

cubic ellipsoid model - improved nuclear mass calculations and spin consideration

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Abstract

In the former paper we constructed the model and found that the excess neutrons shall be found in the ellipsoid envelope.

In this work we find more precisely at what positions these excess neutrons are located, by expanding the drawings to more than 300 stable nuclei and optimizing their mass formula calculations while fixing the mass formula parameters at the values that were found in the last research [2] and require the correct total nuclear spin.

Content

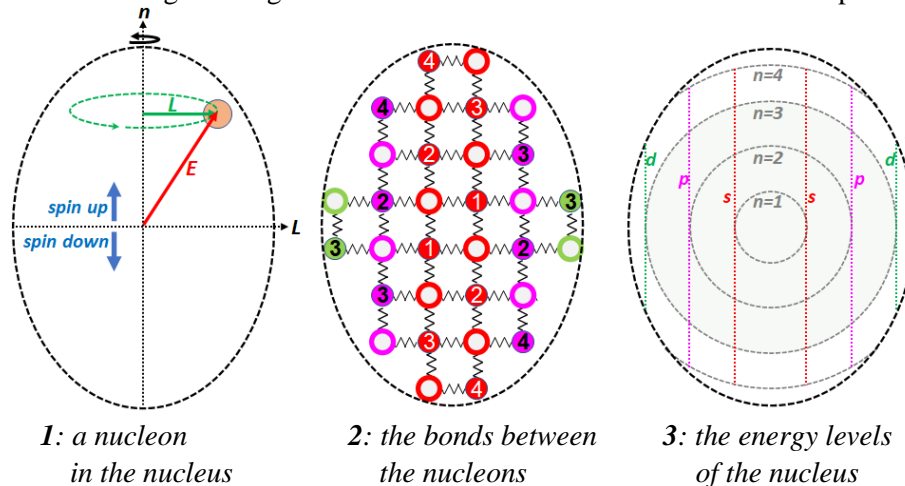
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The model at a glance

A brief description of the model [5]:

- The nucleus has an ellipsoid shape.
- The nucleons are connected in a cubic form.
- Protons are connected to neutrons (**p-n**).
- Neutrons are connected mainly to protons.
- The protons are populated and organized in shells in the nucleus in a full analogy to those of the electrons in the atom.
- The energy layers (principal quantum number **n**) grow along the **z**-axis of the nucleus in its both directions (more precisely **n** grows with its distance from the origin).
- The perpendicular distance from the **z**-axis in the **x-y**-plane reflects the angular momentum (**L**) and so the orbitals.
- The upper half of the ellipsoid is referred to as spin-up and the lower part as spin-down.
- The nucleus possibly rotates around its **z**-axis.

The following drawings describe the idea via cross sections in the **x-z**-plane of the nucleus.



1. One nucleon (orange circle) is observed inside the ellipsoid (dashed line) that encloses the nucleons and schematically defines the nucleus surface:
 - the distance from the origin represents its energy **E**.
 - the distance from the **z**-axis depicts its angular momentum **L**.
 - the nucleons in the upper half have spin up, and in the lower one spin down.
2. The bonds between the nucleons are shown for visibility as springs.
 - **Protons**: full circles of the **s**, **p** and **d** sub-orbitals. **Neutrons**: hollow circles.
3. The circles of equal energy states **n** in the ellipsoid.
 - the lines mark the development of the **s**, **p** and **d** sub-orbitals along the **z**-axis.
 - The **s** line crosses all **n** circles from 1 to 4 (**s1** to **s4**).
 - The **p** line begins by **n=2** and reaches till **n=4** (**p2** to **p4**).
 - The **d** line begins by **n=3** and reaches the ellipsoid border, before it reaches the **n=4** circle, and therefore there are no **d4** states at this stage (only **d3**).

Introduction

The results of the mass formula calculations [2] delivered two parameters:

- d_0 : the minimum distance between two neighboring nucleons in the cubic structure of the nucleus.
- e_b : the binding energy between these neighboring nucleons.

and their best values were found to be:

- $d_0 = 1.62 \text{ fm}$
- $e_b = 5.72 \text{ MeV}$

According to the model the proton positions and their paired neutrons have fixed positions. Unlike these, the excess neutrons can occupy different positions at the envelope of the ellipsoid; the neutrons have no influence on the electric energy, but change the total binding energy and so the results of the mass formula.

We therefore assume that the results for e_b and d_0 are correct and take the opposite direction: we improve the mass formula calculation results by varying the positions of the excess neutrons, while keeping e_b and d_0 fixed and ensuring the correct value of the total spin. This way we aim to better understand where in the envelope and at what sequence the excess neutrons are populated.

Improved calculation by iterations and spin consideration and results

The steps of the process were as follows:

1. drawing more than 300 stable nuclei.
2. changing the drawings, if needed, to fit the spin.
3. calculating the mass formula.
4. while the parameters of the mass formula are kept fixed.
5. varying the positions of the excess neutrons.
6. better understanding the logic of the nucleus composition and returning possibly to point 2 and changing other nuclei in accordance with the knowledge gained.

