32.09 | 0.2% | value 32.16 |
| **Pb** | 82 | stable | 6 | 32.29 | | |
| **Bi** | 83 | y | 6 | 32.40 | | |
| **Po** | 84 | y | 6 | 32.73 | | |
| **At** | 85 | h | 6 | 32.54 | | |
| **Rn** | 86 | d | 6 | 33.15 | | |
| **Fr** | 87 | m | 6 | 33.32 | | |
| **Ra** | 88 | y | 6 | 33.46 | | |
| **Ac** | 89 | y | 6 | 33.78 | | |
| **Th** | 90 | y | 6 | 34.00 | | |
| **Pa** | 91 | y | 6 | 34.50 | | |
| **U** | 92 | y | 6 | 34.81 | | |
| **Np** | 93 | y | 6 | 35.13 | | |
| **Pu** | 94 | y | 6 | 35.35 | | |
| **Am** | 95 | y | 6 | 35.85 | | |
| **Cm** | 96 | y | 6 | 36.21 | | |
| **Bk** | 97 | y | 6 | 36.52 | | |
| **Cf** | 98 | y | 6 | 36.75 | | |
| **Es** | 99 | d | 6 | 37.06 | | |
| **Fm** | 100 | d | 6 | 37.37 | | |
| **Md** | 101 | d | 6 | 37.69 | | |
| **No** | 102 | m | 6 | 37.91 | 1.8% | |
| **Lr** | 103 | h | 6 | 38.14 | 1.2% | deviation |
| **Rf** | 104 | m | 6 | 38.37 | 0.6% | from the limit |
| **Db** | 105 | h | 6< | 38.62 | 0.1% | value 38.60 |
| **Sg** | 106 | m | 6< | 38.92 | | |
| **Bh** | 107 | m | 6< | 39.06 | | |
| **Hs** | 108 | m | 6< | 39.38 | | |
| **Mt** | 109 | s | 6< | 39.60 | | |
| **Ds** | 110 | s | 6< | 39.90 | | |
| **Rg** | 111 | s | 6< | 40.12 | | |
| **Cn** | 112 | s | 6< | 40.36 | | |
| **Nh** | 113 | s | 6< | 40.52 | | |
| **Fl** | 114 | s | 6< | 40.71 | | |
| **Mc** | 115 | ms | 6< | 40.88 | | |
| **Lv** | 116 | ms | 6< | 41.08 | | |
| **Ts** | 117 | ms | 6< | 41.24 | | |
| **Og** | 118 | ms | 6< | 41.43 | | |

**Remark**: **Hg**, **Rn**, **Ra**, **No**, **Cn** and **Og** have closed sub-orbitals (**D**, **P**, **S**, **F**, **D**, **P** respectively) and are marked with colors in accordance with the model convention.

**Results: the number of bonds vs. the relative electric energy**

The following graph illustrates the data from the above table.
We see that the radioactivity is expected to begin around Lead ($Hg_{80}$) and the superheavy nuclei (nuclei that are very unstable with short half-life) are expected to begin around Dubnium ($Db_{105}$).




*Limits of radioactive nuclei (6 nuclear bonds) and superheavy nuclei (beyond 6 bonds)*

## Discussion of the results and conclusion

The electric energy of the center proton in a heavy nucleus seems to determine its stability. We get that the limit of five nuclear bonds (equivalent to a relative electric energy of 32.16) is, as expected, around Lead ($Pb_{86}$).

The maximum electric energy of a single proton must therefore not exceed the value of five bonds, because then if one bond is missing, the nucleus might enter an unstable state.
This phenomenon occurs around $Pb_{82}$ and could be the explanation to why there are no stable nuclei above it.

The main assumption is that the nucleus passes fluctuations (of the bonds between nucleons and possibly also of their locations) that have a finite number of combinations. The character and duration of each of these fluctuations determine the stability of the nucleus.

The total stability of the nucleus depends thus on:

- the probability to reach a five bonds state instead of six in its center.
- the period of time that this state lasts.
- the chain reaction that is caused by this state.

If more than six bonds are required, approximately from Dubnium ($Db_{105}$), then the nucleus is inherently unstable and its half-life is dramatically shorten to the range of hours or much less.

**Remark**: this research is very premature and only provides initial hints or speculation on this mechanism within the framework of the model, yet the results fit to the model in a very elegant way that contributes to its interpretation of radioactivity and also of nuclear fission.

