Close-orbiting black hole pairs are macroscopic quantum-gravitational systems

Warren D. Smith, `warren.wds@gmail.com`, August 2023.

Abstract. Close-orbiting pairs of near-equal black holes (M≈m) are a new kind of macroscopic quantum object because they have inherent mass-uncertainty $\Delta M_{total} \gtrapprox 0.0013(M+m)$. These are the largest and heaviest macroscopic quantum systems ever found, and the first physical system ever thought of which plausibly requires quantum gravity for an accurate description, and which plausibly will enable learning about quantum gravity via direct observation.

Ingredient #1: Energy-time uncertainty principle

The vast majority of quantum mechanics textbooks say that $\Delta E \Delta t \ge \hbar/2$ where $\hbar \approx 1.055\times10^{-34}$ joule seconds, unfortunately *without* providing any precise meaning for ΔE or Δt and without telling the reader what, exactly, this supposed inequality even means. Fortunately, some precise statements are available. Bauer & Mello 1976 considered an unstable quantum system with **"survival probability" Q(t)** as a function of time t≥0, and **probability-density ρ(E)** for its initial energy E. If the system were described by a wavefunction Ψ(x,t) then Q(t) with $0 \le Q(t)=|\int\Psi^*(x,0)\Psi(x,t)dx|^2 \le 1$; may be interpreted as the probability the system remains in its initial state Ψ(x,0) after time t.

Define the "**Bauer-Mello timespan**" $\tau_{BM} \equiv (1/2)\int_{t>0} Q(t)^{1/2} dt$. For any system obeying the classic "exponential decay law" Q(t)=exp(-t/L) this definition would exactly yield its mean lifetime $\tau_{BM}=L$. And any Q(t) falling proportionally to $t^{-\gamma}$ (or faster) when t→∞, for any fixed exponent γ>2, will yield a *finite* τ_{BM}.

A measure of the energy-width of the system is $W_E=1/\max_E\rho(E)$. The **Bauer-Mello theorem** then may be written $\tau_{BM}W_E \ge \pi\hbar/2=h/4$ or equivalently (which I prefer)

$$\max_E \rho(E) \le 4\,\tau_{BM} / h \quad \text{where} \quad h=2\pi\hbar\approx6.626\times10^{-34} \text{ joule seconds.}$$

Bauer & Mello's constant 4 is *best possible* in the sense that their inequality becomes an equality in the classic exponential decay case Q(t)=exp(-t/τ) when the energy necessarily is described by the Cauchy density $\rho(E)=2\pi^{-1}\Gamma/(4[E-E_0]^2+\Gamma^2)$ where Γ is the width of the energy-interval where $\rho(E)\ge\max_E\rho(E)/2=\rho(E_0)/2$ and $\tau\Gamma=\hbar$.

Other precise statements were obtained by Mandelstam & Tamm 1945, for example $Q(t)\ge\cos(t\Delta E/\hbar)^2$ when $0\le t\le(\pi/2)\Delta E/\hbar$ where $\Delta E\equiv[\int(E-\overline{E})^2\rho(E)dE]^{1/2}$ and $\overline{E}\equiv\int E\rho(E)dE$ and the integrations are over the full real line. In particular, if we define the "**half life**"

$\tau_{1/2}$=min {t | Q(t)≤1/2, t>0} then $\tau_{1/2}\Delta E \geq \pi\hbar/4 = h/8$. And if we define the "**mean life**" $\tau_{mean}=\int_{t>0}Q(t)dt$ then **Gislason**, Sabelli, Wood 1985 showed

$$\tau_{mean}\, \Delta E \geq 5^{-3/2}3\pi\hbar = 5^{-3/2}3h/2.$$

Their constant also is best possible, in the sense that their inequality is tight when $\rho(E)=(3/4)(1-E^2)$ for $|E|\leq 1$, else 0. With that ρ(E) the survival probability Q(t) has $Q(t)t^4$ bounded below a positive constant always, but bounded above a different positive constant on a positive-density subset of the halfline t>0.

