Proof of the Collatz conjecture,

3x+1

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Abstract

I will demonstrate that the conjecture 3x+1 is true using a new approach based on an ancient symbol the ENNEAGRAM, and how "numerical gravity" arises from the deterministic divisibility that combinations of integers allow. I will use modular arithmetic. With the help of flow and block diagrams I will find an equation that, applying the 2 conditions, binds all odd numbers and consequently positive numbers to powers of 2. I will find the analytical expression of the function.

I will go up the Collatz graph represented by the inverse function:

$$R(n) = \begin{cases} \{2n, (n-1)/3\} & \text{, if } n \equiv 4 \pmod 6 \\ \{2n\} & \text{otherwise.} \end{cases}$$

which forms a tree with the exception of the cycle 1-4-2-1...

I will show how all **positive integers** are present in the tree, that is connected to the number 1, making extensive use of tables, drawings and colors in order to represent the beauty of mathematics. I will show that for every integer n, $n \equiv 1 \pmod 2$ if and only if $3n+1 \equiv 4 \pmod 6$.

I will follow the exact chronology of the insights.

Careful observation of the numbers will return an elementary (-a)**rithmetic** (double logical negation equals affirmation). I will not omit passages that are obvious, for these are the substrate on which the approach is grounded. I hope you can appreciate the extreme simplicity, harmony and rhythm that the numbers manifest.

Introduction

Let's analyze the algorithm algebraically:

$$f(n) \begin{cases} n/2, & \text{if n is even,} \\ 3n+1 & \text{if n is odd.} \end{cases}$$

The constituent elements will be: 1,2,3, n, the distinction between even and odd numbers, the powers of 2 and modular arithmetic.

It is evident that the second condition, thanks to +1, "forces" the odd numbers to become even in order to make them divisible.

Modular arithmetic:

By placing integers in an array according to a given module, they will be arranged in columns with peculiar characteristics that will return useful information. I will use this method to understand the 3x+1 algorithm.

Module 2: distinguishes EVEN and ODD numbers:

Analytical expression of the succession of even numbers:

an = 0+2n

Analytical expression of the succession of odd numbers:

an = 1+2n

EVEN	ODD
	1
2	3
4	5
6	7
8	9
10	11
12	13
14	15
16	17
18	19

I add beside the basic numbers 10 the corresponding number expressed with the binary positional numbering system:

ODD	binary number		EVEN	binary number
1	1		2	10
3	11		4	100
5	101		6	110
7	111		8	1000
9	1001		10	1010
11	1011		12	1100
13	1101		14	1110
15	1111		16	10000
17	10001		18	10010
19	10011		20	10100
21	10101		22	10110
23	10111		24	11000
25	11001		26	11010
27	11011		28	11100
29	11101		30	11110
31	11111		32	100000
33	100001		34	100010
35	100011		36	100100
37	100101		38	100110
39	100111		40	101000
41	101001		42	101010
43	101011		44	101100
45	101101		46	101110
47	101111		48	110000
49	110001		50	110010
51	110011		52	110100
53	110101		54	110110
55	110111		56	111000
57	111001		58	111010
59	111011		60	111100
61	111101		62	111110
63	111111		64	1000000
127	1111111		128	10000000
255	11111111		256	100000000
511	111111111		512	1000000000

Power numbers of 2 (highlighted in yellow) have a peculiarity:
expressed by the binary system they are represented by a 1 followed by a number of zeros.

The number of zeros present in the binary representation is the exponent of 2 that inserted in the formula, expressed with the decimal system, returns 1:

(N° power of 2)/2^t=1

All EVEN numbers, expressed using the binary system, have 1 or more zeros as the least significant digit.

All ODD numbers, expressed using the binary system, have 1 as the least significant digit.

If we multiply the odd numbers by powers of 2, we get the following formula:

$$(1+2n)*2^t$$

$$n\,,\, t = 0 \div \infty$$

which has as possible solutions:

ODD numbers	with	$t = 0$
1 = power of 2	with	$n\,,\, t = 0$
Powers of 2	with	$n = 0$
EVEN numbers	with	$t > 0$

it follows the equation with $t > 0$:

$$\boxed{\text{ODD numbers} * 2^t = \text{EVEN numbers}}$$

$$1+2n * 2^t = (0+2m)$$

$$\mathbf{1+2n*2^t = 2m}$$

$$\mathbf{1+2n = 2m/2^t}$$

with $m\,,\, t = 1 \div \infty$ and $n=0 \div \infty$

with t = number of zeros less significant than the EVEN number 2m expressed with the binary positional numbering system.

$$\boxed{\text{ODD numbers} = \text{EVEN numbers} / 2^t}$$

Powers of 2 'connect' the number 1, the even numbers and the odd numbers.

Module 3: highlights multiples of 3.

The first 3 numbers of each column can be represented geometrically as the vertices of 3 equilateral triangles inscribed in a circle:

The 3x+1 algorithm seen in optics (mod 3):

1 + 3x

Multiples of 3

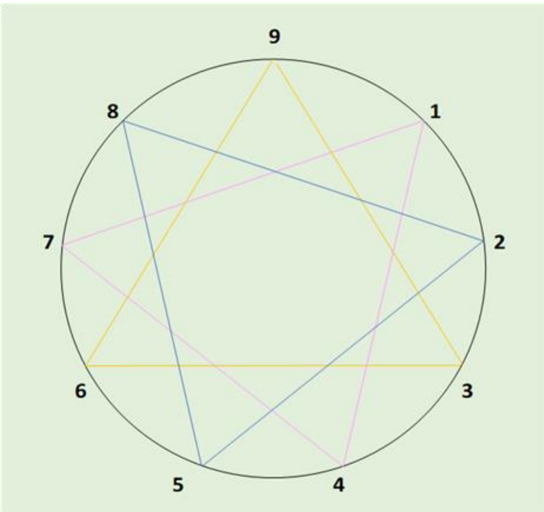
1+3x residue = 1	2+3x residue = 2	3+3x residue = 3
1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30

3 inscribed triangles:

1-4-7

2-5-8

3-6-9



Module 6: distinguish between EVEN and ODD, multiples of 3 ODD and EVEN

ODD	EVEN	multiples of 3 ODD	EVEN	ODD	multiples of 3 EVEN
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60

The EVEN numbers root 4 are ≡ 4 (mod 6) can be reached by the inverse function:

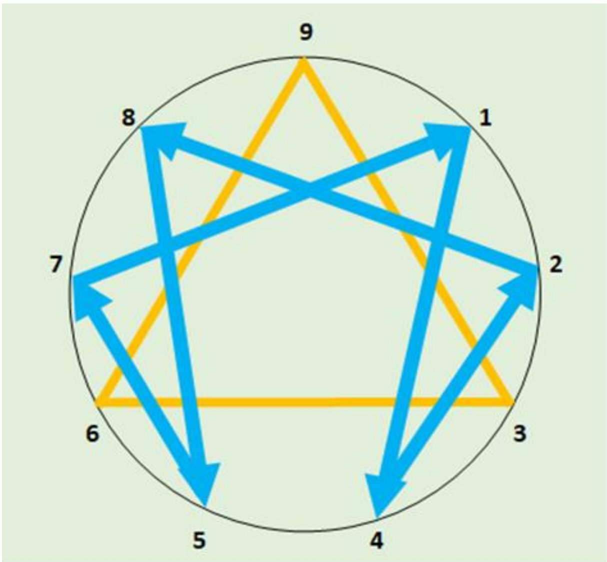
2n, (n-1)/3

Module 9: EVEN numbers that have roots 1-4-7 are ≡ 4 (mod 6), EVEN numbers that have roots 2-5-8 are not. Multiples of 3 have roots 0-3-6.

EVEN ≡ 4 (mod 6)		multiples of 3	EVEN ≡ 4 (mod 6)		multiples of 3	EVEN ≡ 4 (mod 6)		multiples of 3
1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81
82	83	84	85	86	87	88	89	90

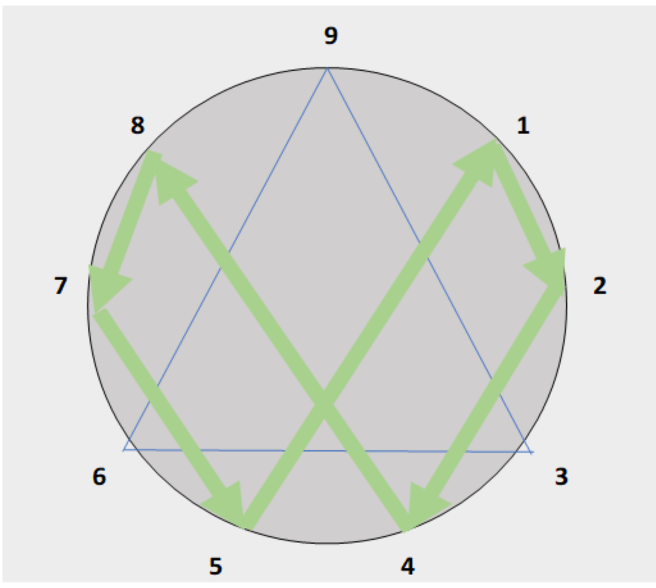
ENNEAGRAM: the ancient symbol is a graphic and geometric representation of arithmetic modulo 9, 6, 3,2 and theosophical reduction:

it is known that the iterative reduction of the result of the sums of the individual digits of an integer leads to the number between 1 and 9, that is, to the residual 1,2,3,4,5,6,7,8,0 (mod9).



The triangle 3-6-9 expressing multiples of 3 that are ≡ 0,3,6 (mod9) is excluded from the root cycle 1-4-2-8-5-7.

I modify the flow of the latter:



Consider the EVEN numbers:

Root cycle 1-2-4-8-7-5:

$$N° \equiv 1(\text{mod}9) * 2 = N° \equiv 2\ (\text{mod}9)$$
$$N° \equiv 2(\text{mod}9) * 2 = N° \equiv 4\ (\text{mod}9)$$
$$N° \equiv 4(\text{mod}9) * 2 = N° \equiv 8\ (\text{mod}9)$$
$$N° \equiv 8(\text{mod}9) * 2 = N° \equiv 7\ (\text{mod}9)$$
$$N° \equiv 7(\text{mod}9) * 2 = N° \equiv 5\ (\text{mod}9)$$
$$N° \equiv 5(\text{mod}9) * 2 = N° \equiv 1\ (\text{mod}9)$$

<u>Now we divide /2 instead of multiplying *2:</u>

$$N° \equiv 1(\text{mod}9)\ / 2 = N° \equiv 5\ (\text{mod}9)$$
$$N° \equiv 5(\text{mod}9)\ / 2 = N° \equiv 7\ (\text{mod}9)$$
$$N° \equiv 7(\text{mod}9)\ / 2 = N° \equiv 8\ (\text{mod}9)$$
$$N° \equiv 8(\text{mod}9)\ / 2 = N° \equiv 4\ (\text{mod}9)$$
$$N° \equiv 4(\text{mod}9)\ / 2 = N° \equiv 2\ (\text{mod}9)$$
$$N° \equiv 2(\text{mod}9)\ / 2 = N° \equiv 1\ (\text{mod}9)$$

Both operations take us back to the starting root.

Root cycle 0-3-6:

$$N° \equiv 0(\text{mod}9)\ * 2 = N° \equiv 0\ (\text{mod}9)$$
$$N° \equiv 0(\text{mod}9)\ / 2 = N° \equiv 0\ (\text{mod}9)$$

$$N° \equiv 3(\text{mod}9)\ * 2 = N° \equiv 6\ (\text{mod}9)$$
$$N° \equiv 6(\text{mod}9)\ / 2 = N° \equiv 3\ (\text{mod}9)$$

$$N° \equiv 6(\text{mod}9)\ * 2 = N° \equiv 3\ (\text{mod}9)$$
$$N° \equiv 3(\text{mod}9)\ / 2 = N° \equiv 6\ (\text{mod}9)$$

Thanks to the cyclicity of modular arithmetic that appears to reach the modulus and manifests itself for the infinity of integers, we can use the inductive method and extend what is stated to all natural numbers.