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**cubic ellipsoid nucleus - part 6: the mechanism of nuclear fission**

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**Abstract**

This paper examines the nuclear fission in the light of the cubic ellipsoid geometric model of the nucleus.
The main outputs of the research are:

- the explanation of the mechanism of the nuclear fission.
- the prediction of the most probable fission products (or fragments).

Both are based on the nuclear model and the nuclear instability as presented and discussed in the former papers in this series of the cubic ellipsoid model of the nucleus. [15]
These results provide additional reinforcement to the model.

**Content**

**Introduction: the fission hypotheses**

We raise the following hypotheses regarding the fission mechanism:

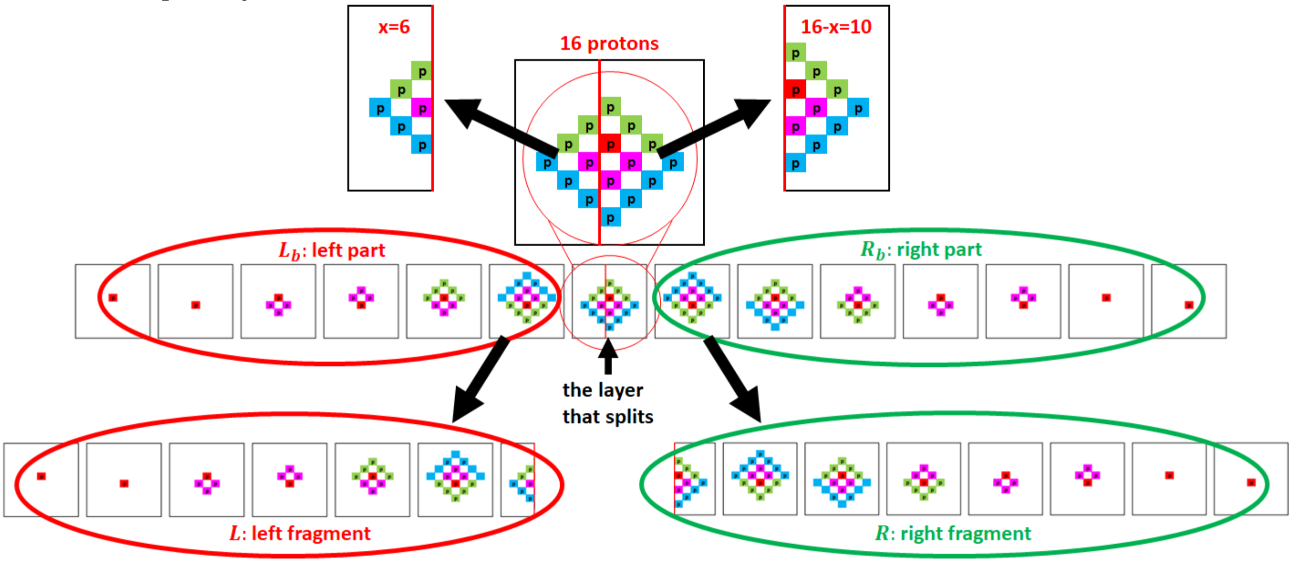
- A necessary but not sufficient condition for fission is that the nucleus is larger than Lead (Pb) and so has an unstable core. (See the previous study in this series on the origin of the instability of heavy nuclei [16]).
- The split of the nucleus occurs in one of the two centers (most inner) layers. [16]
- The number of protons of the products (fragments) is the sum of the protons in the layers from both sides till the split point according to the nucleus description (or illustration) in cross sections along its z-axis.
- The number of neutrons must be a bit lower than the relatively more stable isotope of the nucleus; for example for Uranium the more stable isotope is $U_{92}^{238}$, so the unstable isotope is smaller. The assumption here is that this lack of several neutrons enables some movement of the nucleons in the nucleus and so after radioactivity occurs in the center of the nucleus, a rearrangement of the inner parts enables the creation of the fragments.

In the following sections we describe the fission mechanism according to the model, explain how to calculate the size of the fragments and then demonstrate it with several examples.

## The nuclear fission

### The fission mechanism

The nuclear split occurs according to the model at one of the central layers (see illustration). For nuclei with even number of protons it doesn't matter if we select the right or left center as the one that splits, but for nuclei with odd number of protons, the two possibilities shall be considered separately.