The results of the mass formula calculation for 82 stable nuclei from Ar_{18}^{40} to Pb_{82}^{208} :

maximum	average	st. dev.	$\leq 2\%$ *	$\leq 1\%$	$\leq 0.5\%$
2%	0.6%	0.5%	100%	78%	60%

* the amount of nuclei with relative error smaller than or equal to 2%.

this process led to the following improvement of the results for 296 stable nuclei from Ar_{18}^{40} to Pb_{82}^{210} :

maximum	average	st. dev.	$\leq 2\%$	$\leq 1\%$	$\leq 0.5\%$
2%	0.4%	0.4%	100%	92%	68%

and even when expanding the range to 327 stable nuclei from N_{7}^{14} to Pb_{82}^{210} we get:

maximum	average	st. dev.	$\leq 2\%$	$\leq 1\%$	$\leq 0.5\%$
3%	0.5%	0.5%	98%	88%	64%

so we consider this as an improvement. In the next research we use these results to set the population rules for the excess neutrons of the stable nuclei.

Sources and references

1. *Tables of Nuclear Data: [Japan Atomic Energy Agency \(JAEA\)](#)*
2. *a cubic ellipsoid geometric model of the atomic nucleus and its mass formula - Ronen Yavor - ([viXra](#))*

Appendix: the data

This section shows the data according to which the mass of the nuclei was calculated via the mass formula.

The parameters with which the mass formula are shown here are those of the best results achieved while running the calculation during the first research [2]:

- $d_0 = 1.62 \text{ fm}$
- $e_b = 5.72 \text{ MeV}$

The legend of the table below is given in the order of the columns from left to right:

- *nuc*: the nucleus (name)
- Z_x : atomic number of the nucleus of the element x. the number of protons.
- A_x : mass number of the nucleus of the element x. The number of nucleons.
- $N_x := A_x - Z_x$: the number of neutrons of the nucleus of the element x.
- n_k : the number of nucleon-nucleon bonds in the nucleus as it was calculated by the relevant Excel file. *
- e_c : total relative electric energy of the nucleus as it was calculated by the relevant Excel file. *
- *meas. [amu]*: measured mass of the nucleus in [amu].
- *Base: $Z_x \cdot m_p + N_x \cdot m_n$ [amu]*: base mass in [amu]:

number of protons · proton mass + number of neutrons · neutron mass

- *calc. m [amu]*: calculated mass according to the mass formula in [amu]

$$m_{\text{calc}_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2} .$$

- Δ_1 : [amu] *calc. - meas.*: calculated mass - measured mass in [amu]
- Δ_2 : [amu] *base - meas.*: base mass - measured mass in [amu]
- *rel. err. Δ_1 : Δ_2* : the relative error in percent:

$$\text{rel. err.} = \frac{\text{calculated mass} - \text{measured mass}}{\text{base mass} - \text{measured mass}}$$

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
He	2	3	2	0.7	3.016	3.012	61.4%	3.023	-0.004	0.007
He	2	4	4	0.7	4.003	4.008	18.4%	4.032	0.005	0.029
Li	3	6	6	1.9	6.015	6.013	7.1%	6.048	-0.002	0.033
Li	3	7	7	1.9	7.016	7.015	1.6%	7.056	-0.001	0.040
Be	4	9	11	3.4	9.012	9.008	6.7%	9.072	-0.004	0.060
Be	4	10	12	3.4	10.014	10.011	4.2%	10.081	-0.003	0.068
B	5	10	12	5.7	10.013	10.011	2.2%	10.080	-0.001	0.067
B	5	11	14	5.7	11.009	11.008	1.8%	11.088	-0.001	0.079
C	6	12	16	9.0	12.000	12.006	6.3%	12.096	0.006	0.096
C	6	13	17	9.0	13.003	13.009	5.1%	13.104	0.005	0.101
C	6	14	18	9.0	14.003	14.011	7.1%	14.113	0.008	0.110
N	7	14	20	12.7	14.003	14.001	2.0%	14.112	-0.002	0.109
N	7	15	21	12.7	15.000	15.003	2.8%	15.120	0.003	0.120
O	8	16	24	16.7	15.995	15.996	0.9%	16.128	0.001	0.133
O	8	17	25	16.7	16.999	16.999	0.3%	17.136	0.000	0.137
O	8	18	26	16.7	17.999	18.001	1.4%	18.145	0.002	0.146
F	9	19	29	20.8	18.998	18.994	2.9%	19.152	-0.004	0.154
Ne	10	20	32	26.1	19.992	19.988	2.7%	20.159	-0.005	0.167
Ne	10	21	33	26.1	20.994	20.990	2.0%	21.168	-0.003	0.174
Ne	10	22	34	26.1	21.991	21.993	0.8%	22.177	0.002	0.185
Na	11	22	34	30.0	21.994	21.995	0.4%	22.175	0.001	0.181
Na	11	23	36	30.0	22.990	22.992	0.9%	23.184	0.002	0.194
Mg	12	24	38	34.1	23.985	23.991	2.7%	24.191	0.005	0.206
Mg	12	25	40	34.1	24.986	24.987	0.5%	25.200	0.001	0.214
Mg	12	26	42	34.1	25.983	25.983	0.3%	26.209	0.001	0.226
Al	13	26	42	39.3	25.987	25.987	0.0%	26.207	0.000	0.220
Al	13	27	44	39.3	26.982	26.983	0.8%	27.216	0.002	0.234
Si	14	28	46	44.8	27.977	27.984	2.7%	28.223	0.007	0.246
Si	14	29	48	44.8	28.976	28.980	1.3%	29.232	0.003	0.255
Si	14	30	50	44.8	29.974	29.976	1.0%	30.241	0.003	0.267
Si	14	32	54	44.8	31.974	31.969	1.8%	32.258	-0.005	0.284
P	15	31	52	53.0	30.974	30.979	1.9%	31.248	0.005	0.274
S	16	32	56	59.7	31.972	31.968	1.4%	32.255	-0.004	0.283
S	16	33	57	59.7	32.971	32.971	0.2%	33.264	-0.001	0.292
S	16	34	58	59.7	33.968	33.973	1.8%	34.272	0.005	0.305
S	16	36	62	59.7	35.967	35.966	0.3%	36.290	-0.001	0.323
Cl	17	35	61	66.4	34.969	34.969	0.1%	35.280	0.000	0.311
Cl	17	36	63	66.4	35.968	35.965	1.1%	36.288	-0.003	0.320
Cl	17	37	65	66.4	36.966	36.961	1.4%	37.297	-0.005	0.331
Ar	18	36	64	74.4	35.968	35.965	0.8%	36.287	-0.003	0.319
Ar	18	38	68	74.4	37.963	37.958	1.5%	38.304	-0.005	0.342
Ar	18	39	69	74.4	38.964	38.960	1.2%	39.313	-0.004	0.349
Ar	18	40	70	74.4	39.962	39.963	0.1%	40.322	0.000	0.359
Ar	18	42	74	74.4	41.963	41.956	2.0%	42.339	-0.007	0.376
K	19	39	69	79.9	38.964	38.964	0.1%	39.312	0.000	0.348
K	19	40	71	79.9	39.964	39.961	1.0%	40.320	-0.003	0.356
K	19	41	73	79.9	40.962	40.957	1.3%	41.329	-0.005	0.367