Matrix (mod9)

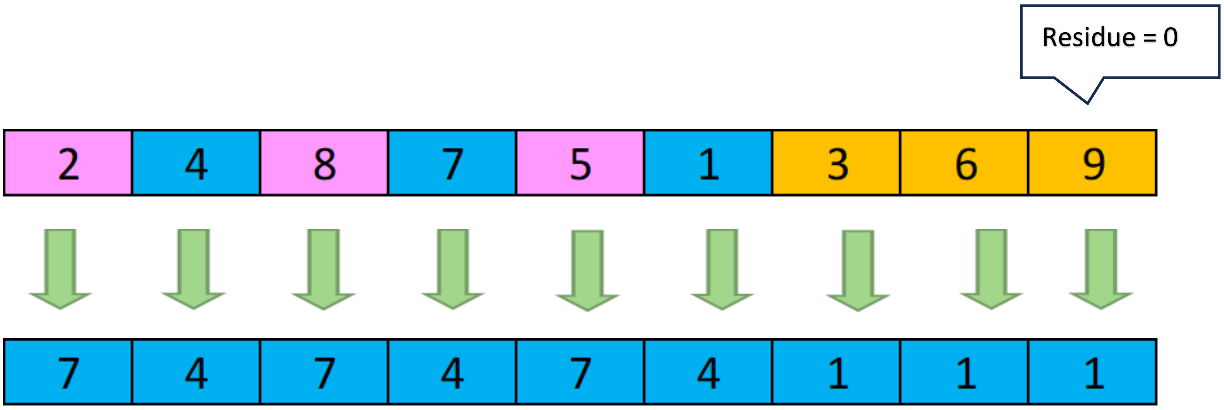
<u>The matrix modulo 9 aligns multiples of 3 in 3 columns: 3-6-9</u>

I change the order of the 9 columns to better highlight the 3-6-0 root triangle:

2	4	8	7	5	1	3	6	9
11	13	17	16	14	10	12	15	18
20	22	26	25	23	19	21	24	27
29	31	35	34	32	28	30	33	36
38	40	44	43	41	37	39	42	45
47	49	53	52	50	46	48	51	54
56	58	62	61	59	55	57	60	63
65	67	71	70	68	64	66	69	72
74	76	80	79	77	73	75	78	81
83	85	89	88	86	82	84	87	90
92	94	98	97	95	91	93	96	99
101	103	107	106	104	100	102	105	108
110	112	116	115	113	109	111	114	117
119	121	125	124	122	118	120	123	126
128	130	134	133	131	127	129	132	135
137	139	143	142	140	136	138	141	144
146	148	152	151	149	145	147	150	153
155	157	161	160	158	154	156	159	162
164	166	170	169	167	163	165	168	171
173	175	179	178	176	172	174	177	180
182	184	188	187	185	181	183	186	189
191	193	197	196	194	190	192	195	198
200	202	206	205	203	199	201	204	207
209	211	215	214	212	208	210	213	216
218	220	224	223	221	217	219	222	225
227	229	233	232	230	226	228	231	234
236	238	242	241	239	235	237	240	243
245	247	251	250	248	244	246	249	252
254	256	260	259	257	253	255	258	261

The matrix continues to infinity and contains all positive numbers.

I apply the 3x+1 condition to all the odd numbers in the matrix (mod9):



The 3x+1 condition has 3 effects:

1) Converts ODD numbers x to multiples of $3 = 3x$ and thanks to the sum returns EVEN numbers.

2) The EVEN numbers generated by the condition, which are $\equiv 4 \pmod 6$, pour into the root triangle 1, 4, 7 (mod9) only.

Generalizing:

$3*(9t+0)+1 = 27t+1 \equiv 1 \pmod 9$

$3*(9t+1)+1 = 27t+4 \equiv 4 \pmod 9$

$3*(9t+2)+1 = 27t+7 \equiv 7 \pmod 9$

$3*(9t+3)+1 = 27t+10 \equiv 1 \pmod 9$; $10 \equiv 1 \pmod 9$

$3*(9t+4)+1 = 27t+13 \equiv 4 \pmod 9$; $13 \equiv 4 \pmod 9$

$3*(9t+5)+1 = 27t+16 \equiv 7 \pmod 9$; $16 \equiv 7 \pmod 9$

$3*(9t+6)+1 = 27t+19 \equiv 1 \pmod 9$; $19 \equiv 1 \pmod 9$

$3*(9t+7)+1 = 27t+22 \equiv 4 \pmod 9$; $22 \equiv 4 \pmod 9$

$3*(9t+8)+1 = 27t+25 \equiv 7 \pmod 9$; $25 \equiv 7 \pmod 9$

3) Merges into the column with root 1 multiples of 3 ODD:

$$(3k)*3+1 = 9k+1$$

which are ≡ 3 (mod6) and ≡ 0,3,6 (mod9) and we can write as:

$$(1+2n)*3 = 3+6n$$

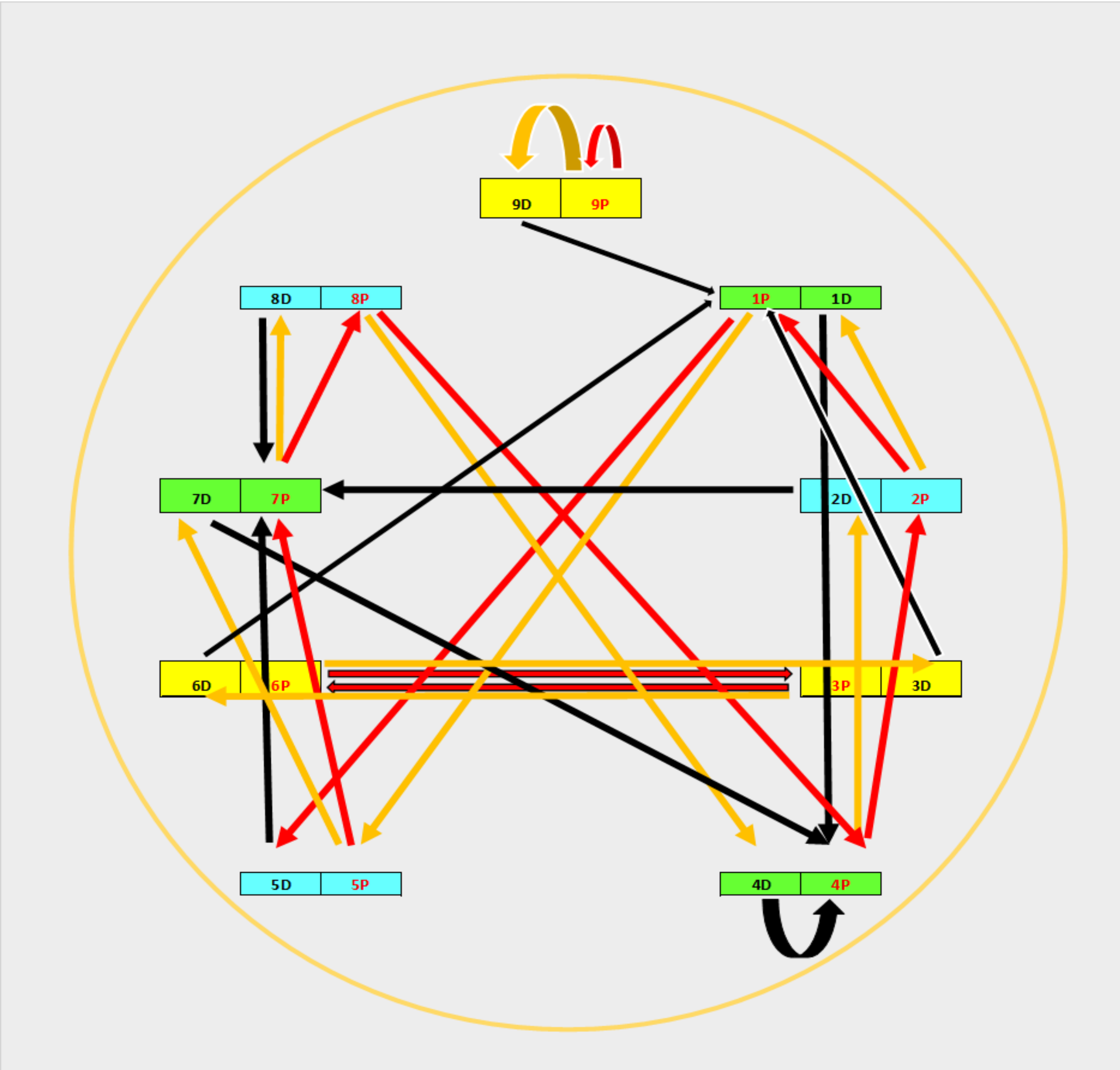
summing 1 we get: $\mathbf{3+6n+1 = 4+6n}$

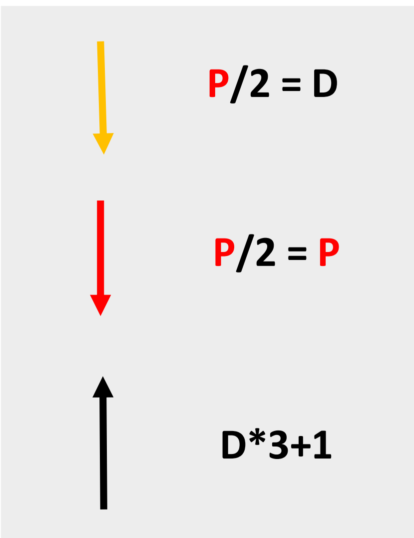
Thanks to modular arithmetic we can use the inductive method and extend what is stated to all natural numbers.

It is important to understand how the condition connects the cycle of multiples of 3 (triangle 3-6-9 and root cycle 3-6-0) with the cycle of multiples of 2 (root cycle 1-2-4-8-7-5), which, thanks to the /2 condition, leads to 1. It effectively eliminates the multiples of 3 from subsequent cycles thus reconciling the numbers 2 and 3 that are known to be coprime.

The following are 4 flow patterns (mod9) that, while maintaining the same functioning, highlight the circulation of numbers:

Flow diagram 1: distinguishing between roots and EVEN and ODD numbers:

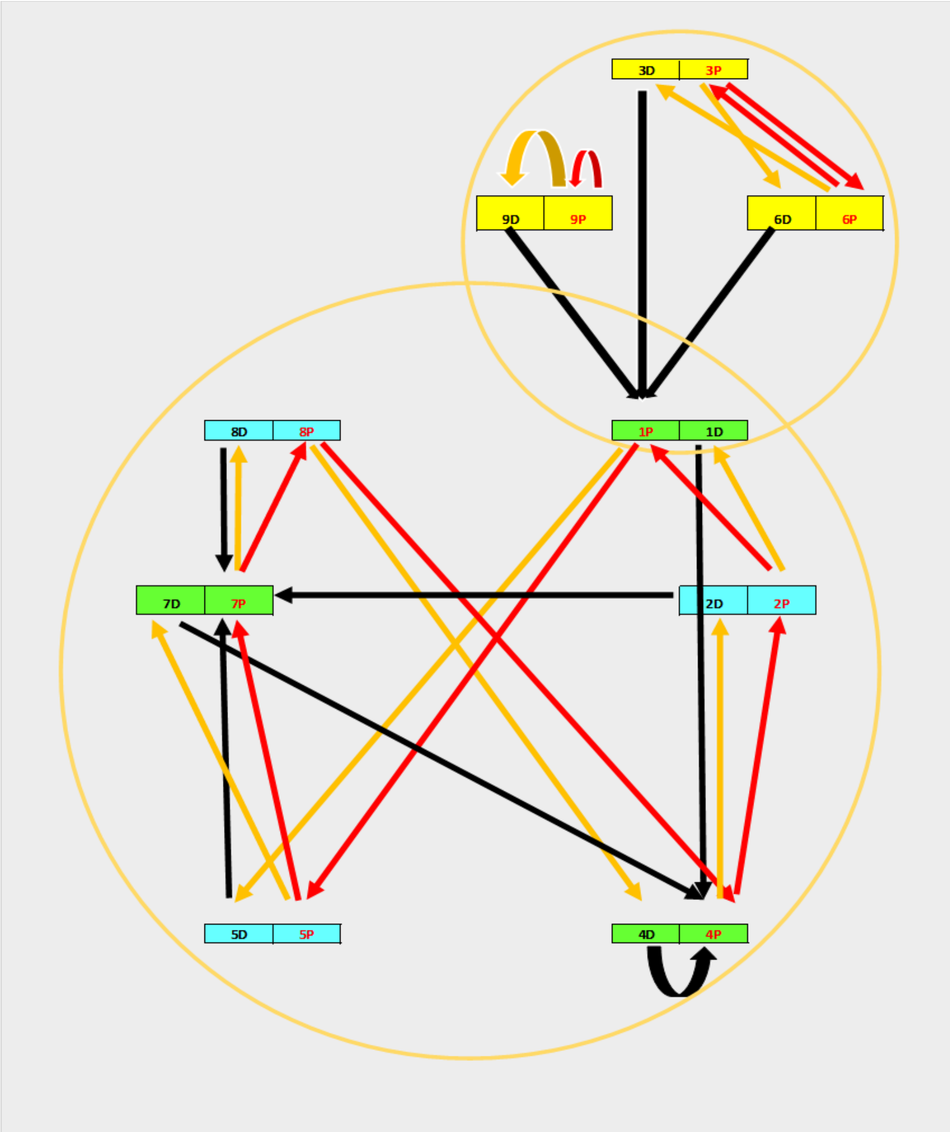




D = DISPARI = ODD number

P = PARI = EVEN number

Flow diagram 2 where I highlight the transition from **Enneagram to Hexagram** in the form of ∞ or form similar to the Lorenz attractor ∞ (red arrows inside the large circle):



All numbers ≡ 0,3,6 (mod9) are multiples of 3 and enter the hexagram cycle and are connected to the condition / 2 and *2 going up the graph.