*The fission and the definition of the fragments*

We define (see illustration):

- $P$ : number of protons of the nucleus that undergos fission.
- $R_b$: the number of protons of the right part of the nucleus till its center.
- $L_b$: as $R_b$, for the left part without its most inner layer (of 16 protons).
- $x$: the number of protons (out of 16) from the left side of the layer that splits.
- $R$: the number of protons of the right fragment.
- $L$: the number of protons of the left fragment.

and their values:

- $P := \begin{cases} 2m+1 & P\ odd \\ 2m & P\ even \end{cases}$ with $m$ integer
- $R_b := \frac{P}{2} = m$ (integer division)
- $L_b := \frac{P}{2} - 16 + remainder\left(\frac{P}{2}\right) = \begin{cases} m-16+1 & P\ odd \\ m-16 & P\ even \end{cases}$
- $R := R_b + 16 - x$
- $L := L_b + x$

and get that the sum of the fragments is equal, as required, to the total number of protons $P$:

- $R + L = (m + 16 - x) + \begin{cases} m-16+1+x \\ m-16+x \end{cases} = \begin{cases} 2m+1 & P\ odd \\ 2m & P\ even \end{cases} = P$

We take $x \in [6,16-6] = [6,10]$ and get the most probable fission product. We could possibly expand it to $x \in [1,15]$ if we want to get additional potential fission products.

## Fission products (fragments)

The following table shows the results of the above calculation for the nuclei from Thorium ($Th_{90}$) to Fermium ($Fm_{100}$) with the $x$ values $x \in [6{,}10]$ (and near it the *16-x* values ).

| nucleon | x=6 | 10 | x=7 | 9 | x=8 | 8 | x=9 | 7 | x=10 | 6 |
|---|---|---|---|---|---|---|---|---|---|---|
| $Th_{90}$ | $Sb_{51}$ | $Y_{39}$ | $Te_{52}$ | $Sr_{38}$ | $I_{53}$ | $Rb_{37}$ | $Xe_{54}$ | $Kr_{36}$ | $Cs_{55}$ | $Br_{35}$ |
| $Pa_{91}$ | $Sb_{51}$ | $Zr_{40}$ | $Te_{52}$ | $Y_{39}$ | $I_{53}$ | $Sr_{38}$ | $Xe_{54}$ | $Rb_{37}$ | $Cs_{55}$ | $Kr_{36}$ |
| $U_{92}$ | $Te_{52}$ | $Zr_{40}$ | $I_{53}$ | $Y_{39}$ | $Xe_{54}$ | $Sr_{38}$ | $Cs_{55}$ | $Rb_{37}$ | $Ba_{56}$ | $Kr_{36}$ |
| $Np_{93}$ | $Te_{52}$ | $Nb_{41}$ | $I_{53}$ | $Zr_{40}$ | $Xe_{54}$ | $Y_{39}$ | $Cs_{55}$ | $Sr_{38}$ | $Ba_{56}$ | $Rb_{37}$ |
| $Pu_{94}$ | $I_{53}$ | $Nb_{41}$ | $Xe_{54}$ | $Zr_{40}$ | $Cs_{55}$ | $Y_{39}$ | $Ba_{56}$ | $Sr_{38}$ | $La_{57}$ | $Rb_{37}$ |
| $Am_{95}$ | $I_{53}$ | $Mo_{42}$ | $Xe_{54}$ | $Nb_{41}$ | $Cs_{55}$ | $Zr_{40}$ | $Ba_{56}$ | $Y_{39}$ | $La_{57}$ | $Sr_{38}$ |
| $Cm_{96}$ | $Xe_{54}$ | $Mo_{42}$ | $Cs_{55}$ | $Nb_{41}$ | $Ba_{56}$ | $Zr_{40}$ | $La_{57}$ | $Y_{39}$ | $Ce_{58}$ | $Sr_{38}$ |
| $Bk_{97}$ | $Xe_{54}$ | $Tc_{43}$ | $Cs_{55}$ | $Mo_{42}$ | $Ba_{56}$ | $Nb_{41}$ | $La_{57}$ | $Zr_{40}$ | $Ce_{58}$ | $Y_{39}$ |
| $Cf_{98}$ | $Cs_{55}$ | $Tc_{43}$ | $Ba_{56}$ | $Mo_{42}$ | $La_{57}$ | $Nb_{41}$ | $Ce_{58}$ | $Zr_{40}$ | $Pr_{59}$ | $Y_{39}$ |
| $Es_{99}$ | $Cs_{55}$ | $Ru_{44}$ | $Ba_{56}$ | $Tc_{43}$ | $La_{57}$ | $Mo_{42}$ | $Ce_{58}$ | $Nb_{41}$ | $Pr_{59}$ | $Zr_{40}$ |
| $Fm_{100}$ | $Ba_{56}$ | $Ru_{44}$ | $La_{57}$ | $Tc_{43}$ | $Ce_{58}$ | $Mo_{42}$ | $Pr_{59}$ | $Nb_{41}$ | $Nd_{60}$ | $Zr_{40}$ |