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
Ca	20	40	72	85.6	39.963	39.958	1.2%	40.319	-0.004	0.356
Ca	20	41	73	85.6	40.962	40.961	0.4%	41.327	-0.001	0.365
Ca	20	42	74	85.6	41.959	41.963	1.3%	42.336	0.005	0.378
Ca	20	43	75	85.6	42.959	42.966	1.9%	43.345	0.007	0.386
Ca	20	44	78	85.6	43.955	43.956	0.2%	44.353	0.001	0.398
Ca	20	45	80	85.6	44.956	44.953	0.9%	45.362	-0.004	0.406
Ca	20	46	82	85.6	45.954	45.949	1.1%	46.371	-0.005	0.417
Ca	20	48	84	85.6	47.953	47.954	0.4%	48.388	0.002	0.436
Sc	21	45	80	92.8	44.956	44.958	0.6%	45.361	0.002	0.405
Ti	22	44	80	103.7	43.960	43.959	0.3%	44.351	-0.001	0.391
Ti	22	46	84	103.7	45.953	45.951	0.3%	46.368	-0.001	0.415
Ti	22	47	85	103.7	46.952	46.954	0.5%	47.377	0.002	0.425
Ti	22	48	88	103.7	47.948	47.944	0.9%	48.385	-0.004	0.437
Ti	22	49	89	103.7	48.948	48.947	0.3%	49.394	-0.001	0.446
Ti	22	50	90	103.7	49.945	49.949	1.0%	50.403	0.004	0.458
V	23	50	92	112.4	49.947	49.944	0.8%	50.401	-0.003	0.454
V	23	51	93	112.4	50.944	50.946	0.5%	51.410	0.002	0.466
Cr	24	50	94	123.1	49.946	49.940	1.3%	50.400	-0.006	0.454
Cr	24	52	96	123.1	51.941	51.945	1.0%	52.417	0.005	0.477
Cr	24	53	100	132.2	52.941	52.938	0.5%	53.426	-0.003	0.485
Cr	24	54	102	132.2	53.939	53.934	0.9%	54.435	-0.004	0.496
Mn	25	53	100	141.5	52.941	52.946	0.9%	53.425	0.004	0.483
Mn	25	55	103	141.5	54.938	54.945	1.3%	55.442	0.006	0.504
Fe	26	54	104	153.3	53.940	53.940	0.0%	54.432	0.000	0.492
Fe	26	55	106	153.3	54.938	54.936	0.5%	55.440	-0.002	0.502
Fe	26	56	108	153.3	55.935	55.932	0.5%	56.449	-0.003	0.514
Fe	26	57	108	153.3	56.935	56.941	1.1%	57.458	0.006	0.522
Fe	26	58	110	153.3	57.933	57.937	0.8%	58.466	0.004	0.533
Fe	26	60	112	153.3	59.934	59.942	1.5%	60.484	0.008	0.550
Co	27	59	113	164.5	58.933	58.937	0.7%	59.474	0.004	0.541
Ni	28	58	112	164.5	57.935	57.933	0.4%	58.464	-0.002	0.528
Ni	28	59	113	164.5	58.934	58.936	0.2%	59.472	0.001	0.538
Ni	28	60	114	164.5	59.931	59.938	1.3%	60.481	0.007	0.550
Ni	28	61	116	164.5	60.931	60.934	0.6%	61.490	0.003	0.559
Ni	28	62	118	164.5	61.928	61.931	0.4%	62.498	0.002	0.570
Ni	28	63	120	164.5	62.930	62.927	0.4%	63.507	-0.002	0.577
Ni	28	64	122	164.5	63.928	63.924	0.7%	64.516	-0.004	0.588
Cu	29	63	121	174.6	62.930	62.929	0.1%	63.506	0.000	0.576
Cu	29	65	124	174.6	64.928	64.928	0.1%	65.523	0.000	0.595
Zn	30	64	122	185.1	63.929	63.941	1.9%	64.513	0.011	0.584
Zn	30	66	126	185.1	65.926	65.933	1.2%	66.530	0.007	0.604
Zn	30	67	128	185.1	66.927	66.930	0.4%	67.539	0.003	0.612
Zn	30	68	130	185.1	67.925	67.926	0.2%	68.548	0.001	0.623
Zn	30	70	132	185.1	69.925	69.931	0.9%	70.565	0.006	0.640
Ga	31	69	132	195.0	68.926	68.931	0.8%	69.555	0.005	0.629
Ga	31	71	136	195.0	70.925	70.923	0.2%	71.572	-0.001	0.647
Ge	32	70	134	205.1	69.924	69.935	1.7%	70.562	0.011	0.638