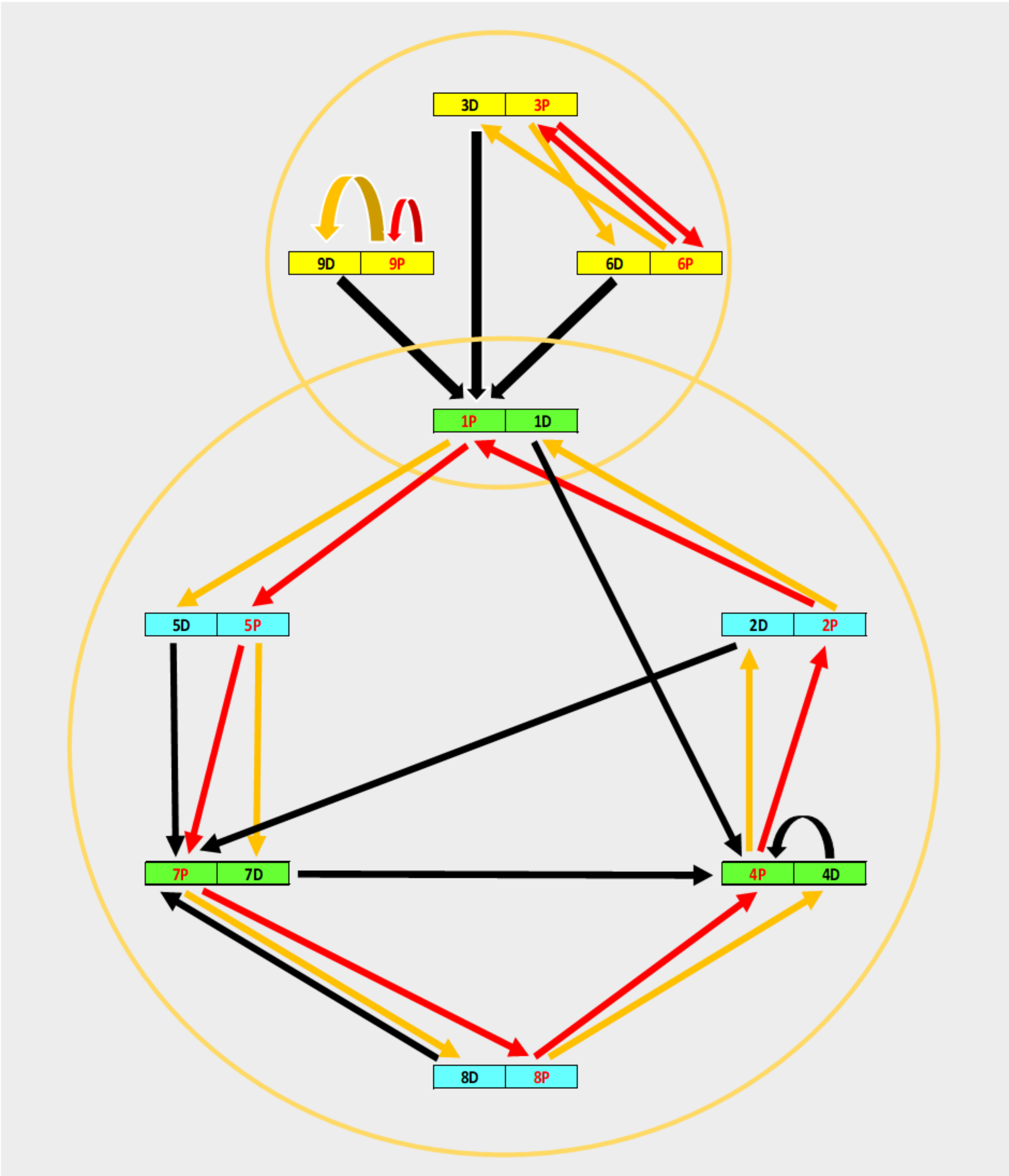
Multiples of 3, after applying the 2 conditions, become

EVEN numbers **≡ 1 (mod9) and are divided by 2^t** generating an

ODD number **≡ 1,4,7 (mod9) ≡ 1 (mod6)** and generating an

ODD number **≡ 2,5,8 (mod9) ≡ 5 (mod6).**

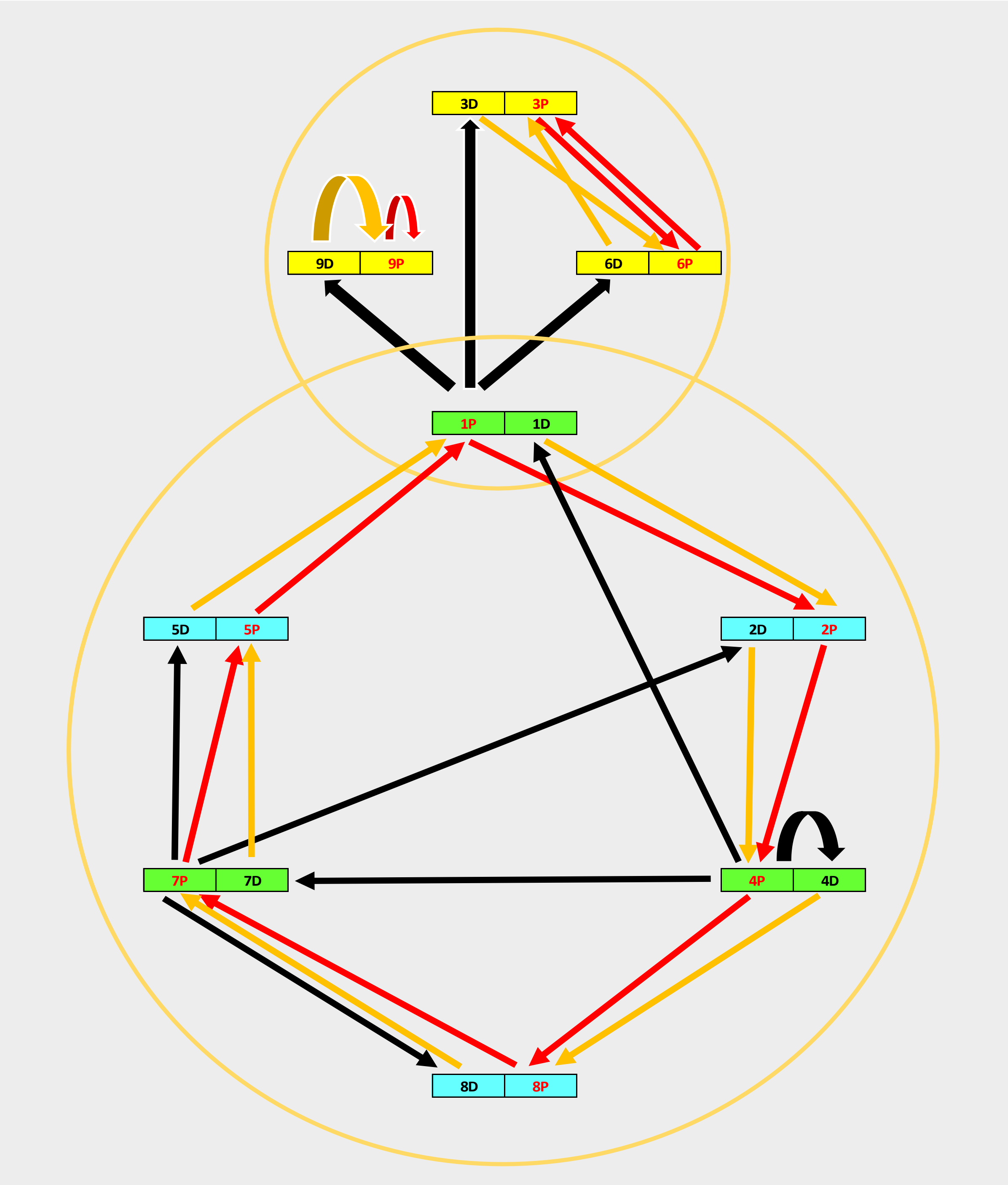
Flow diagram 3: i arrange the Hexagram in the form of zero (red arrows inside the large circle):

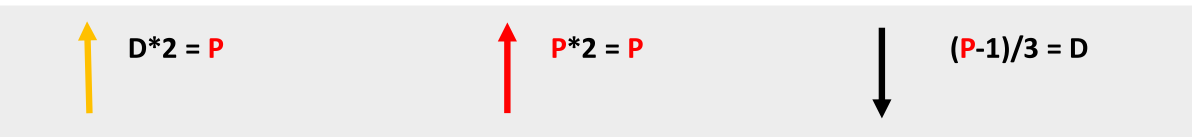


The numbers powers of 2 follow the red /2 path and reach 1.

The EVEN numbers, multiples of 2, follow the red path until they deviate (orange arrow) on an ODD number which will become, after the condition 3x+1, EVEN ≡ 4.7 (mod9).

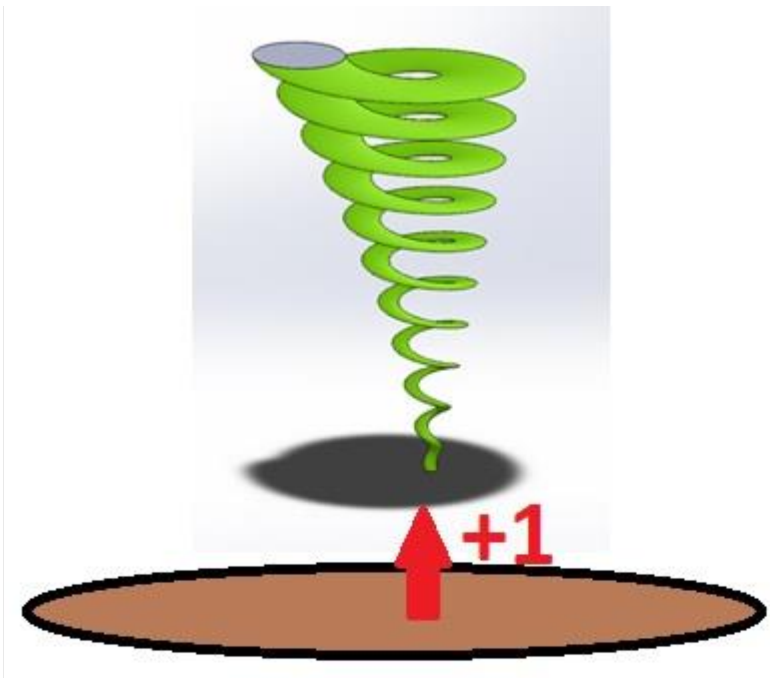
Flow pattern 4. By reversing the direction of the arrows and replacing the conditions with those of the inverse function**, the 3 patterns become the Collatz graph:**





The graph tree and flow diagram in 3 dimensions become a 3D roller coaster (flow diagram 2) or a **3D spiral** (flow diagram 3):

we could allegorically describe them as a huge slide where numerical gravity, represented by divisibility by 2, pushes all positive integers towards the foreground at +1.

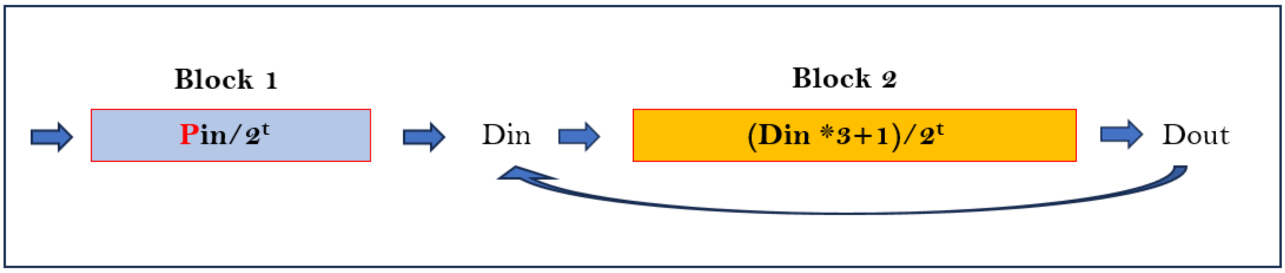


The powers of 2 are the fulcrum, the constraint, through which the algorithm links all positive integers.

Powers of 2:

Positive feedback block diagram.

I highlight the fundamental action of the powers of 2:



All positive numbers are present in the binary positional numbering system, where the numbers 1 (high level) represent a power of 2 with exponent, starting from 0, increasing from right to left. The sum of the values obtained by elevating 2 to its exponent returns the base number 10. An EVEN number, expressed with binary numbering, will have one or more, less significant digits marked by a 0 (low level). Eliminating the less significant zeros is equivalent to dividing by 2 times as many times as there are zeros. The number thus divided is an ODD number.

All EVEN numbers subjected to the /2 condition, one or more times, become ODD.

An EVEN number is written 2^t *y with t>0 and y is always ODD, **so I can write EVEN /2^t = ODD.**

The number 1 is an ODD number and is related to EVEN numbers thanks to the condition /2.

If the chosen number is EVEN, it will be processed by block 1, which will deliver an ODD number to block 2.

Block 2 applies the 2 conditions: 3*x+1 and /2^t.

The positive feedback returns the ODD number at the output, at the input of the block itself and adds it to 0, since the EVEN number has already been processed and has given up its energy. The subsequent cycles will be exclusively those of block 2.

Module 32: has the peculiarity of columning the powers of 2.

In the search for **regularities in the distribution of powers of 2**, we observe how the ODD input numbers placed in a 16-column matrix. Applying the condition 3*x+1, generate an EVEN number that is divisible by 2^t.