*Table of the expected fission fragments* from $Th_{90}$ *to* $Fm_{100}$ *for* $x \in [6{,}10]$

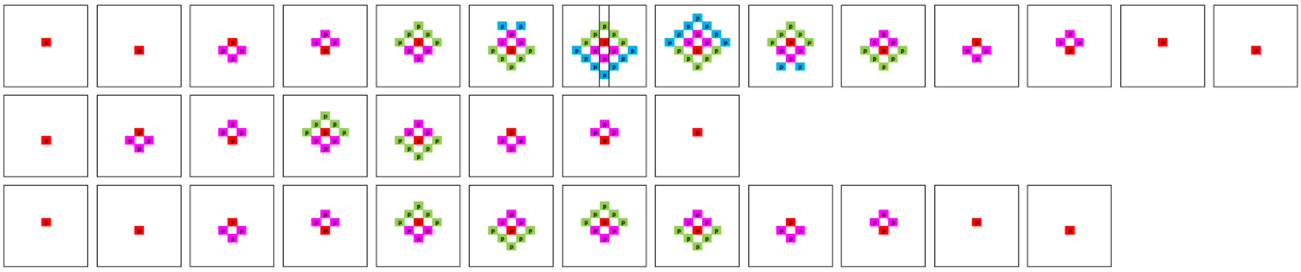
These results show the main fragments [13]; in order to get other fragments $x$ could be taken from a wider range (e.g $x \in [3{,}13]$ or even $x \in [1{,}15]$).

**Fission examples: observation of the protons solely**

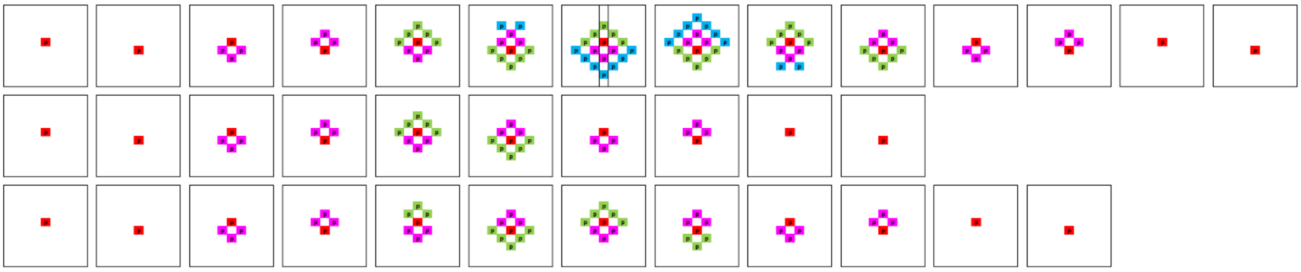
First we want to observe only the protons involved in the process; we choose for Uranium and Plutonium products [11], that have higher probability to appear and see first that according to the number of protons, the fission must occur at one of the two center layers as the model predicts.
The area of the split in one of the center layers is marked with two lines.