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
Ge	32	72	138	205.1	71.922	71.928	0.9%	72.579	0.006	0.657
Ge	32	73	139	205.1	72.923	72.930	1.0%	73.588	0.007	0.665
Ge	32	74	142	205.1	73.921	73.921	0.1%	74.597	0.000	0.676
Ge	32	76	144	205.1	75.921	75.926	0.6%	76.614	0.004	0.693
As	33	75	144	218.5	74.922	74.928	1.0%	75.604	0.007	0.682
Se	34	74	146	230.0	73.922	73.917	0.8%	74.594	-0.005	0.672
Se	34	76	148	230.0	75.919	75.922	0.4%	76.611	0.003	0.692
Se	34	77	150	230.0	76.920	76.918	0.2%	77.620	-0.001	0.700
Se	34	78	152	230.0	77.917	77.915	0.3%	78.629	-0.002	0.711
Se	34	79	153	230.0	78.918	78.917	0.2%	79.637	-0.001	0.719
Se	34	80	154	230.0	79.917	79.920	0.5%	80.646	0.003	0.729
Se	34	82	158	230.0	81.917	81.913	0.5%	82.663	-0.004	0.747
Br	35	79	155	241.7	78.918	78.915	0.5%	79.636	-0.003	0.718
Br	35	81	157	241.7	80.916	80.920	0.5%	81.653	0.004	0.737
Kr	36	78	154	253.5	77.920	77.922	0.3%	78.626	0.002	0.706
Kr	36	80	158	253.5	79.916	79.915	0.2%	80.643	-0.001	0.727
Kr	36	81	159	253.5	80.917	80.918	0.1%	81.652	0.001	0.735
Kr	36	82	162	253.5	81.913	81.908	0.8%	82.661	-0.006	0.747
Kr	36	83	163	253.5	82.914	82.910	0.5%	83.669	-0.004	0.755
Kr	36	84	164	253.5	83.911	83.913	0.2%	84.678	0.001	0.766
Kr	36	85	166	253.5	84.913	84.909	0.4%	85.687	-0.003	0.774
Kr	36	86	166	253.5	85.911	85.918	0.9%	86.695	0.007	0.785
Rb	37	85	167	264.1	84.912	84.912	0.0%	85.685	0.000	0.773
Rb	37	87	169	264.1	86.909	86.917	1.0%	87.702	0.008	0.793
Sr	38	84	166	271.1	83.913	83.915	0.2%	84.675	0.001	0.762
Sr	38	86	170	271.1	85.909	85.907	0.2%	86.692	-0.002	0.783
Sr	38	87	172	271.1	86.909	86.904	0.6%	87.701	-0.005	0.792
Sr	38	88	174	271.1	87.906	87.900	0.7%	88.710	-0.005	0.804
Sr	38	90	176	271.1	89.908	89.905	0.3%	90.727	-0.002	0.819
Y	39	89	175	283.2	88.906	88.913	0.9%	89.717	0.007	0.811
Zr	40	90	178	295.5	89.905	89.913	1.1%	90.724	0.009	0.820
Zr	40	91	180	295.5	90.906	90.910	0.5%	91.733	0.004	0.827
Zr	40	92	182	295.5	91.905	91.906	0.1%	92.742	0.001	0.837
Zr	40	93	184	295.5	92.906	92.903	0.5%	93.750	-0.004	0.844
Zr	40	94	184	295.5	93.906	93.911	0.6%	94.759	0.005	0.853
Zr	40	96	186	295.5	95.908	95.916	0.9%	96.776	0.008	0.868
Nb	41	92	184	313.4	91.907	91.910	0.3%	92.740	0.002	0.833
Nb	41	93	185	313.4	92.906	92.912	0.7%	93.749	0.006	0.843
Nb	41	94	187	313.4	93.907	93.909	0.1%	94.758	0.001	0.850
Mo	42	92	186	329.6	91.907	91.911	0.6%	92.739	0.005	0.832
Mo	42	93	187	329.6	92.907	92.914	0.8%	93.748	0.007	0.841
Mo	42	94	190	329.6	93.905	93.904	0.1%	94.756	-0.001	0.851
Mo	42	95	191	329.6	94.906	94.907	0.1%	95.765	0.001	0.859
Mo	42	96	194	329.6	95.905	95.897	0.9%	96.774	-0.008	0.869
Mo	42	97	195	329.6	96.906	96.899	0.7%	97.782	-0.007	0.876
Mo	42	98	196	329.6	97.905	97.902	0.4%	98.791	-0.003	0.885
Mo	42	100	198	329.6	99.907	99.907	0.0%	100.808	0.000	0.901