Exponent of 2

2	(mod9)	1	(mod9)	4	(mod9)	1	(mod9)	2	(mod9)	1	(mod9)	3	(mod9)	1	(mod9)	2	(mod9)	1	(mod9)	>=5	(mod9)	1	(mod9)	2	(mod9)	1	(mod9)	3	(mod9)	1	(mod9)
1	1	3	3	5	5	7	7	9	0	11	2	13	4	15	6	17	8	19	1	21	3	23	5	25	7	27	0	29	2	31	4
33	6	35	8	37	1	39	3	41	5	43	7	45	0	47	2	49	4	51	6	53	8	55	1	57	3	59	5	61	7	63	0
65	2	67	4	69	6	71	8	73	1	75	3	77	5	79	7	81	0	83	2	85	4	87	6	89	8	91	1	93	3	95	5
97	7	99	0	101	2	103	4	105	6	107	8	109	1	111	3	113	5	115	7	117	0	119	2	121	4	123	6	125	8	127	1
129	3	131	5	133	7	135	0	137	2	139	4	141	6	143	8	145	1	147	3	149	5	151	7	153	0	155	2	157	4	159	6
161	8	163	1	165	3	167	5	169	7	171	0	173	2	175	4	177	6	179	8	181	1	183	3	185	5	187	7	189	0	191	2
193	4	195	6	197	8	199	1	201	3	203	5	205	7	207	0	209	2	211	4	213	6	215	8	217	1	219	3	221	5	223	7
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257	5	259	7	261	0	263	2	265	4	267	6	269	8	271	1	273	3	275	5	277	7	279	0	281	2	283	4	285	6	287	8
289	1	291	3	293	5	295	7	297	0	299	2	301	4	303	6	305	8	307	1	309	3	311	5	313	7	315	0	317	2	319	4
321	6	323	8	325	1	327	3	329	5	331	7	333	0	335	2	337	4	339	6	341	8	343	1	345	3	347	5	349	7	351	0
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385	7	387	0	389	2	391	4	393	6	395	8	397	1	399	3	401	5	403	7	405	0	407	2	409	4	411	6	413	8	415	1
417	3	419	5	421	7	423	0	425	2	427	4	429	6	431	8	433	1	435	3	437	5	439	7	441	0	443	2	445	4	447	6
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609	6	611	8	613	1	615	3	617	5	619	7	621	0	623	2	625	4	627	6	629	8	631	1	633	3	635	5	637	7	639	0
641	2	643	4	645	6	647	8	649	1	651	3	653	5	655	7	657	0	659	2	661	4	663	6	665	8	667	1	669	3	671	5
673	7	675	0	677	2	679	4	681	6	683	8	685	1	687	3	689	5	691	7	693	0	695	2	697	4	699	6	701	8	703	1
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737	8	739	1	741	3	743	5	745	7	747	0	749	2	751	4	753	6	755	8	757	1	759	3	761	5	763	7	765	0	767	2
769	4	771	6	773	8	775	1	777	3	779	5	781	7	783	0	785	2	787	4	789	6	791	8	793	1	795	3	797	5	799	7
801	0	803	2	805	4	807	6	809	8	811	1	813	3	815	5	817	7	819	0	821	2	823	4	825	6	827	8	829	1	831	3
833	5	835	7	837	0	839	2	841	4	843	6	845	8	847	1	849	3	851	5	853	7	855	0	857	2	859	4	861	6	863	8
865	1	867	3	869	5	871	7	873	0	875	2	877	4	879	6	881	8	883	1	885	3	887	5	889	7	891	0	893	2	895	4
897	6	899	8	901	1	903	3	905	5	907	7	909	0	911	2	913	4	915	6	917	8	919	1	921	3	923	5	925	7	927	0
929	2	931	4	933	6	935	8	937	1	939	3	941	5	943	7	945	0	947	2	949	4	951	6	953	8	955	1	957	3	959	5
961	7	963	0	965	2	967	4	969	6	971	8	973	1	975	3	977	5	979	7	981	0	983	2	985	4	987	6	989	8	991	1
993	3	995	5	997	7	999	0	1001	2	1003	4	1005	6	1007	8	1009	1	1011	3	1013	5	1015	7	1017	0	1019	2	1021	4	1023	6
1025	8	1027	1	1029	3	1031	5	1033	7	1035	0	1037	2	1039	4	1041	6	1043	8	1045	1	1047	3	1049	5	1051	7	1053	0	1055	2
1057	4	1059	6	1061	8	1063	1	1065	3	1067	5	1069	7	1071	0	1073	2	1075	4	1077	6	1079	8	1081	1	1083	3	1085	5	1087	7
1089	0	1091	2	1093	4	1095	6	1097	8	1099	1	1101	3	1103	5	1105	7	1107	0	1109	2	1111	4	1113	6	1115	8	1117	1	1119	3
1121	5	1123	7	1125	0	1127	2	1129	4	1131	6	1133	8	1135	1	1137	3	1139	5	1141	7	1143	0	1145	2	1147	4	1149	6	1151	8
1153	1	1155	3	1157	5	1159	7	1161	0	1163	2	1165	4	1167	6	1169	8	1171	1	1173	3	1175	5	1177	7	1179	0	1181	2	1183	4
1185	6	1187	8	1189	1	1191	3	1193	5	1195	7	1197	0	1199	2	1201	4	1203	6	1205	8	1207	1	1209	3	1211	5	1213	7	1215	0
1217	2	1219	4	1221	6	1223	8	1225	1	1227	3	1229	5	1231	7	1233	0	1235	2	1237	4	1239	6	1241	8	1243	1	1245	3	1247	5
1249	7	1251	0	1253	2	1255	4	1257	6	1259	8	1261	1	1263	3	1265	5	1267	7	1269	0	1271	2	1273	4	1275	6	1277	8	1279	1
1281	3	1283	5	1285	7	1287	0	1289	2	1291	4	1293	6	1295	8	1297	1	1299	3	1301	5	1303	7	1305	0	1307	2	1309	4	1311	6
1313	8	1315	1	1317	3	1319	5	1321	7	1323	0	1325	2	1327	4	1329	6	1331	8	1333	1	1335	3	1337	5	1339	7	1341	0	1343	2

By vertically displacing the columns of the matrix of ODD numbers (mod32), I obtain rows of data that have the same (mod9):

2	(mod9)	1	(mod9)	>=5	(mod9)	1	(mod9)	2	(mod9)	1	(mod9)	3	(mod9)	1	(mod9)	2	(mod9)	1	(mod9)	4	(mod9)	1	(mod9)	2	(mod9)	1	(mod9)	3	(mod9)	1	(mod9)
																												13	4	31	4
																								9	0	27	0	45	0	63	0
																				5	5	23	5	41	5	59	5	77	5	95	5
																1	1	19	1	37	1	55	1	73	1	91	1	109	1	127	1
														15	6	33	6	51	6	69	6	87	6	105	6	123	6	141	6	159	6
										11	2	29	2	47	2	65	2	83	2	101	2	119	2	137	2	155	2	173	2	191	2
						7	7	25	7	43	7	61	7	79	7	97	7	115	7	133	7	151	7	169	7	187	7	205	7	223	7
		3	3	21	3	39	3	57	3	75	3	93	3	111	3	129	3	147	3	165	3	183	3	201	3	219	3	237	3	255	3
17	8	35	8	53	8	71	8	89	8	107	8	125	8	143	8	161	8	179	8	197	8	215	8	233	8	251	8	269	8	287	8
49	4	67	4	85	4	103	4	121	4	139	4	157	4	175	4	193	4	211	4	229	4	247	4	265	4	283	4	301	4	319	4
81	0	99	0	117	0	135	0	153	0	171	0	189	0	207	0	225	0	243	0	261	0	279	0	297	0	315	0	333	0	351	0
113	5	131	5	149	5	167	5	185	5	203	5	221	5	239	5	257	5	275	5	293	5	311	5	329	5	347	5	365	5	383	5
145	1	163	1	181	1	199	1	217	1	235	1	253	1	271	1	289	1	307	1	325	1	343	1	361	1	379	1	397	1	415	1
177	6	195	6	213	6	231	6	249	6	267	6	285	6	303	6	321	6	339	6	357	6	375	6	393	6	411	6	429	6	447	6
209	2	227	2	245	2	263	2	281	2	299	2	317	2	335	2	353	2	371	2	389	2	407	2	425	2	443	2	461	2	479	2
241	7	259	7	277	7	295	7	313	7	331	7	349	7	367	7	385	7	403	7	421	7	439	7	457	7	475	7	493	7	511	7
273	3	291	3	309	3	327	3	345	3	363	3	381	3	399	3	417	3	435	3	453	3	471	3	489	3	507	3	525	3	543	3
305	8	323	8	341	8	359	8	377	8	395	8	413	8	431	8	449	8	467	8	485	8	503	8	521	8	539	8	557	8	575	8
337	4	355	4	373	4	391	4	409	4	427	4	445	4	463	4	481	4	499	4	517	4	535	4	553	4	571	4	589	4	607	4
369	0	387	0	405	0	423	0	441	0	459	0	477	0	495	0	513	0	531	0	549	0	567	0	585	0	603	0	621	0	639	0
401	5	419	5	437	5	455	5	473	5	491	5	509	5	527	5	545	5	563	5	581	5	599	5	617	5	635	5	653	5	671	5
433	1	451	1	469	1	487	1	505	1	523	1	541	1	559	1	577	1	595	1	613	1	631	1	649	1	667	1	685	1	703	1
465	6	483	6	501	6	519	6	537	6	555	6	573	6	591	6	609	6	627	6	645	6	663	6	681	6	699	6	717	6	735	6
497	2	515	2	533	2	551	2	569	2	587	2	605	2	623	2	641	2	659	2	677	2	695	2	713	2	731	2	749	2	767	2
529	7	547	7	565	7	583	7	601	7	619	7	637	7	655	7	673	7	691	7	709	7	727	7	745	7	763	7	781	7	799	7
561	3	579	3	597	3	615	3	633	3	651	3	669	3	687	3	705	3	723	3	741	3	759	3	777	3	795	3	813	3	831	3
593	8	611	8	629	8	647	8	665	8	683	8	701	8	719	8	737	8	755	8	773	8	791	8	809	8	827	8	845	8	863	8
625	4	643	4	661	4	679	4	697	4	715	4	733	4	751	4	769	4	787	4	805	4	823	4	841	4	859	4	877	4	895	4
657	0	675	0	693	0	711	0	729	0	747	0	765	0	783	0	801	0	819	0	837	0	855	0	873	0	891	0	909	0	927	0
689	5	707	5	725	5	743	5	761	5	779	5	797	5	815	5	833	5	851	5	869	5	887	5	905	5	923	5	941	5	959	5
721	1	739	1	757	1	775	1	793	1	811	1	829	1	847	1	865	1	883	1	901	1	919	1	937	1	955	1	973	1	991	1
753	6	771	6	789	6	807	6	825	6	843	6	861	6	879	6	897	6	915	6	933	6	951	6	969	6	987	6	1005	6	1023	6
785	2	803	2	821	2	839	2	857	2	875	2	893	2	911	2	929	2	947	2	965	2	983	2	1001	2	1019	2	1037	2	1055	2
817	7	835	7	853	7	871	7	889	7	907	7	925	7	943	7	961	7	979	7	997	7	1015	7	1033	7	1051	7	1069	7	1087	7
849	3	867	3	885	3	903	3	921	3	939	3	957	3	975	3	993	3	1011	3	1029	3	1047	3	1065	3	1083	3	1101	3	1119	3
881	8	899	8	917	8	935	8	953	8	971	8	989	8	1007	8	1025	8	1043	8	1061	8	1079	8	1097	8	1115	8	1133	8	1151	8
913	4	931	4	949	4	967	4	985	4	1003	4	1021	4	1039	4	1057	4	1075	4	1093	4	1111	4	1129	4	1147	4	1165	4	1183	4
945	0	963	0	981	0	999	0	1017	0	1035	0	1053	0	1071	0	1089	0	1107	0	1125	0	1143	0	1161	0	1179	0	1197	0	1215	0
977	5	995	5	1013	5	1031	5	1049	5	1067	5	1085	5	1103	5	1121	5	1139	5	1157	5	1175	5	1193	5	1211	5	1229	5	1247	5
1009	1	1027	1	1045	1	1063	1	1081	1	1099	1	1117	1	1135	1	1153	1	1171	1	1189	1	1207	1	1225	1	1243	1	1261	1	1279	1
1041	6	1059	6	1077	6	1095	6	1113	6	1131	6	1149	6	1167	6	1185	6	1203	6	1221	6	1239	6	1257	6	1275	6	1293	6	1311	6
1073	2	1091	2	1109	2	1127	2	1145	2	1163	2	1181	2	1199	2	1217	2	1235	2	1253	2	1271	2	1289	2	1307	2	1325	2	1343	2

I look for cyclicality in the manifestation of the algorithm through powers of 2:

$$\text{Din } {*}3+1 = \text{P1-P4-P7}$$
$$\text{P1-P4-P7} / 2^t = \text{P}/2^t$$
$$\text{P}/2^t = \text{Dout}$$

D = odd number

P = even number

Din(mod9)	Din	Din(mod32)	P1-P4-P7	P(mod9)	t	$P/2^t$
1	1	1	4	4	2	1
3	3	3	10	1	1	5
5	5	5	16	7	4	1
7	7	7	22	4	1	11
0	9	9	28	1	2	7
2	11	11	34	7	1	17
4	13	13	40	4	3	5
6	15	15	46	1	1	23
8	17	17	52	7	2	13
1	19	19	58	4	1	29
3	21	21	64	1	6	1
5	23	23	70	7	1	35
7	25	25	76	4	2	19
0	27	27	82	1	1	41
2	29	29	88	7	3	11
4	31	31	94	4	1	47
6	33	1	100	1	2	25
8	35	3	106	7	1	53
1	37	5	112	4	4	7
3	39	7	118	1	1	59
5	41	9	124	7	2	31
7	43	11	130	4	1	65
0	45	13	136	1	3	17
2	47	15	142	7	1	71
4	49	17	148	4	2	37
6	51	19	154	1	1	77
8	53	21	160	7	5	5
1	55	23	166	4	1	83
3	57	25	172	1	2	43
5	59	27	178	7	1	89
7	61	29	184	4	3	23
0	63	31	190	1	1	95
2	65	1	196	7	2	49
4	67	3	202	4	1	101
6	69	5	208	1	4	13
8	71	7	214	7	1	107
1	73	9	220	4	2	55
3	75	11	226	1	1	113

The array continues to infinity and contains all ODD numbers.