**Uranium**

$$U_{92} \Rightarrow Kr_{36} + Ba_{56}$$

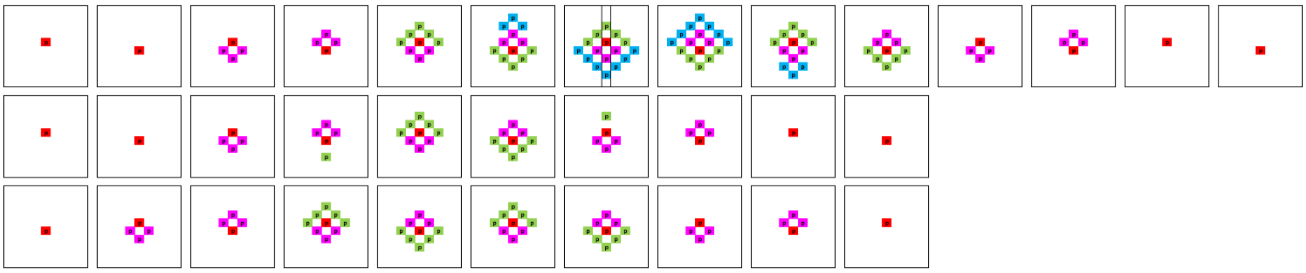


$$U_{92} \Rightarrow Zr_{38} + Te_{52}$$



**Plutonium**

$$Pu_{94} \Rightarrow Zr_{40} + Xe_{54}$$

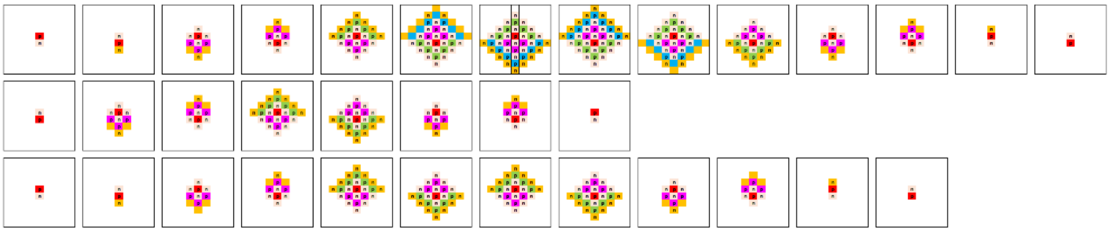


### Fission examples: the full fragments

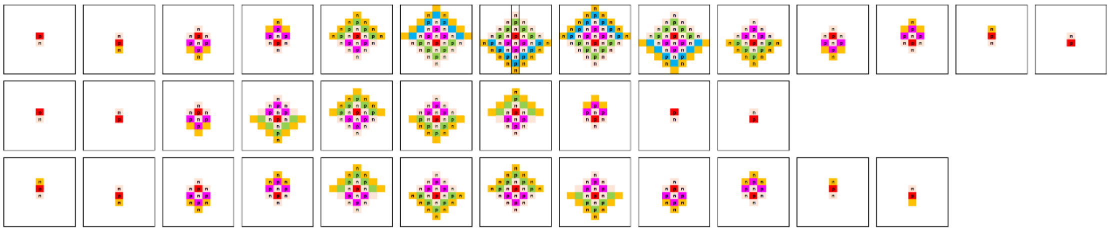
Now we consider the above nuclei as a whole [11].
We see that for the nucleus that undergoes fission and also for the product nuclei almost all potential excess neutron positions are occupied.
The area of the split in one of the center layers is marked with two lines.
We see that the number of neutrons in the fission region corresponds to the number of neutrons in the fission products, even though we only chose it based on the number of protons in it. This is another reinforcement for the fission hypothesis.

### Uranium

$$U_{92}^{235} + n_0^1 \Rightarrow Kr_{36}^{90} + Ba_{56}^{144} + 2 \cdot n_0^1$$

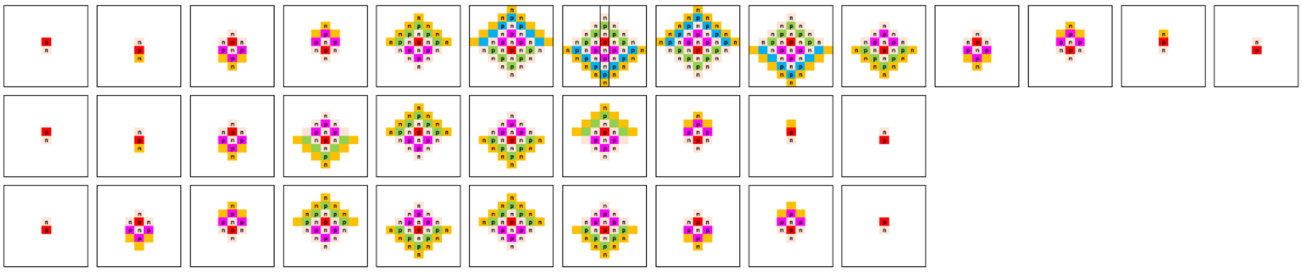


$$U_{92}^{235} + n_0^1 \Rightarrow Zr_{38}^{94} + Te_{52}^{139} + 3 \cdot n_0^1$$



### Plutonium

$$Pu_{94}^{239} + n_0^1 \Rightarrow Xe_{54}^{134} + Zr_{40}^{103} + 3 \cdot n_0^1$$



## Discussion of the results and conclusion

The main results of this research are:

- a tangible description of the fission mechanism.
- the prediction of the most probable products of the nuclear fission.

These are not a proof to the model, but strengthen its assumptions, just as the former research of this series did regarding instability of heavy nuclei.

We have up to this stage several results and all support the model on the one hand and none of them contradicts the common nuclear theory or physics in general, so maybe it is worth continuing to study the model and deepen its understanding.

## Sources and references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