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
Tc	43	97	195	343.5	96.906	96.911	0.6%	97.781	0.005	0.874
Tc	43	98	197	343.5	97.907	97.908	0.1%	98.789	0.000	0.882
Tc	43	99	198	343.5	98.906	98.910	0.4%	99.798	0.004	0.892
Ru	44	96	196	357.5	95.908	95.909	0.1%	96.771	0.001	0.863
Ru	44	98	198	357.5	97.905	97.914	0.9%	98.788	0.008	0.883
Ru	44	99	200	357.5	98.906	98.910	0.5%	99.797	0.004	0.891
Ru	44	100	202	357.5	99.904	99.906	0.2%	100.805	0.002	0.901
Ru	44	101	202	357.5	100.906	100.915	1.0%	101.814	0.009	0.908
Ru	44	102	204	357.5	101.904	101.911	0.8%	102.823	0.007	0.918
Ru	44	104	208	357.5	103.905	103.904	0.1%	104.840	-0.001	0.935
Rh	45	103	209	373.9	102.906	102.904	0.2%	103.830	-0.002	0.925
Pd	46	102	208	384.8	101.906	101.910	0.5%	102.820	0.005	0.914
Pd	46	104	212	384.8	103.904	103.903	0.1%	104.837	-0.001	0.933
Pd	46	105	215	384.8	104.905	104.893	1.3%	105.846	-0.012	0.941
Pd	46	106	216	384.8	105.903	105.896	0.8%	106.855	-0.008	0.951
Pd	46	107	217	384.8	106.905	106.898	0.7%	107.863	-0.007	0.958
Pd	46	108	218	384.8	107.904	107.901	0.3%	108.872	-0.003	0.968
Ag	47	107	219	405.6	106.905	106.904	0.1%	107.862	-0.001	0.957
Ag	47	108	220	405.6	107.906	107.907	0.1%	108.871	0.001	0.965
Ag	47	109	222	405.6	108.905	108.903	0.2%	109.879	-0.002	0.974
Cd	48	106	220	421.0	105.906	105.903	0.4%	106.852	-0.004	0.945
Cd	48	108	222	421.0	107.904	107.908	0.4%	108.869	0.004	0.965
Cd	48	110	226	421.0	109.903	109.901	0.2%	110.886	-0.002	0.983
Cd	48	111	226	421.0	110.904	110.909	0.5%	111.895	0.005	0.991
Cd	48	112	230	421.0	111.903	111.893	0.9%	112.904	-0.009	1.001
Cd	48	113	230	421.0	112.904	112.902	0.2%	113.912	-0.002	1.008
Cd	48	114	232	421.0	113.903	113.899	0.5%	114.921	-0.005	1.018
Cd	48	116	234	421.0	115.905	115.904	0.1%	116.938	-0.001	1.034
In	49	113	232	433.9	112.904	112.901	0.3%	113.911	-0.003	1.007
In	49	115	234	433.9	114.904	114.906	0.2%	115.928	0.002	1.025
Sn	50	112	232	447.1	111.905	111.903	0.1%	112.901	-0.001	0.996
Sn	50	114	234	447.1	113.903	113.908	0.6%	114.918	0.006	1.016
Sn	50	115	236	447.1	114.903	114.905	0.1%	115.927	0.001	1.024
Sn	50	116	238	447.1	115.902	115.901	0.1%	116.936	-0.001	1.034
Sn	50	117	238	447.1	116.903	116.910	0.7%	117.944	0.007	1.041
Sn	50	118	240	447.1	117.902	117.906	0.4%	118.953	0.005	1.051
Sn	50	119	242	447.1	118.903	118.903	0.1%	119.962	-0.001	1.058
Sn	50	120	244	447.1	119.902	119.899	0.3%	120.970	-0.003	1.068
Sn	50	122	246	447.1	121.903	121.904	0.1%	122.988	0.001	1.084
Sn	50	124	248	447.1	123.905	123.909	0.3%	125.005	0.004	1.100
Sn	50	126	252	447.1	125.908	125.902	0.5%	127.022	-0.006	1.115
Sb	51	121	248	463.9	120.904	120.898	0.6%	121.978	-0.006	1.074
Sb	51	123	249	463.9	122.904	122.909	0.4%	123.995	0.005	1.091
Sb	51	125	253	463.9	124.905	124.902	0.3%	126.012	-0.004	1.107
Te	52	120	248	478.4	119.904	119.901	0.2%	120.968	-0.003	1.064
Te	52	122	250	478.4	121.903	121.907	0.3%	122.985	0.004	1.082
Te	52	123	250	478.4	122.904	122.915	1.0%	123.994	0.011	1.089