Cycle 16 of powers of 2.

The column t follows a cycle 16 that repeats with the sole exception for the eleventh position that alternates the exponent of 2 = 5 with > of 5.

The average of the exponents of 2 is minimal: 31/16 = 1,9375

equal, or > 2 when the exponent >5 ex. 34/16 = 2.125

Din (mod9)	Din	Din (mod32)	P1-P4-P7	P(mod9)	t	$P/2^t$
2	65	1	196	7	2	49
4	67	3	202	4	1	101
6	69	5	208	1	4	13
8	71	7	214	7	1	107
1	73	9	220	4	2	55
3	75	11	226	1	1	113
5	77	13	232	7	3	29
7	79	15	238	4	1	119
0	81	17	244	1	2	61
2	83	19	250	7	1	125
4	85	21	256	4	8	1
6	87	23	262	1	1	131
8	89	25	268	7	2	67
1	91	27	274	4	1	137
3	93	29	280	1	3	35
5	95	31	286	7	1	143

Filter the powers of 2 from 1 to 4.

The increase in value of the ODD number input (Din-Din$_{-1}$), starting from 4, is an expression of the powers of 2, so it multiplies *2 as for the musical octaves, up to 32.

Din-Din$_{-1}$	Din (mod9)	Din	Din (mod32)	P1-P4-P7	P(mod9)	t	P/2^t
4	3	3	3	10	1	1	5
4	7	7	7	22	4	1	11
4	2	11	11	34	7	1	17
4	6	15	15	46	1	1	23
4	1	19	19	58	4	1	29
4	5	23	23	70	7	1	35
8	1	1	1	4	4	2	1
8	0	9	9	28	1	2	7
8	8	17	17	52	7	2	13
8	7	25	25	76	4	2	19
8	6	33	1	100	1	2	25
8	5	41	9	124	7	2	31
16	4	13	13	40	4	3	5
16	2	29	29	88	7	3	11
16	0	45	13	136	1	3	17
16	7	61	29	184	4	3	23
16	5	77	13	232	7	3	29
16	3	93	29	280	1	3	35
32	5	5	5	16	7	4	1
32	1	37	5	112	4	4	7
32	6	69	5	208	1	4	13
32	2	101	5	304	7	4	19
32	7	133	5	400	4	4	25
32	3	165	5	496	1	4	31

Filter the powers of 2 >4.

The numbers D in are ≡21 (mod32). Every 32 numbers starting from 21 we find an exponent of 2 >4

$Din\text{-}Din_{-1}$	Din (mod9)	Din	Din (mod32)	P1-P4-P7	P(mod9)	t	$P/2^t$
	3	21	21	64	1	6	1
32	8	53	21	160	7	5	5
32	4	85	21	256	4	8	1
32	0	117	21	352	1	5	11
32	5	149	21	448	7	6	7
32	1	181	21	544	4	5	17
32	6	213	21	640	1	7	5
32	2	245	21	736	7	5	23
32	7	277	21	832	4	6	13
32	3	309	21	928	1	5	29
32	8	341	21	1024	7	10	1
32	4	373	21	1120	4	5	35
32	0	405	21	1216	1	6	19
32	5	437	21	1312	7	5	41
32	1	469	21	1408	4	7	11
32	6	501	21	1504	1	5	47
32	2	533	21	1600	7	6	25
32	7	565	21	1696	4	5	53
32	3	597	21	1792	1	8	7
32	8	629	21	1888	7	5	59
32	4	661	21	1984	4	6	31
32	0	693	21	2080	1	5	65
32	5	725	21	2176	7	7	17
32	1	757	21	2272	4	5	71
32	6	789	21	2368	1	6	37
32	2	821	21	2464	7	5	77
32	7	853	21	2560	4	9	5
32	3	885	21	2656	1	5	83
32	8	917	21	2752	7	6	43
32	4	949	21	2848	4	5	89
32	0	981	21	2944	1	7	23
32	5	1013	21	3040	7	5	95
32	1	1045	21	3136	4	6	49
32	6	1077	21	3232	1	5	101
32	2	1109	21	3328	7	8	13
32	7	1141	21	3424	4	5	107
32	3	1173	21	3520	1	6	55
32	8	1205	21	3616	7	5	113
32	4	1237	21	3712	4	7	29
32	0	1269	21	3808	1	5	119
32	5	1301	21	3904	7	6	61

Filter the powers of 2 >5.

Every 64 numbers starting from 21 we find an exponent of 2 >5.

The ODD number input is ≡ 21 (mod32)

Din-Din-1	Din (mod9)	Din	Din (mod32)	P1-P4-P7	P(mod9)	t	$P/2^t$
	3	21	21	64	1	6	1
64	4	85	21	256	4	8	1
64	5	149	21	448	7	6	7
64	6	213	21	640	1	7	5
64	7	277	21	832	4	6	13
64	8	341	21	1024	7	10	1
64	0	405	21	1216	1	6	19
64	1	469	21	1408	4	7	11
64	2	533	21	1600	7	6	25
64	3	597	21	1792	1	8	7
64	4	661	21	1984	4	6	31
64	5	725	21	2176	7	7	17
64	6	789	21	2368	1	6	37
64	7	853	21	2560	4	9	5
64	8	917	21	2752	7	6	43
64	0	981	21	2944	1	7	23
64	1	1045	21	3136	4	6	49
64	2	1109	21	3328	7	8	13
64	3	1173	21	3520	1	6	55
64	4	1237	21	3712	4	7	29
64	5	1301	21	3904	7	6	61
64	6	1365	21	4096	1	12	1
64	7	1429	21	4288	4	6	67
64	8	1493	21	4480	7	7	35
64	0	1557	21	4672	1	6	73
64	1	1621	21	4864	4	8	19
64	2	1685	21	5056	7	6	79
64	3	1749	21	5248	1	7	41
64	4	1813	21	5440	4	6	85
64	5	1877	21	5632	7	9	11
64	6	1941	21	5824	1	6	91
64	7	2005	21	6016	4	7	47
64	8	2069	21	6208	7	6	97
64	0	2133	21	6400	1	8	25
64	1	2197	21	6592	4	6	103
64	2	2261	21	6784	7	7	53
64	3	2325	21	6976	1	6	109
64	4	2389	21	7168	4	10	7
64	5	2453	21	7360	7	6	115

“Pump-up” mechanism.

As can be seen from table mod 32, exponent 1 and exponents > 1 alternate every 18 numbers. In order to achieve a trend that causes the number to plummet towards 1, one must find the **exponents of 2 'virtuous' numbers that alternate every 36**.

This happens when you reach, in an increasing phase, the number "Peak" (the highest value reached in all cycles related to the assigned number) which is always an EQUAL ≡ 7 (mod9). This number is obtained thanks to the mechanism **of "pump upwards"** implemented by D8*3+1=P7, P7/2=D8 which follows one another several times, alternating gearbox cycles with approach cycles ex. D1*3+1=P4, P4/2 = D2, D2*3+1=P7, P7/2=D8.

The only possibility that the algorithm has to grow the number is to divide by 2 with exponent 1. **However, this condition is not arbitrary but dictated by numerical possibilities**. The deterministic alternation of amplifier and reducer cycles determines the course of the function. It can be said that the algorithm "searches" the way back to 1 in the highest numbers

n° cycles (3x+1)/2	D8	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2
1	17	26						
2	35	53	80					
1	53	80						
3	71	107	161	242				
1	89	134						
2	107	161	242					
1	125	188						
4	143	215	323	485	728			
1	161	242						
2	179	269	404					
1	197	296						
3	215	323	485	728				
1	233	350						
2	251	377	566					
1	269	404						
5	287	431	647	971	1457	2186		
1	305	458						
2	323	485	728					
1	341	512						
3	359	539	809	1214				
1	377	566						
2	395	593	890					
1	413	620						
4	431	647	971	1457	2186			
1	449	674						
2	467	701	1052					
1	485	728						
3	503	755	1133	1700				
1	521	782						
2	539	809	1214					
1	557	836						
6	575	863	1295	1943	2915	4373	6560	
1	593	890						
2	611	917	1376					
1	629	944						
3	647	971	1457	2186				
1	665	998						

The cycle D8*3+1=P7/2=D8

Repeats until P8 is generated

Examples of upward pump:

n° cycles (3x+1)/2	D8	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2	(3x+1)/2
6	883583	1325375	1988063	2982095	4473143	6709715	10064573									
1	883601	1325402														
2	883619	1325429	1988144													
1	883637	1325456														
3	883655	1325483	1988225	2982338												
1	883673	1325510														
2	883691	1325537	1988306													
1	883709	1325564														
4	883727	1325591	1988387	2982581	4473872											
1	883745	1325618														
2	883763	1325645	1988468													
1	883781	1325672														
3	883799	1325699	1988549	2982824												
1	883817	1325726														
2	883835	1325753	1988630													
1	883853	1325780														
5	883871	1325807	1988711	2983067	4474601	6711902										
1	883889	1325834														
2	883907	1325861	1988792													
1	883925	1325888														
3	883943	1325915	1988873	2983310												
1	883961	1325942														
2	883979	1325969	1988954													
1	883997	1325996														
4	884015	1326023	1989035	2983553	4475330											
1	884033	1326050														
2	884051	1326077	1989116													
1	884069	1326104														
3	884087	1326131	1989197	2983796												
1	884105	1326158														
2	884123	1326185	1989278													
1	884141	1326212														
6	884159	1326239	1989359	2984039	4476059	6714089	10071134									
1	884177	1326266														
2	884195	1326293	1989440													
1	884213	1326320														
3	884231	1326347	1989521	2984282												
1	884249	1326374														
2	884267	1326401	1989602													
1	884285	1326428														
4	884303	1326455	1989683	2984525	4476788											
1	884321	1326482														
2	884339	1326509	1989764													
1	884357	1326536														
3	884375	1326563	1989845	2984768												
1	884393	1326590														
2	884411	1326617	1989926													
1	884429	1326644														
5	884447	1326671	1990007	2985011	4477517	6716276										
1	884465	1326698														
2	884483	1326725	1990088													
1	884501	1326752														
3	884519	1326779	1990169	2985254												
1	884537	1326806														
2	884555	1326833	1990250													
1	884573	1326860														
4	884591	1326887	1990331	2985497	4478246											
1	884609	1326914														
2	884627	1326941	1990412													
1	884645	1326968														
3	884663	1326995	1990493	2985740												
1	884681	1327022														
2	884699	1327049	1990574													
1	884717	1327076														
15	884735	1327103	1990655	2985983	4478975	6718463	10077695	15116543	22674815	34012223	51018335	76527503	114791255	172186883	258280325	387420488

The first column that explains the number of cycles D8*3+1=P7, P7/2=D8 shows that this procedure also follows **cycle 16**.

It stops reached an EVEN ≡ 8(MOD9) which is divisible by a power of 2 with exponent > of 1.

There are no cycles that make the number grow faster than the “pump up“ mechanism.

When upward pump cycles become frequent, although alternating with /2 cycles with exponent >1, the number grows faster.

We have seen how the aforementioned routine follows the deterministic cycle 16 which is applicable only to ODD numbers ≡ 8 (mod9).

Of these, however, only 50% have as divisor 2^1, while all ODD numbers have a 50% chance that, after applying the condition 3x+1, they become an EVEN divisible by a power of 2 > 1

The ODD ≡ 8 (mod9) are 1/9 of the ODD numbers = $11.\bar{1}$%, so there are $5,\bar{5}$% of ODD which are D8 that activate the pump mechanism.

Din (mod9)	Din	Din (mod32)	P1-P4-P7	P(mod9)	t	$P/2^t$
8	17	17	52	7	2	13
8	35	3	106	7	1	53
8	53	21	160	7	5	5
8	71	7	214	7	1	107
8	89	25	268	7	2	67
8	107	11	322	7	1	161
8	125	29	376	7	3	47
8	143	15	430	7	1	215
8	161	1	484	7	2	121
8	179	19	538	7	1	269
8	197	5	592	7	4	37
8	215	23	646	7	1	323
8	233	9	700	7	2	175
8	251	27	754	7	1	377
8	269	13	808	7	3	101
8	287	31	862	7	1	431

One possible way to validate the conjecture is to prove:

- There are no loops except for 1-4-2-1...
- There are no routines that lead to infinity.