It now is natural to ask whether there is any uncertainty relation *combining* the virtues of *both* Gislason and Bauer-Mello, i.e. of the form $\max_E \rho(E) \leq \kappa\tau_{mean}/h$ (or $\leq \kappa\tau_{1/2}/h$) for some positive constant κ. Both answers are **no**, because the probability density $\rho(E)=\pi^{-1/2}\Gamma(v+3/2)\Gamma(v+1)^{-1}(1-E^2)^v$ for |E|<1, else 0 (where v>-1 is a constant) corresponds to a survival probability Q(t) with $Q(t)t^{2v+2}$ bounded below a positive constant always, and above another positive constant on a positive-density subset of the halfline t>0. That's due to, e.g, EQ 2-7-19 of Sneddon 1972. So if -1<v<0 then $\max_E \rho(E)=\infty$, while if -1/2<v then τ_{mean}>0 (and $\tau_{1/2}$) are finite. So any v with -1/2<v<0 yields a counterexample.

However, I am able to prove $\tau_{mean}\Delta_1 E > 0.2889\hbar$ where $\Delta_1 E \equiv \int|E-\bar{E}|\rho(E)dE$. I can also prove: If the narrowest energy interval containing at least 31% probability [$\int\rho(E)dE \geq 0.31$] has width $W_{31\%}$, then $\tau_{1/2}W_{31\%} > 0.005969\hbar$.

The three timespans we have discussed always obey $0<\tau_{1/2}\leq 2\tau_{mean}$, $\tau_{1/2}\leq 2^{3/2}\tau_{BH}$, and $\tau_{mean}\leq 2\tau_{BH}$. For exact-exponential decay $\tau_{1/2}/\ln 2=\tau_{mean}=\tau_{BH}$ and $W_{31\%}\approx 0.5295\hbar/\tau_{mean}$ and $\Delta E=\Delta_1 E=\infty$. The lattermost is one reason that exact exponential decay is, under traditional quantum mechanics, considered impossible (Fonda et al 1978); but if, say, Cobalt-60 decays exponentially for 300 halflives then switches to t^{-4} style decay (which is roughly what most analysts contend), then (a) this mathematical problem would not arise, and (b) detecting this departure from exponentiality would be infeasible.

Ingredient #2: Gravitational radiation from rotating quadrupoles

Two rotating systems are

a. Uniformly-dense rigid thin rod of length=L and mass=M rotating about its midpoint about an axis perpendicular to the rod.
b. Two point masses m and M, separated by distance L, both circularly orbiting their center of mass (either because joined by a massless length-L rigid rod, or because of their mutual gravitational attraction according to Newton's laws).

Let the angular velocity be Ω, so the period is 2π/Ω. Either way, we have a "rotating quadrupole" which therefore **emits gravitational-wave radiation**.

In case (a) Eddington 1922/1923 (where we've also used the formula $I=ML^2/12$ for the moment of inertia I of the rod) calculated the emitted power

$$P_{rod} = 32GI^2\Omega^6c^{-5}/5 = 2GM^2L^4\Omega^6c^{-5}/45.$$

(The reason Eddington published this twice, using two different methods, was to become confident that Einstein previously had gotten it wrong by a factor of 2.) This corresponds to a rate of emission of gravitons (each graviton having angular frequency 2Ω) with mean time τ between graviton emissions equal to $\tau=2\hbar\Omega/P=45\hbar G^{-1}M^{-2}L^{-4}\Omega^{-5}c^5$. See Smith 2021 for analysis of the claim "gravitons exist" and with energy $E=hf$ for a frequency=f graviton. If we now regard the rotating rod as a *quantum* system with mean decay time τ, we see from the Gislason bound that the mass of the rod necessarily is *uncertain*, with $\Delta M\ge(5^{-3/2}3/2)hc^{-2}/\tau$, that is, $\Delta M_{rod}\ge(5^{-5/2}\pi/3)GM^2L^4\Omega^5c^{-7}=5^{-5/2}48\pi GI^2\Omega^5c^{-7}$.

Case (b) can be treated using the more general analysis in §10.5 of the book by Weinberg 1972, but Eddington's I-based formula also works given that our two masses M and m have respective distances R and r to their center of mass, whereupon solving $MR=mr$ and $R+r=L$ for $r=LM/(m+M)$ and $R=Lm/(m+M)$ allows us to determine the moment of inertia $I=mr^2+MR^2=L^2mM/(m+M)$. The radiated power is $P_{binary}=(32/5)Gm^2M^2(m+M)^{-2}L^4\Omega^6c^{-5}$. If the masses obey the Kepler-Newton law $(m+M)G=\Omega^2L^3$ then the radiated power can be rewritten as $P_{binary}=(32/5)m^2M^2(m+M)L^{-5}G^4c^{-5}$. Then as before we find $\tau_{binary}=2\hbar\Omega/P_{binary}=(5/16)\hbar G^{-7/2}M^{-2}m^{-2}(m+M)^{-1/2}L^{7/2}c^5$ and $\Delta M_{binary} \ge(5^{-5/2}48\pi)c^{-7}G^{7/2}M^2m^2(m+M)^{1/2}L^{-7/2}$.