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
Te	52	124	252	478.4	123.903	123.912	0.8%	125.002	0.009	1.099
Te	52	125	254	478.4	124.904	124.908	0.3%	126.011	0.004	1.106
Te	52	126	256	478.4	125.903	125.904	0.1%	127.020	0.001	1.116
Te	52	128	260	478.4	127.904	127.897	0.6%	129.037	-0.007	1.132
Te	52	130	262	478.4	129.906	129.902	0.4%	131.054	-0.004	1.148
I	53	127	258	493.4	126.904	126.914	0.8%	128.027	0.009	1.122
I	53	129	262	493.4	128.905	128.906	0.1%	130.044	0.001	1.139
Xe	54	124	256	508.0	123.906	123.912	0.6%	124.999	0.007	1.094
Xe	54	126	260	508.0	125.904	125.905	0.1%	127.017	0.001	1.113
Xe	54	128	264	508.0	127.904	127.898	0.5%	129.034	-0.005	1.131
Xe	54	129	264	508.0	128.905	128.907	0.2%	130.043	0.002	1.138
Xe	54	130	266	508.0	129.904	129.903	0.0%	131.051	0.000	1.148
Xe	54	131	266	508.0	130.905	130.912	0.6%	132.060	0.007	1.155
Xe	54	132	268	508.0	131.904	131.908	0.3%	133.069	0.004	1.165
Xe	54	134	272	508.0	133.905	133.901	0.4%	135.086	-0.004	1.181
Xe	54	136	274	508.0	135.907	135.906	0.1%	137.103	-0.001	1.196
Cs	55	133	271	519.5	132.905	132.908	0.2%	134.076	0.003	1.171
Cs	55	134	272	519.5	133.907	133.910	0.3%	135.085	0.004	1.178
Cs	55	135	274	519.5	134.906	134.907	0.1%	136.093	0.001	1.187
Cs	55	137	276	519.5	136.907	136.912	0.4%	138.111	0.005	1.204
Ba	56	130	268	531.0	129.906	129.910	0.3%	131.049	0.004	1.142
Ba	56	132	272	531.0	131.905	131.903	0.2%	133.066	-0.002	1.161
Ba	56	133	272	531.0	132.906	132.911	0.5%	134.075	0.005	1.169
Ba	56	134	274	531.0	133.905	133.908	0.3%	135.083	0.003	1.179
Ba	56	135	276	531.0	134.906	134.904	0.1%	136.092	-0.002	1.186
Ba	56	136	278	531.0	135.905	135.901	0.3%	137.101	-0.004	1.196
Ba	56	137	278	531.0	136.906	136.909	0.3%	138.109	0.003	1.204
Ba	56	138	280	531.0	137.905	137.906	0.0%	139.118	0.000	1.213
La	57	137	279	545.4	136.906	136.915	0.7%	138.108	0.009	1.201
La	57	138	281	545.4	137.907	137.912	0.4%	139.117	0.005	1.210
La	57	139	283	545.4	138.906	138.908	0.2%	140.125	0.002	1.219
Ce	58	136	280	566.6	135.907	135.919	1.0%	137.098	0.012	1.191
Ce	58	138	284	566.6	137.906	137.912	0.5%	139.115	0.006	1.209
Ce	58	140	288	566.6	139.905	139.905	0.0%	141.133	0.000	1.227
Ce	58	142	292	566.6	141.909	141.898	0.9%	143.150	-0.011	1.241
Pr	59	141	290	582.7	140.908	140.915	0.6%	142.140	0.008	1.232
Nd	60	142	296	603.2	141.908	141.905	0.2%	143.147	-0.002	1.239
Nd	60	143	297	603.2	142.910	142.908	0.2%	144.156	-0.002	1.246
Nd	60	144	298	603.2	143.910	143.910	0.0%	145.164	0.000	1.254
Nd	60	145	299	603.2	144.913	144.913	0.0%	146.173	0.000	1.261
Nd	60	146	300	603.2	145.913	145.915	0.2%	147.182	0.002	1.269
Nd	60	148	302	603.2	147.917	147.921	0.3%	149.199	0.004	1.282
Nd	60	150	304	603.2	149.921	149.926	0.4%	151.216	0.005	1.296
Pm	61	145	302	621.6	144.913	144.911	0.2%	146.172	-0.002	1.259
Pm	61	146	302	621.6	145.915	145.919	0.4%	147.180	0.005	1.266
Pm	61	147	304	621.6	146.915	146.916	0.1%	148.189	0.001	1.274
Sm	62	144	302	636.1	143.912	143.915	0.2%	145.162	0.003	1.250

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
Sm	62	146	306	636.1	145.913	145.907	0.5%	147.179	-0.006	1.266
Sm	62	147	306	636.1	146.915	146.916	0.1%	148.188	0.001	1.273
Sm	62	148	308	636.1	147.915	147.912	0.2%	149.196	-0.002	1.281
Sm	62	149	308	636.1	148.917	148.921	0.3%	150.205	0.004	1.288
Sm	62	150	310	636.1	149.917	149.917	0.0%	151.214	0.000	1.296
Sm	62	151	311	636.1	150.920	150.920	0.0%	152.222	0.000	1.302
Sm	62	152	312	636.1	151.920	151.922	0.2%	153.231	0.003	1.311
Sm	62	154	316	636.1	153.922	153.915	0.5%	155.248	-0.007	1.326
Eu	63	150	313	659.5	149.920	149.920	0.0%	151.212	0.000	1.293
Eu	63	151	314	659.5	150.920	150.922	0.2%	152.221	0.003	1.301
Eu	63	152	315	659.5	151.922	151.925	0.2%	153.230	0.003	1.308
Eu	63	153	316	659.5	152.921	152.927	0.5%	154.238	0.006	1.317
Eu	64	154	318	659.5	153.923	153.922	0.0%	155.246	-0.001	1.323
Eu	65	155	320	659.5	154.923	154.917	0.4%	156.253	-0.005	1.330
Gd	64	148	314	683.3	147.918	147.918	0.0%	149.194	0.000	1.275
Gd	64	150	316	683.3	149.919	149.923	0.3%	151.211	0.004	1.292
Gd	64	152	318	683.3	151.920	151.928	0.6%	153.228	0.008	1.308
Gd	64	154	322	683.3	153.921	153.921	0.0%	155.246	0.000	1.325
Gd	64	155	324	683.3	154.923	154.917	0.4%	156.254	-0.006	1.332
Gd	64	156	324	683.3	155.922	155.926	0.3%	157.263	0.004	1.341
Gd	64	157	326	683.3	156.924	156.922	0.1%	158.272	-0.002	1.348
Gd	64	158	328	683.3	157.924	157.918	0.4%	159.280	-0.006	1.356
Gd	64	160	330	683.3	159.927	159.924	0.3%	161.298	-0.004	1.370
Tb	65	159	331	700.8	158.925	158.924	0.1%	160.287	-0.001	1.362
Dy	66	154	326	718.4	153.924	153.927	0.2%	155.243	0.002	1.318
Dy	66	156	330	718.4	155.924	155.920	0.4%	157.260	-0.005	1.336
Dy	66	158	332	718.4	157.924	157.925	0.0%	159.277	0.000	1.353
Dy	66	160	334	718.4	159.925	159.930	0.3%	161.295	0.004	1.370
Dy	66	161	336	718.4	160.927	160.926	0.1%	162.303	-0.001	1.376
Dy	66	162	338	718.4	161.927	161.922	0.3%	163.312	-0.004	1.385
Dy	66	163	339	718.4	162.929	162.925	0.3%	164.321	-0.004	1.392
Dy	66	164	340	718.4	163.929	163.928	0.1%	165.329	-0.002	1.400
Ho	67	163	341	735.8	162.929	162.928	0.1%	164.319	-0.001	1.391
Ho	67	165	344	735.8	164.930	164.927	0.3%	166.337	-0.004	1.406
Er	68	162	342	756.6	161.929	161.931	0.2%	163.309	0.003	1.381
Er	68	164	346	756.6	163.929	163.924	0.4%	165.327	-0.005	1.397
Er	68	166	348	756.6	165.930	165.929	0.1%	167.344	-0.001	1.414
Er	68	167	349	756.6	166.932	166.932	0.0%	168.353	0.000	1.421
Er	68	168	350	756.6	167.932	167.934	0.1%	169.361	0.002	1.429
Er	68	170	352	756.6	169.935	169.939	0.3%	171.379	0.004	1.443
Tm	69	169	354	777.9	168.934	168.937	0.2%	170.369	0.003	1.434
Tm	69	171	358	777.9	170.936	170.930	0.4%	172.386	-0.006	1.449
Yb	70	168	356	796.0	167.934	167.932	0.1%	169.359	-0.001	1.425
Yb	70	170	358	796.0	169.935	169.937	0.2%	171.376	0.003	1.441
Yb	70	171	360	796.0	170.936	170.934	0.2%	172.385	-0.003	1.448
Yb	70	172	362	796.0	171.936	171.930	0.4%	173.393	-0.006	1.457
Yb	70	173	363	796.0	172.938	172.933	0.4%	174.402	-0.005	1.464