<u>Given the flow scheme I can write</u>:

D1*3+1=P4 , P4/2=P2 , P2/2=P4/4=D1

1*3+1 =4 , 4/2=2 , 2/2 = 4/4 = 1

(D1*3+1)/4 = D1*3/4+1/4 = D1

D1-D1*3/4-1/4 = 0

D1* (1-3/4)-1/4 = 0

D1*1/4 -1/4 = 0

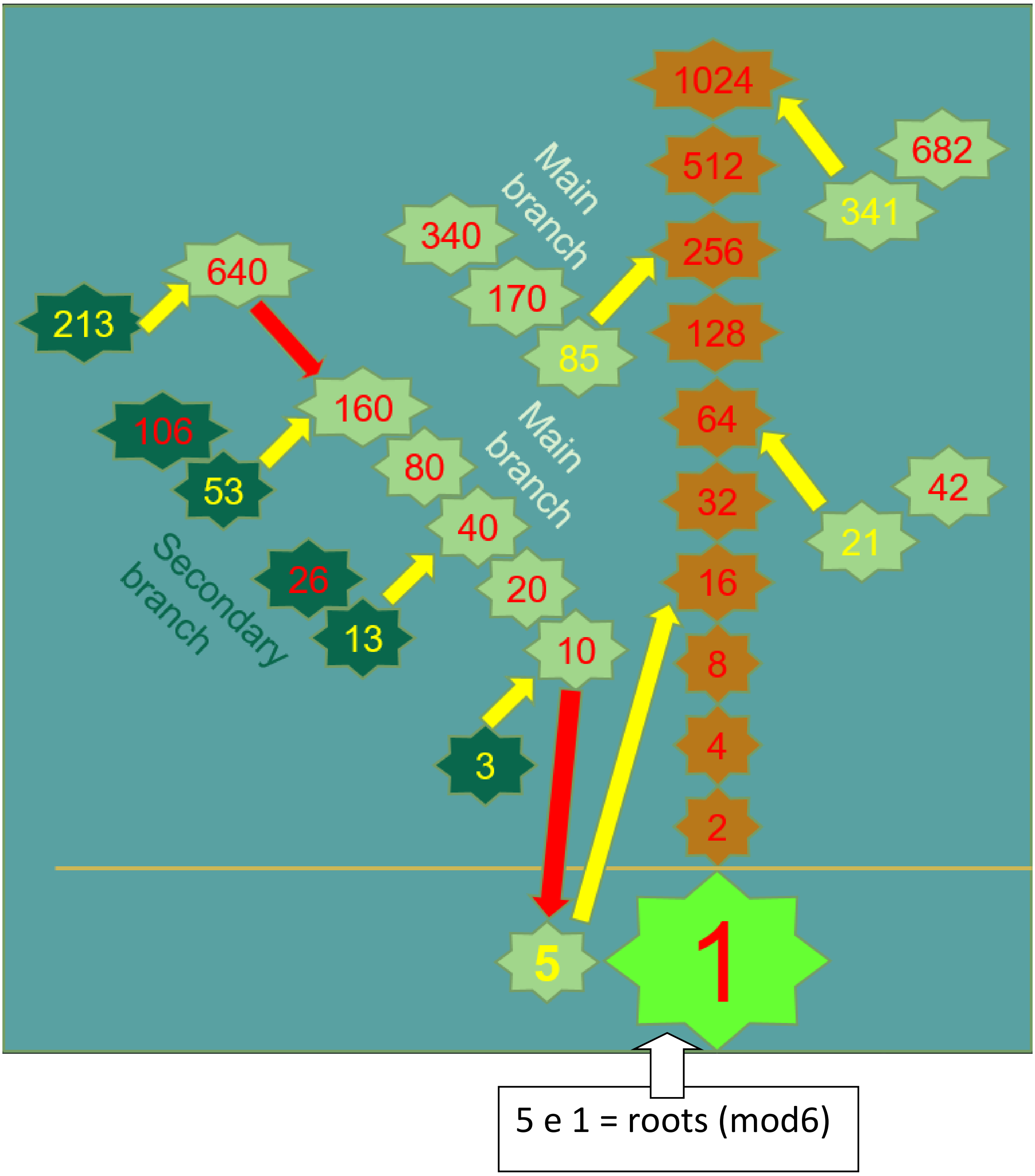
D1*1/4 = 1/4

D1=(1/4)/(1/4) = 1 $\Longrightarrow$ D1= 1 is the only possible solution of this routine. (x*3+1)/4 = x has solution x=1

I will not follow the statistical route or chasing possible routines.

I will formulate below the equations that will show how all positive numbers are present in the Collatz tree, connected to 1, then reachable using the 2 conditions.

The Collatz graph tree:



EVEN numbers not reachable by 3*x+1 (ex. n° 26) after the division by 2^t they become ODD and enter Block 2 and graph.

I derive the equations by looking at the data of the tables that highlight the powers of 2. I can write for the exponents of 2 ODD:

First ODD number input where the exponent of 2 ODD t = 1 appears for the first time

Succession: 3,7,11,15... ; (3+4n)= 3 with n=0
(3+4n)= 7 with n=1 ; (3+4n)= 11 with n=2

$$\frac{(3+4n)*3+1}{2^1}$$

I can write 2^{t+1}

$$\frac{(13+16n)*3+1}{2^3}$$

$$\frac{(53+64n)*3+1}{2^5}$$

I am looking for an increase:

$213 - 53 = 160 = 10*2^{t-1} = 10*2^4 = 10*16$

$53 - 13 = 40 = 10*2^{t-1} = 10*2^2 = 10*4$

$13 - 3 = 10 = 10*2^{t-1} = 10*2^0 = 10*1$

$$\frac{(213+256n)*3+1}{2^7}$$

t= exponent of 2

EQUATION IN BLOCK 2 for ODD POWERS OF 2 with n from 0 to ∞:

$$\frac{\left(\left(\frac{10*2^{t-1}-1}{3}+2^{t+1}*n\right)*3+1\right)}{2^t} = 5+6n$$

$$\left(\frac{10*2^{t-1}-1}{3}+2^{t+1}*n\right) = \text{Din}$$

Din*3+1= P

$P/2^t$ = Dout

Root $= \frac{10*2^{t-1}-1}{3} = \frac{5*2^t-1}{3}$ obtained by the inverse function becomes the secondary branch of the tree

Module $= 2^{t+1}$

$n = 0$

t= D	inverse formula (**P** -1)/3 =	Din*3+1 =	(Din * 3+1)/2^t =
t	**Din**	**P**	**Dout**
1	3	10	5
3	13	40	5
5	53	160	5
7	213	640	5
9	853	2560	5
11	3413	10240	5
13	13653	40960	5
15	54613	163840	5
17	218453	655360	5
19	873813	2621440	5
21	3495253	10485760	5
23	13981013	41943040	5
25	55924053	167772160	5
27	223696213	671088640	5
29	894784853	2684354560	5
31	3579139413	10737418240	5

concise way to generate Din numbers from 13 from the table,

$$3 + \sum_{t:0}^{\infty} 2^{2t} * 10$$

or the sequence can be generated recursively:

$a_n = a_{n-1} * q + 1$ with $q = 4$ and $a_1 = 3$

I can write for the exponents of 2 EVEN:

First ODD number input where the exponent of 2 EVEN t = 2 appears for the first time

I can write 2^{t+1}

I am looking for an increase:

$85 - 21 = 64 = 2^6$

$21 - 5 = 16 = 2^4$

$5 - 1 = 4 = 2^2$

$$\frac{(1+8n)*3+1}{2^2}$$

$$\frac{(5+32n)*3+1}{2^4}$$

$$\frac{(21+128n)*3+1}{2^6}$$

$$\frac{(85+512n)*3+1}{2^8}$$

EQUATION IN BLOCK 2 for POWERS OF 2 EVEN with n from 0 to ∞:

$$\frac{\left(\frac{2^t-1}{3}+2^{t+1}*n\right)*3+1}{2^t} = 1+6n$$

$$\left(\frac{2^t-1}{3}+2^{t+1}*n\right) = \text{Din}$$

Din*3+1= P

P/2^t = Dout

Root = $\frac{2^t-1}{3}$ obtained by the inverse function becomes the main branch of the tree

Module = 2^{t+1}

n=0

t= P t	inverse formula (P -1)/3 = Din	Din*3+1 = powers of 2 n=0 P	(Din * 3+1)/ = 2^t Dout
2	1	4	1
4	5	16	1
6	21	64	1
8	85	256	1
10	341	1024	1
12	1365	4096	1
14	5461	16384	1
16	21845	65536	1
18	87381	262144	1
20	349525	1048576	1
22	1398101	4194304	1
24	5592405	16777216	1
26	22369621	67108864	1
28	89478485	268435456	1
30	357913941	1073741824	1
32	1431655765	4294967296	1

synthetic way to generate the Din numbers from the table,

$$\sum_{t:0}^{\infty} 2^{2t}$$

or the sequence can be generated recursively:

$a_n = a_{n-1} * q + 1$ with $q = 4$ and $a_1 = 1$

Both equations determining the number Din have as modulo 2^{t+1}

The 2 unfolding equations can be written as follows:

$$\frac{Din*3+1}{2^t} = 1 + 6n \qquad \text{with} \qquad t = \text{EVEN}$$

$$\text{Din} = \frac{2^t(1+6n)-1}{3}$$

where (1+6n)= ODD output

and 2^t(1+6n)= EVEN inserted in the inverse formula

$$\text{Din} = \frac{2^t-1}{3} + 2^{t+1} * n = \frac{2^t(1+6n)-1}{3}$$

verification:

$$2^t - 1 + 2^{t+1} * 3n = 2^t + 2^t * 6n - 1$$

$$2^{t+1} * 3 = 2^t * 6$$

$$2^t * 2 * 3 = 2^t * 6$$

$$1 = 1$$

$$\frac{Din*3+1}{2^t} = 5 + 6n \qquad \text{with} \qquad t=\text{ODD}$$

$$\text{Din} = \frac{2^t(5+6n)-1}{3}$$

where (5+6n)= ODD output

and 2^t(5+6n)= EVEN inserted in the inverse formula

$$\text{Din} = \left(\frac{10*2^{t-1}-1}{3} + 2^{t+1} * n\right) = \frac{2^t(5+6n)-1}{3}$$

verification:

$$10*2^{t-1} - 1 + 2^{t+1} * 3n = 2^t * 5 + 2^t * 6 * n - 1$$

$$5*2^t + 3*2^{t+1} = 5*2^t + 2^t * 6$$

$$2^t * 2 * 3 = 2^t * 6$$

$$1 = 1$$

All ODD numbers can be written in the form:

$$\frac{2^t(1+6n)-1}{3} \quad \text{with even t}$$

$$\frac{2^t(5+6n)-1}{3} \quad \text{with odd t}$$

$$\text{Din} = 1+2\text{p} = \frac{2^t(1+6n)-1}{3} \quad \text{with even t}$$

$$\text{Din} = 1+2\text{p} = \frac{2^t(5+6n)-1}{3} \quad \text{with odd t}$$

I can express Dout ≡ 1,5 (mod6) as:

$$1+6\text{n} = \mathbf{1+3*2n}$$

$$5+6\text{n} = 2+3+3*2\text{n} = \mathbf{2+3*(1+2n)}$$

$$\text{Dout} \equiv 1 \pmod 6 \;=\; \text{Dout} \equiv 1 \pmod 3$$

$$\text{Dout} \equiv 5 \pmod 6 \;=\; \text{Dout} \equiv 2 \pmod 3$$

$$\text{k}= 2\text{n} \quad ; \quad \text{k}=1+2\text{n}$$

and also:

$$\mathbf{Dout = \; r+3k}$$

$$\text{con k} = 0 \div \infty \quad ; \quad \text{r} = 1+ \text{rest of } \text{k}/2 = 1 \div 2$$

by going from (mod 6) to (mod 3) I can reduce the two equations to one:

$$\text{Din} = 1+2\text{n} = \frac{2^t(r+3k)-1}{3}$$

$$1+3+6n = 2^t(r+3k)$$

$$\boxed{\frac{\mathbf{4+6n}}{\mathbf{2^t}} = \mathbf{r+3k}}$$

Which is equivalent to saying:

$$\boxed{\frac{\mathbf{(1+2n)*3+1}}{\mathbf{2^t}} = \mathbf{r+3k}}$$

Unfolding the equation:

$$\frac{3+6n+1}{2^t} = r+3k$$

$$\text{with} \quad \begin{cases} n = 0 \div \infty \\ t = 1 \div \infty \\ k = 0 \div \infty \\ r = 1 + \text{rest} \quad \text{of } k/2 = 1 \div 2 \end{cases}$$

$$k = \frac{\frac{4+6n}{2^t} - r}{3} = \text{integer result}$$

<u>The number EVEN = **4+6n**, can be reached by the inverse function</u>

Powers of 2 bind all ODD input numbers to ODD output numbers.