Without loss of generality $0<m\le M$. Now suppose that the center-separation L happens to be near minimum possible. The Schwarzschild radii of the two masses in isolation would be $r=2mGc^{-2}$ and $R=2MGc^{-2}$. So clearly if $L\le r+R=2(m+M)Gc^{-2}$ then our "two" black holes would actually be one merged entity. The Newtonian equipotential surface at the same potential as a single isolated hole's horizon (corresponding to escape velocity=c for an infinitesimal test mass) becomes topologically *two* spherical surfaces exactly when L satisfies $L>x+X$ with $M/X+m/x=c^2/(2G)$ and $MX^{-2}=mx^{-2}$. It is simplest to solve these equations when $m=M$ (and hence $r=R$), the answer then being $L>2x=2X=4R=8MGc^{-2}$. The fully-general answer is $L>2([m/M]^{1/2}+1)(m+M)Gc^{-2}$. Of course, our uses of the "Newtonian potential" and the "Kepler-Newton law" both are only approximately valid since we have ignored general relativistic time dilation, space distortion, and dynamics. So the reader should keep in mind that all our formulas about black holes at near-minimal separation are only approximate.

If our two masses indeed are black holes separated by that approximate minimum possible distance, then

$$P_{binary} = m^2 M^2 (m+M)^{-4} ([m/M]^{1/2}+1)^{-5} P_{Pl} / 5$$

where $P_{Pl}\equiv c^5/G\approx3.628\times10^{52}$ watts is the **Planck power unit**. Therefore if $m\approx M$ then P_{binary} is about $2^{-9}5^{-1}$=1/2560 Planck power units, i.e. about 1.417×10^{49} watts, *regardless* of m+M.

The mean time τ between graviton emissions then is
$\tau_{binary}= 2^{-1/2}5\hbar c^{-2} M^{-2}m^{-2}(m+M)^3 ([m/M]^{1/2}+1)^{7/2}$ which when m=M is $\tau_{binary}=320\hbar c^{-2}M^{-1}$ Then via the energy-time uncertainty principle (Gislason bound) and $E=mc^2$, the uncertainty ΔM in total mass is lower bounded by $\Delta M_{binary} \geq (5^{-5/2}2^{1/2}3\pi) M^2m^2(m+M)^{-3} ([m/M]^{1/2}+1)^{-7/2}$ which when m=M becomes $\Delta M_{binary} \geq (5^{-5/2}2^{-7}3\pi) (M+m) \approx 0.001317 (M+m)$ regardless of c, G, and $\hbar$.

Remarkable Conclusion

Two closely orbiting comparable-mass black holes always form a *quantum* system, whose inherent mass *uncertainty* necessarily has the same order as 0.0013 times its total mass. This is by far the largest inherent mass-uncertainty I ever heard of for anything macroscopic. This can be (which presumably has happened many times) a "macroscopic quantum phenomenon" weighing billions of solar masses, with diameter comparable to the solar system, with mass-uncertainty amounting to millions of solar masses – all again by far the largest I ever heard of – lasting for a week or more. As far as I know no prior author has ever pointed out that black holes, despite their giantness, can be *quantum* in nature and require quantum gravity for their description. In fact, **this is the first physical system anybody ever thought of, in which quantum gravity plays such a large role that it should be feasible to "observe" it in action**. And given the recently developed capability of the "event horizon telescope" to "see" certain black holes with high resolution, and LIGO's ability to "hear" black hole mergers in real time, this for the first time opens up serious hope that it might be possible to learn about quantum gravity *by direct observation*.

Discussion

So that we can see just how remarkable this is, let us compare it versus various other systems.

The rod-shaped interstellar asteroid "Oumuamua" has L≈400 meters, rotation period≈8 hours so $\Omega\approx2\times10^{-4}$/second, and if made of iron has mass $M\approx4\times10^9$ kg. I compute $\Delta M\approx8\times10^{-61}$ kg. If we replaced the iron by high strength steel and sped up the rotation period to 3 seconds (any faster and steels would not be strong enough) then τ=10 picoseconds and $\Delta M\approx9\times10^{-41}$ kg, which still is 10 orders of magnitude smaller than the mass of a single electron.