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
Yb	70	174	364	796.0	173.939	173.935	0.2%	175.411	-0.004	1.472
Yb	70	175	366	796.0	174.941	174.932	0.7%	176.419	-0.010	1.478
Yb	70	176	368	796.0	175.943	175.928	1.0%	177.428	-0.015	1.485
Lu	71	173	364	815.1	172.939	172.943	0.3%	174.400	0.005	1.462
Lu	71	174	365	815.1	173.940	173.946	0.4%	175.409	0.006	1.469
Lu	71	175	367	815.1	174.941	174.942	0.1%	176.418	0.002	1.477
Lu	71	176	369	815.1	175.943	175.939	0.3%	177.426	-0.004	1.484
Hf	72	172	366	834.0	171.939	171.939	0.0%	173.390	0.000	1.451
Hf	72	174	370	834.0	173.940	173.932	0.6%	175.408	-0.008	1.468
Hf	72	176	370	834.0	175.941	175.949	0.5%	177.425	0.008	1.484
Hf	72	177	372	834.0	176.943	176.946	0.2%	178.434	0.002	1.490
Hf	72	178	374	834.0	177.944	177.942	0.1%	179.442	-0.002	1.499
Hf	72	179	377	834.0	178.946	178.932	0.9%	180.451	-0.014	1.505
Hf	72	180	378	834.0	179.947	179.935	0.8%	181.460	-0.012	1.513
Hf	72	182	382	834.0	181.951	181.928	1.5%	183.477	-0.023	1.526
Ta	73	179	380	859.3	178.946	178.937	0.6%	180.450	-0.009	1.504
Ta	73	180	382	859.3	179.947	179.933	1.0%	181.458	-0.014	1.511
Ta	73	181	383	859.3	180.948	180.936	0.8%	182.467	-0.012	1.519
W	74	180	382	882.6	179.947	179.954	0.5%	181.457	0.007	1.510
W	74	182	386	882.6	181.948	181.947	0.1%	183.474	-0.002	1.526
W	74	183	388	882.6	182.950	182.943	0.5%	184.483	-0.007	1.533
W	74	184	390	882.6	183.951	183.939	0.7%	185.492	-0.012	1.541
W	74	186	392	882.6	185.954	185.944	0.6%	187.509	-0.010	1.555
Re	75	185	391	903.1	184.953	184.960	0.5%	186.499	0.007	1.546
Re	75	187	395	903.1	186.956	186.953	0.2%	188.516	-0.003	1.560
Os	76	184	394	923.6	183.952	183.951	0.1%	185.489	-0.001	1.536
Os	76	186	394	923.6	185.954	185.969	0.9%	187.506	0.015	1.552
Os	76	187	396	923.6	186.956	186.965	0.6%	188.515	0.009	1.559
Os	76	188	398	923.6	187.956	187.961	0.3%	189.523	0.005	1.568
Os	76	189	400	923.6	188.958	188.958	0.0%	190.532	0.000	1.574
Os	76	190	402	923.6	189.958	189.954	0.3%	191.541	-0.004	1.582
Os	76	192	406	923.6	191.961	191.947	0.9%	193.558	-0.015	1.597
Os	76	194	408	923.6	193.965	193.952	0.8%	195.575	-0.013	1.610
Ir	77	191	406	947.2	190.961	190.959	0.1%	192.548	-0.001	1.588
Ir	77	193	409	947.2	192.963	192.958	0.3%	194.565	-0.005	1.602
Pt	78	190	408	971.0	189.960	189.960	0.0%	191.538	0.000	1.578
Pt	78	192	410	971.0	191.961	191.965	0.2%	193.555	0.004	1.594
Pt	78	193	412	971.0	192.963	192.961	0.1%	194.564	-0.002	1.601
Pt	78	194	412	971.0	193.963	193.970	0.4%	195.573	0.007	1.610
Pt	78	195	414	971.0	194.965	194.966	0.1%	196.581	0.001	1.617
Pt	78	196	416	971.0	195.965	195.963	0.2%	197.590	-0.002	1.625
Pt	78	198	420	971.0	197.968	197.955	0.8%	199.607	-0.013	1.639
Au	79	197	419	992.4	196.967	196.972	0.3%	198.597	0.005	1.631
Hg	80	198	424	1,014.0	197.967	197.969	0.1%	199.605	0.002	1.638
Hg	80	200	426	1,014.0	199.968	199.974	0.3%	201.622	0.006	1.654
Hg	80	202	432	1,014.0	201.971	201.955	1.0%	203.639	-0.016	1.669
Tl	81	203	431	1,032.2	202.972	202.985	0.8%	204.647	0.013	1.674