Below we look at the tables by varying the exponent t:

equation for exponents of 2 EVEN equazione per esponenti di 2 PARI	t=>2	n=	0

$$\frac{\left(\frac{2^t-1}{3}+2^{t+1}*n\right)*3+1}{2^t}=1+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = P/2^t		Dout = 1+6*n	(mod6)	t
1	1	4	4	1	=	1	1	2
5	5	16	16	1	=	1	1	4
3	21	64	64	1	=	1	1	6
1	85	256	256	1	=	1	1	8
5	341	1024	1024	1	=	1	1	10
3	1365	4096	4096	1	=	1	1	12
1	5461	16384	16384	1	=	1	1	14
5	21845	65536	65536	1	=	1	1	16
3	87381	262144	262144	1	=	1	1	18
1	349525	1048576	1048576	1	=	1	1	20
5	1398101	4194304	4194304	1	=	1	1	22
3	5592405	16777216	16777216	1	=	1	1	24
1	22369621	67108864	67108864	1	=	1	1	26
5	89478485	268435456	268435456	1	=	1	1	28
3	357913941	1073741824	1073741824	1	=	1	1	30
1	1431655765	4294967296	4294967296	1	=	1	1	32
5	5726623061	17179869184	17179869184	1	=	1	1	34
3	22906492245	68719476736	68719476736	1	=	1	1	36
1	91625968981	274877906944	274877906944	1	=	1	1	38
5	366503875925	1099511627776	1099511627776	1	=	1	1	40
1	1466015503701	4398046511104	4398046511104	1	=	1	1	42
5	5864062014805	17592186044416	17592186044416	1	=	1	1	44
3	23456248059221	70368744177664	70368744177664	1	=	1	1	46
1	93824992236885	281474976710656	281474976710656	1	=	1	1	48
5	375299968947541	1125899906842620	1125899906842620	1	=	1	1	50

equation for exponents of 2 ODD equazione per esponenti di 2 DISPARI	t=>1	n=	0

$$\frac{\left(\left(\frac{10*2^{t-1}-1}{3}+2^{t+1}*n\right)*3+1\right)}{2^t}=5+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = P/2^t		Dout = 5+6*n	(mod6)	t
3	3	10	2	5	=	5	5	1
1	13	40	8	5	=	5	5	3
5	53	160	32	5	=	5	5	5
3	213	640	128	5	=	5	5	7
1	853	2560	512	5	=	5	5	9
5	3413	10240	2048	5	=	5	5	11
3	13653	40960	8192	5	=	5	5	13
1	54613	163840	32768	5	=	5	5	15
5	218453	655360	131072	5	=	5	5	17
3	873813	2621440	524288	5	=	5	5	19
1	3495253	10485760	2097152	5	=	5	5	21
5	13981013	41943040	8388608	5	=	5	5	23
3	55924053	167772160	33554432	5	=	5	5	25
1	223696213	671088640	134217728	5	=	5	5	27
5	894784853	2684354560	536870912	5	=	5	5	29
3	3579139413	10737418240	2147483648	5	=	5	5	31
1	14316557653	42949672960	8589934592	5	=	5	5	33
5	57266230613	171798691840	34359738368	5	=	5	5	35
3	229064922453	687194767360	137438953472	5	=	5	5	37
3	916259689813	2748779069440	549755813888	5	=	5	5	39
1	3665038759253	10995116277760	2199023255552	5	=	5	5	41
5	14660155037013	43980465111040	8796093022208	5	=	5	5	43
3	58640620148053	175921860444160	35184372088832	5	=	5	5	45
1	234562480592213	703687441776640	140737488355328	5	=	5	5	47
5	938249922368853	2814749767106560	562949953421312	5	=	5	5	49

Varying n:

equation for exponents of 2 EVEN		t=>2
equazione per esponenti di 2 PARI	t=	2

$$\frac{\left(\frac{2^t-1}{3}+2^{t+1}*n\right)*3+1}{2^t}=1+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = P/2^t		Dout = 1+6*n	(mod6)	n
1	1	4	4	1	=	1	1	0
3	9	28	4	7	=	7	1	1
5	17	52	4	13	=	13	1	2
1	25	76	4	19	=	19	1	3
3	33	100	4	25	=	25	1	4
5	41	124	4	31	=	31	1	5
1	49	148	4	37	=	37	1	6
3	57	172	4	43	=	43	1	7
5	65	196	4	49	=	49	1	8
1	73	220	4	55	=	55	1	9
3	81	244	4	61	=	61	1	10
5	89	268	4	67	=	67	1	11
1	97	292	4	73	=	73	1	12

equation for exponents of 2 EVEN		t=>2
equazione per esponenti di 2 PARI	t=	18

$$\frac{\left(\frac{2^t-1}{3}+2^{t+1}*n\right)*3+1}{2^t}=1+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = P/2^t		Dout = 1+6*n	(mod6)	n
3	87381	262144	262144	1	=	1	1	0
5	611669	1835008	262144	7	=	7	1	1
1	1135957	3407872	262144	13	=	13	1	2
3	1660245	4980736	262144	19	=	19	1	3
5	2184533	6553600	262144	25	=	25	1	4
1	2708821	8126464	262144	31	=	31	1	5
3	3233109	9699328	262144	37	=	37	1	6
5	3757397	11272192	262144	43	=	43	1	7
1	4281685	12845056	262144	49	=	49	1	8
3	4805973	14417920	262144	55	=	55	1	9
5	5330261	15990784	262144	61	=	61	1	10
1	5854549	17563648	262144	67	=	67	1	11
3	6378837	19136512	262144	73	=	73	1	12

equation for exponents of 2 ODD		t=>1
equazione per esponenti di 2 DISPARI	t=	3

$$\frac{\left(\left(\frac{10*2^{t-1}-1}{3}+2^{t+1}*n\right)*3+1\right)}{2^t}=5+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = $P/2^t$		Dout = 5+6*n	(mod6)	n
1	13	40	8	5	=	5	5	0
5	29	88	8	11	=	11	5	1
3	45	136	8	17	=	17	5	2
1	61	184	8	23	=	23	5	3
5	77	232	8	29	=	29	5	4
3	93	280	8	35	=	35	5	5
1	109	328	8	41	=	41	5	6
5	125	376	8	47	=	47	5	7
3	141	424	8	53	=	53	5	8
1	157	472	8	59	=	59	5	9
5	173	520	8	65	=	65	5	10
3	189	568	8	71	=	71	5	11
1	205	616	8	77	=	77	5	12

equation for exponents of 2 ODD		t=>1
equazione per esponenti di 2 DISPARI	t=	11

$$\frac{\left(\left(\frac{10*2^{t-1}-1}{3}+2^{t+1}*n\right)*3+1\right)}{2^t}=5+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = $P/2^t$		Dout = 5+6*n	(mod6)	n
5	3413	10240	2048	5	=	5	5	0
3	7509	22528	2048	11	=	11	5	1
1	11605	34816	2048	17	=	17	5	2
5	15701	47104	2048	23	=	23	5	3
3	19797	59392	2048	29	=	29	5	4
1	23893	71680	2048	35	=	35	5	5
5	27989	83968	2048	41	=	41	5	6
3	32085	96256	2048	47	=	47	5	7
1	36181	108544	2048	53	=	53	5	8
5	40277	120832	2048	59	=	59	5	9
3	44373	133120	2048	65	=	65	5	10
1	48469	145408	2048	71	=	71	5	11
5	52565	157696	2048	77	=	77	5	12

Distribution of equations.

Cycle 32 of equations:

equation for exponents of 2 EVEN	equation for exponents of 2 ODD
equazione per esponenti di 2 PARI	equazione per esponenti di 2 DISPARI

$$\frac{\left(\frac{2^t-1}{3}+2^{t+1}*n\right)*3+1}{2^t}=1+6n$$

$$\frac{\left(\left(\frac{10*2^{t-1}-1}{3}+2^{t+1}*n\right)*3+1\right)}{2^t}=5+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = P/ 2^t		1+6*n 5+6*n	(mod6)	(mod3)	n
1	1	4	4	1	=	1	1	1	0
3	3	10	2	5	=	5	5	2	0
5	5	16	16	1	=	1	1	1	0
1	7	22	2	11	=	11	5	2	1
3	9	28	4	7	=	7	1	1	1
5	11	34	2	17	=	17	5	2	2
1	13	40	8	5	=	5	5	2	0
3	15	46	2	23	=	23	5	2	3
5	17	52	4	13	=	13	1	1	2
1	19	58	2	29	=	29	5	2	4
3	21	64	64	1	=	1	1	1	0
5	23	70	2	35	=	35	5	2	5
1	25	76	4	19	=	19	1	1	3
3	27	82	2	41	=	41	5	2	6
5	29	88	8	11	=	11	5	2	1
1	31	94	2	47	=	47	5	2	7
3	33	100	4	25	=	25	1	1	4
5	35	106	2	53	=	53	5	2	8
1	37	112	16	7	=	7	1	1	1
3	39	118	2	59	=	59	5	2	9
5	41	124	4	31	=	31	1	1	5
1	43	130	2	65	=	65	5	2	10
5	45	136	8	17	=	17	5	2	2
1	47	142	2	71	=	71	5	2	11
3	49	148	4	37	=	37	1	1	6
5	51	154	2	77	=	77	5	2	12
3	53	160	32	5	=	5	5	2	0
5	55	166	2	83	=	83	5	2	13
1	57	172	4	43	=	43	1	1	7
3	59	178	2	89	=	89	5	2	14
5	61	184	8	23	=	23	5	2	3
1	63	190	2	95	=	95	5	2	15

In the succession of equations, with EVEN and ODD exponent following each other following a cycle 32, there is a variation starting with the number 213 as the EVEN>5 exponent becomes ODD>5 and this occurs at the <u>eleventh ordinal number of cycle 32</u> which is ≡ 21 (mod32) with $Din-Din_{-1}$ cadence varying between 128 and 256.

$Din-Din_{-1}$	Din (mod9)	Din	Din (mod32)	P1-P4-P7	P(mod9)	t	$P/2^t$
	6	213	21	640	1	7	5
256	1	469	21	1408	4	7	11
256	5	725	21	2176	7	7	17
128	7	853	21	2560	4	9	5
128	0	981	21	2944	1	7	23
256	4	1237	21	3712	4	7	29
256	8	1493	21	4480	7	7	35
256	3	1749	21	5248	1	7	41
128	5	1877	21	5632	7	9	11
128	7	2005	21	6016	4	7	47
256	2	2261	21	6784	7	7	53
256	6	2517	21	7552	1	7	59
256	1	2773	21	8320	4	7	65
128	3	2901	21	8704	1	9	17
128	5	3029	21	9088	7	7	71
256	0	3285	21	9856	1	7	77
128	2	3413	21	10240	7	11	5
128	4	3541	21	10624	4	7	83
256	8	3797	21	11392	7	7	89
128	1	3925	21	11776	4	9	23
128	3	4053	21	12160	1	7	95
256	7	4309	21	12928	4	7	101
256	2	4565	21	13696	7	7	107
256	6	4821	21	14464	1	7	113
128	8	4949	21	14848	7	9	29

The following equation applies for all exponents of 2 and all n:

$$\mathrm{Din}_{+1} - \mathrm{Din} = \mathrm{Din}*3+1$$

⇩

$$\mathrm{Din}_{+1} = \mathrm{Din} + \mathrm{Din}*3+1 = \mathbf{Din*4+1}$$

ex.

equation for exponents of 2 EVEN

equazione per esponenti di 2 PARI

t=>2

n= 295

$$\frac{\left(\frac{2^t-1}{3}+2^{t+1}*n\right)*3+1}{2^t}=1+6n$$

(mod6)	Din	P = Din*3+1	2^t	Dout = P/2^t		Dout = 1+6*n	(mod6)	t	Din_{+1}-Din
3	2361	7084	4	1771	=	1771	1	2	7084
1	9445	28336	16	1771	=	1771	1	4	28336
5	37781	113344	64	1771	=	1771	1	6	113344
3	151125	453376	256	1771	=	1771	1	8	453376
1	604501	1813504	1024	1771	=	1771	1	10	1813504
5	2418005	7254016	4096	1771	=	1771	1	12	7254016
3	9672021	29016064	16384	1771	=	1771	1	14	29016064
1	38688085	116064256	65536	1771	=	1771	1	16	116064256
5	154752341	464257024	262144	1771	=	1771	1	18	464257024
3	619009365	1857028096	1048576	1771	=	1771	1	20	1857028096
1	2476037461	7428112384	4194304	1771	=	1771	1	22	7428112384
5	9904149845	29712449536	16777216	1771	=	1771	1	24	29712449536

$$\mathrm{Din}_{+1} = 4\left(\frac{2^t-1}{3}+2^{t+1}*n\right)+1 = \frac{2^{t+2}-4+3\cdot2^{t+3}\cdot n+3}{3} = \frac{1*2^{t+2}-1}{3}+2^{t+3}*n$$

t even

$$\mathrm{Din}_{+1} = 4\left(\frac{10*2^{t-1}-1}{3}+2^{t+1}*n\right)+1 = \frac{5\cdot2^{t+2}-4+3\cdot2^{t+3}\cdot n+3}{3} = \frac{5*2^{t+2}-1}{3}+2^{t+3}*n$$

t odd

4*x+1 generates the sequence of ODD numbers that, applied the 2 conditions, share the same ODD outgoing.

Thanks to this equation **I get infinite ODD input that have in common the same ODD output. <u>The 3x+1 interval is the "measure",</u> the distance between an incoming ODD and the next.**

The interval 3x+1 is a power of 2 or a multiple of a power of 2

Numbers in and out of Block 2 ODD-ODD.

The algorithm using the formula: $(3*x+1)/2^t$ eliminates the ODD numbers ≡ 3 (mod6) (multiples of 3 highlighted in yellow), which will not be repeated as input in the next cycle.

The same as can be seen from the table below are ODD ≡ 0,3,6 (mod9) which become EVEN ≡ 1 (mod9) (highlighted in red) after applying the condition 3x+1.

Din	(mod6)	(mod9)	P= Din*3+1	(mod6)	(mod9)	Dout = P/2^t	(mod6)	(mod9)	t
1	1		4	4	4	1	1	1	2
3	3	3	10	4	1	5	5	5	1
5	5	5	16	4	7	1	1	1	4
7	1	7	22	4	4	11	5	2	1
9	3	0	28	4	1	7	1	7	2
11	5	2	34	4	7	17	5	8	1
13	1	4	40	4	4	5	5	5	3
15	3	6	46	4	1	23	5	5	1
17	5	8	52	4	7	13	1	4	2
19	1	1	58	4	4	29	5	2	1
21	3	3	64	4	1	1	1	1	6
23	5	5	70	4	7	35	5	8	1
25	1	7	76	4	4	19	1	1	2
27	3	0	82	4	1	41	5	5	1
29	5	2	88	4	7	11	5	2	3
31	1	4	94	4	4	47	5	2	1
33	3	6	100	4	1	25	1	7	2
35	5	8	106	4	7	53	5	8	1
37	1	1	112	4	4	7	1	7	4
39	3	3	118	4	1	59	5	5	1
41	5	5	124	4	7	31	1	4	2
43	1	7	130	4	4	65	5	2	1
45	3	0	136	4	1	17	5	8	3
47	5	2	142	4	7	71	5	8	1
49	1	4	148	4	4	37	1	1	2
51	3	6	154	4	1	77	5	5	1
53	5	8	160	4	7	5	5	5	5
55	1	1	166	4	4	83	5	2	1
57	3	3	172	4	1	43	1	7	2
59	5	5	178	4	7	89	5	8	1
61	1	7	184	4	4	23	5	5	3
63	3	0	190	4	1	95	5	5	1

We will have 2 possible output roots: 1,5 (mod6) as deduced from the table and by what is stated on pages 10 to 14 and the following equations:

$$x = 1+2n$$

$$\mathbf{(1+2n)*3+1 = 4+6n}$$

$$\frac{\mathbf{4+6n}}{\mathbf{2^t}} = \mathbf{1+6*m}$$

after dividing a power of 2 EVEN

$$\frac{\mathbf{4+6n}}{\mathbf{2^t}} = \mathbf{5+6*m}$$

after dividing a power of 2 ODD.

Leading to the second member of equation 2^t we can write for the

n = ODD = $1+2^t m + c$

with t=1 ; m=0÷∞ ; c=0

$$4+6*(1+2^t m + c) = 2^t*(\mathbf{5+6m})$$

$$4+6+12m = 10+12m$$

$$10+12m = 10+12m$$

$$10*2^0 +(2^1+10*2^0)*m = 10*2^0 +(2+10*2^0)*m$$

We can write for the n = EVEN = $1+2^t m + c$

with t=3 ; m=0÷∞ ; c=5

$$4+6*(1+2^3 m+5) = 2^3*(\mathbf{5+6m})$$

$$40+48m=40+48m$$

$$10*2^2 +(2^3+10*2^2)*m = 10*2^2 +(2^3+10*2^2)*m$$

with $t=5$; $m=0 \div \infty$; $c=25$

$$4+6*(1+2^5m+25) = 2^5*(\mathbf{5+6m})$$

$$160+192m = 160+192m$$

$$10*2^4+(2^5+10*2^4)*m = 10*2^4+(2^5+10*2^4)*m$$

with $t=7$; $m=0 \div \infty$; $c=105$

$$4+6*(1+2^7m+105) = 2^7*(\mathbf{5+6m})$$

$$640+768m = 640+768m$$

$$10*2^6+(2^7+10*2^6)*m = 10*2^6+(2^7+10*2^6)*m$$

with $t=9$; $m=0 \div \infty$; $c=425$

$$4+6*(1+2^9m+425) = 2^9*(\mathbf{5+6m})$$

$$2560+3072m = 2560+3072m$$

$$10*2^8+(2^9+10*2^8)*m = 10*2^8+(2^9+10*2^8)*m$$

with $t=11$; $m=0 \div \infty$; $c=1705$

$$4+6*(1+2^{11}m+1705) = 2^{11}*(\mathbf{5+6m})$$

$$10240+12288m = 10240+12288m$$

$$10*2^{10}+(2^{11}+10*2^{10})*m = 10*2^{10}+(2^{11}+10*2^{10})*m$$

$$10*2^{t-1}+(2^t+10*2^{t-1})*m = 10*2^{t-1}+(2^t+10*2^{t-1})*m$$

$$\mathbf{5*2^t+(2^t+5*2^t)*m = 5*2^t+(2^t+5*2^t)*m}$$

$$5*2^t+(2^t*(1+5))*m = 5*2^t+2^t*6m = \mathbf{2^t*(5+6m)}$$

$$\mathbf{4+6*n} = 4+6*(1+2^tm+c) = 5*2^t+(2^t+5*2^t)*m \mathbf{= 2^t*(5+6m)}$$

with t = ODD ; $m=0 \div \infty$

We can write for the n, t = EVEN and $n = 2^t m + d$

$$4+6*n = 2^t*(\mathbf{1+6m})$$

$$4+6*(2^t m + d) = 2^t*(\mathbf{1+6m})$$

with t=2 : m=0÷∞ ; d=0

$$4+6*2^2 m = 2^2*(\mathbf{1+6m})$$

$$4+24m=4+24m$$

$$2^2+(2^3+2^4)*m = 2^2+(2^3+2^4)*m$$

with t=4 ; m=0÷∞ ; d=2

$$4+6*(2^4 m+2) = 2^4*(\mathbf{1+6m})$$

$$16+96m=16+96m$$

$$2^4+(2^5+2^6)*m = 2^4+(2^5+2^6)*m$$

with t=6 ; m=0÷∞ ; d=10

$$4+6*(2^6 m+10) = 2^6*(\mathbf{1+6m})$$

$$64+384m=64+384m$$

$$2^6+(2^7+2^8)*m = 2^6+(2^7+2^8)*m$$

with t=8 ; m=0÷∞ ; d=42

$$4+6*(2^8 m+42) = 2^8*(\mathbf{1+6m})$$

$$256+1536m=256+1536m$$

$$2^8+(2^9+2^{10})*m = 2^8+(2^9+2^{10})*m$$

with $t=10$; $m=0\div\infty$; $d=170$

$$4+6*(2^{10}m+170) = 2^{10}*(\mathbf{1+6m})$$

$$1024+6144m=1024+6144m$$

$$2^{10}+(2^{11}+2^{12})*m = 2^{10}+(2^{11}+2^{12})*m$$

$$\mathbf{2^{t}+(2^{t+1}+2^{t+2})*m = 2^{t}+(2^{t+1}+2^{t+2})*m}$$

$$2^{t}+(2^{t}*2^{1}+2^{t}*2^{2})*m = 2^{t}+(2^{t}*2+2^{t}*4)*m$$

$$2^{t}+(2^{t}*(2+4))*m = 2^{t}+(2^{t}*6)*m = \mathbf{2^{t}*(1+6m)}$$

$$4+6*n = 4+6*2^{t}m + d = 2^{t}+(2^{t+1}+2^{t+2})*m = \mathbf{2}^{t}*(\mathbf{1+6m})$$

with t = EVEN ; $m=0\div\infty$

$4+6n \equiv 4(\mathrm{mod}6)$

$$\left.\begin{array}{l} 2^{t}*(5+6m) \equiv 4(\mathrm{mod}6) \\ 2^{1+2k} * 6m \equiv 0(\mathrm{mod}6) \\ 5*2^{1+2k} \equiv 4(\mathrm{mod}6) \end{array}\right\} \quad \begin{array}{l} t = \mathrm{ODD} \\ m = 0 \div \infty \\ k = 0 \div \infty \end{array}$$

In accordance with the properties of congruences module n:

$$[a]+[b] = [a+b]$$

$$n° \equiv 4(\mathrm{mod}6) + n° \equiv 0(\mathrm{mod}6) = n° \equiv 4(\mathrm{mod}6)$$

$$\left.\begin{array}{l} 2^{t}*(1+6m) \equiv 4(\mathrm{mod}6) \\ 2^{2k} * 6m \equiv 0(\mathrm{mod}6) \\ 1*2^{2k} \equiv 4(\mathrm{mod}6) \end{array}\right\} \quad \begin{array}{l} t = \mathrm{EVEN} \\ m = 0 \div \infty \\ k = 1 \div \infty \end{array}$$

$3*2^{t}$ with $t = 0 \div \infty$

it will never be $\equiv 4(\mathrm{mod}6)$

$4+6n \neq 2^{t}*(3+6m)$

To sum up:

1. All positive numbers are present in the binary positional numbering system, where the numbers 1 (high level) represent a power of 2 with exponent, starting from 0, increasing from right to left. The sum of the values obtained by elevating 2 to its exponent returns the decimal number. An EVEN number, expressed with binary numbering, will have one or more less significant digits marked by a 0 (low level). (page 3-4)
2. Eliminate less significant zeros is equivalent to dividing by 2 times as many times as there are zeros. The number thus obtained is ODD. Applying the condition /2 one or more times, i.e. divided by a power of 2, all EVEN numbers become ODD.
   ex. EVEN = 1408_{10} = 10110000000_2 t = 7
   ODD = $1408/2^t$ = $1408/2^7$ = 11_{10} = 1011_2
3. The number **1** is an ODD number and is related to EVEN numbers thanks to the condition /2
4. The flow scheme expressed with (mod9) shows the behavior of all positive numbers as a function of the 2 conditions. It shows how the condition /2 can be reversed allowing the application of the inverse function *2. (page 8÷14)
5. The 3x+1 condition "converts" ODD numbers to EVEN numbers so that the /2 and $/2^t$ conditions can be applied. The same can be reached by the inverse function with 2n and (n-1)/3 and are EVEN numbers ≡ 4 (mod6) and ≡ 1,4,7 (mod9).
6. All multiples of 3, thanks to the 2 conditions, become EVEN numbers ≡ 1 (mod9) and then Dout ≡ 1,4,7, (mod9) ≡ 1 (mod6) or Dout ≡ 2,5,8 (mod9) ≡ 5 (mod6). By doing so, the algorithm connects all multiples of 3 to the hexagram and graph.
   ODD numbers are ≡ 1,3,5 (mod6), ODD numbers ≡ 3 (mod6) are multiples of 3.
   ODD numbers – ODD numbers ≡ 3 (mod6) = ODD numbers ≡ 1.5 (mod6).
   ODD numbers*3+1 = 2^t * ODD numbers ≡ 1.5 (mod6).
   (page 42÷47)

7. The powers of 2 connect the ODD numbers in the 2 equations:

1) $\frac{Din*3+1}{2^t} = 1 + 6n$ with t = EVEN

2) $\frac{Din*3+1}{2^t} = 5 + 6n$ with t=ODD

8. The third equation links the ODD input and the same to the ODD output:

$$\mathbf{Din_{+1}} = Din + Din*3+1 = \mathbf{Din*4+1}$$

4*x+1 generates the sequence of **the infinite ODD inputs** that follow one another with the interval 3x+1 and **share the same ODD output**.

Successions can be expressed recursively:

$$\begin{cases} a_0 = 1+2n & \text{with } n=0\div\infty \\ a_n = a_{n-1} * 4 + 1 & \end{cases}$$

Analytic expression: $\mathbf{(1+2n)*\ 2^{2a} + \Sigma 2^{2i}}$

with $n = 0 \div \infty$; $a = 0 \div \infty$ (if $a=0 \Rightarrow \Sigma 2^{2i}=0$) ; $i = 0 \div a-1$

I verify by entering $\mathbf{Din_{+1}}$ = Din*4+1 in the equation $(Din*3+1)/2^t = Dout$, where 2^t becomes $2^{t+2} = 4*2^t$:

$$((Din*4+1)*3+1)/2^{t+2} = Dout$$
$$(Din*12+3+1)/2^{t+2} = Dout$$
$$(Din*12+4)/(4*2^t) = Dout$$
$$((Din*3+1)*4)/(4*2^t) = Dout$$
$$(Din*3+1)/2^t = Dout$$

I can express the ODD output ≡ 1.5 (mod6) as:

1+6n = 1+3*2n ; 5+6n = 2+3+3*2n = 2+3*(1+2n)

Dout = r+3k

k = 0÷ ∞
r = 1+ rest of k/2 = 1÷2

with k = 0 and EVEN ⟹ r = 1
with k = ODD ⟹ r = 2

The analytic expression of the function becomes:

$$\boxed{\frac{\left((1+2n)\cdot 2^{2a}+\sum_{i:0}^{a-1}2^{2i}\right)*3+1}{2^t}=r+3k}$$

with

n = 0÷∞
a = 0÷ ∞ (if a=0 ⟹ $\Sigma 2^{2i}$=0)
k = 0÷ ∞

i = 0 ÷ a -1
t = 1÷ ∞ = 2a + t_0
t_0 = t calculated with a =0
r = 1+ rest of k/2 = 1÷2

Varying **n** and **a** generation the sequence of the ODD input that share the same ODD output and varying **k** all possible sequences.

$$\text{Din}=1+2n=\frac{\left(\frac{(r+3k)\cdot 2^t-1}{3}\right)-\sum_{i:0}^{a-1}2^{2i}}{2^{2a}}$$

9. The equation 7. 1) and 2) become:

$$\frac{(1+2n)*3+1}{2^t} = r+3k$$

unfolding the equation:

$$\frac{3+6n+1}{2^t} = r+3k$$

$$\boxed{\frac{4+6n}{2^t} = r+3k}$$

with $\begin{cases} n = 0 \div \infty \\ t = 1 \div \infty \\ k = 0 \div \infty \\ r = 1 + \text{rest of } k/2 = 1 \div 2 \end{cases}$

$$k = \frac{\frac{4+6n}{2^t} - r}{3}$$

if t = 1 ⇨ n = K = ODD to 1÷∞ ⇨ r=2
if n = 0 ⇨ k = 0 ⇨ t = 2 ⇨ r = 1

All ODD numbers *3+1 become an EVEN number ≡ 4 (mod 6), reachable by the inverse function,_which expressed with the binary numbering system highlights the exponent variable of 2:

t = the number of consecutive zeros, starting from the least significant figure, up to the least significant 1.

Conclusion:

All positive integers are present in the Collatz graph and can be reached by applying the 2 conditions and then connected to 1 thanks to the powers of 2.

<u>The Collatz conjecture is true</u>.

Reference:

https://en.wikipedia.org/wiki/Collatz_conjecture

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