nuc.	Z _x	A _x	n _k	e _c	meas. [amu]	calc. m [amu]	rel. error Δ ₁ :Δ ₂	base Z _x *m _p +N _x *m _n [amu]	Δ ₁ : calc - meas.	Δ ₂ : base - meas.
Tl	81	204	434	1,032.2	203.974	203.976	0.1%	205.655	0.002	1.681
Tl	81	205	436	1,032.2	204.974	204.972	0.1%	206.664	-0.002	1.689
Pb	82	202	434	1,050.5	201.972	201.974	0.1%	203.636	0.002	1.664
Pb	82	204	438	1,050.5	203.973	203.967	0.4%	205.654	-0.006	1.681
Pb	82	205	439	1,050.5	204.974	204.970	0.3%	206.662	-0.005	1.688
Pb	82	206	440	1,050.5	205.974	205.972	0.1%	207.671	-0.002	1.697
Pb	82	207	440	1,050.5	206.976	206.981	0.3%	208.680	0.005	1.704
Pb	82	208	442	1,050.5	207.977	207.977	0.0%	209.688	0.001	1.712
Pb	82	210	444	1,050.5	209.984	209.982	0.1%	211.706	-0.002	1.722
Bi	83	207	443	1,072.4	206.978	206.982	0.2%	208.678	0.003	1.700
Bi	83	208	444	1,072.4	207.980	207.984	0.3%	209.687	0.005	1.707
Bi	83	209	445	1,072.4	208.980	208.987	0.4%	210.696	0.007	1.715
Bi	83	210	446	1,072.4	209.984	209.989	0.3%	211.704	0.005	1.720
Po	84	208	446	1,092.0	207.981	207.989	0.5%	209.686	0.008	1.704
Po	84	209	448	1,092.0	208.982	208.986	0.2%	210.694	0.003	1.712
At	85	209	451	1,111.1	208.986	208.984	0.1%	210.693	-0.002	1.707
Rn	86	222	470	1,132.0	222.018	221.999	1.1%	223.804	-0.019	1.787
Fr	87	223	473	1,146.8	223.020	223.002	1.0%	224.811	-0.018	1.792
Ra	88	226	478	1,162.0	226.025	226.010	0.8%	227.836	-0.015	1.811
Ac	89	227	479	1,183	227.028	227.032	0.2%	228.843	0.004	1.816
Th	90	228	484	1,205	228.029	228.029	0.0%	229.851	0.000	1.822
Th	90	230	486	1,205	230.033	230.034	0.0%	231.868	0.001	1.835
Th	90	232	488	1,205	232.038	232.039	0.0%	233.885	0.001	1.847
Pa	91	231	491	1,235	231.036	231.039	0.2%	232.875	0.003	1.839
U	92	232	496	1,263	232.037	232.043	0.3%	233.883	0.006	1.845
U	92	233	499	1,263	233.040	233.033	0.3%	234.891	-0.006	1.852
U	92	234	498	1,263	234.041	234.048	0.4%	235.900	0.007	1.859
U	92	235	499	1,263	235.044	235.051	0.4%	236.909	0.007	1.865
U	92	236	500	1,263	236.046	236.053	0.4%	237.917	0.007	1.872
U	92	237	501	1,263	237.049	237.056	0.4%	238.926	0.007	1.877
U	92	238	502	1,263	238.051	238.058	0.4%	239.935	0.007	1.884
Np	93	236	506	1,290	236.047	236.040	0.4%	237.916	-0.007	1.869
Np	93	237	507	1,290	237.048	237.042	0.3%	238.924	-0.006	1.876
Pu	94	236	506	1,310	236.046	236.058	0.6%	237.914	0.012	1.868
Pu	94	238	508	1,310	238.050	238.063	0.7%	239.932	0.013	1.882
Pu	94	239	508	1,310	239.052	239.071	1.0%	240.940	0.019	1.888
Pu	94	240	510	1,310	240.054	240.068	0.7%	241.949	0.014	1.895
Pu	94	241	511	1,310	241.057	241.070	0.7%	242.958	0.013	1.901
Pu	94	242	512	1,310	242.059	242.073	0.7%	243.966	0.014	1.908
Pu	94	244	516	1,310	244.064	244.066	0.1%	245.984	0.001	1.920