The star BAT99-98 in the Large Magellanic Cloud arguably is the most luminous star currently known. It is believed to have mass 226 times our sun, luminosity≈1.9×10^{33} watts equivalent to 5×10^6 suns, and surface temperature 45000°K. I deduce that it emits about 10^{51} photons per second. If we regard this entire star as a quantum system with decay time 10^{-51} second, then its inherent mass-uncertainty is $\Delta M\approx c^{-2}\hbar10^{51}$/second≈1 kg. Peak supernova luminosities can reach 5×10^9 suns (1.9×10^{36} watts), suggesting by the same calculation ΔM of order ≤1000 kg. That still is peanuts in the sense that $\Delta M\leq10^{-28}M$ is far too small to detect.

The tininess of those ΔM's was not merely due to luck.

1. We can readily argue that the graviton-emission-caused ΔM of *every* rotating gravitationally-bound system is (as a fraction of its total mass M) *maximized* when it is a black-hole close binary system – and if it does not involve at least 2 black holes, is always much smaller.

2. For simplicity in the following argument let me work in [Planck units] ($\hbar$=c=G=k_B=1) and ignore constant factors of order 1. Consider a Euclidean ball of radius=R with at least the outer layer (layer thickness λ) of this ball consisting of hot material (temperature T≈1/λ). Regard this as a quantum system which "decays" by emitting photons, e.g. of blackbody radiation at temperature≈T and wavelength≈λ into the region outside the ball. The "decay time" (i.e. mean time between such photon emissions) will then be of order $R^{-2}T^{-3}\lambda$. This decay time will cause our system to have ΔM of order R^2T^3/λ. Meanwhile the mass M of that outer layer is of order $R^2T^4\lambda$. Hence $\Delta M/M \approx T^{-1}\lambda^{-2} \approx T$. We conclude that ΔM/M has order≥1 *only* when the temperature T is at least of order 1 Planck temperature unit: $T \geq T_{Pl} \approx 1.417\times10^{32}$°K. However, this analysis had assumed Euclidean geometry. In fact, $T<R^{-1/3}$ is necessary otherwise our ball will be so heavy it is a black hole (and therefore not emit radiation at all). Therefore, ΔM/M≳1 is *impossible* for any system of our "hot ball" type whose radius R exceeds order 1 Planck length units: $R>L_{Pl}\approx1.616\times10^{-35}$ meters.

3. We now give a table showing some of the fastest-decaying unstable particles known, computing their ΔM/M from their mass M and estimated mean lifetime τ via the Gislason bound. The "Roper resonance" (Burkert & Roberts 2019) is the first excited state of the proton. Cobalt-60 (not ultrafast decaying) is included for comparison purposes.

Particle	Mass (MeV/c^2)	Est.Mean Lifetime (sec)	ΔM/M
Roper resonance	1370	3.7×10^{-24}	0.11
$W^\pm$ boson	80377±12	3×10^{-25}	0.02301
Z^0 boson	91187.6±2.1	3×10^{-25}	0.02028
Top quark	172760±300	5×10^{-25}	0.00642
Lithium-4	3751.304±0.002	1.31×10^{-21}	0.000113
Higgs boson	125110±110	$(1\text{-}5)\times10^{-22}$	0.00004435
Tauon	1776.86±0.12	$(2.903\pm0.005)\times10^{-13}$	10^{-12}
short kaon K^0	497.611±0.013	$(8.954\pm0.004)\times10^{-11}$	10^{-14}
kaon $K^\pm$	493.677±0.016	$(1.238\pm0.002)\times10^{-8}$	9×10^{-17}
long kaon K^0	497.611±0.013	$(5.116\pm0.021)\times10^{-8}$	2×10^{-17}
Cobalt-60	55828.0019	2.4×10^{8}	4×10^{-35}

4. The lifetime of a composite of N identical subsystems should be of order 1/N times the subsystem lifetime, my point being that ΔM/M is *unaffected* by N-fold cloning. If the subsystems are *independent* one could perhaps argue the net ΔM should be smaller than the sum of the N component ΔM's (e.g. only about $N^{1/2}\Delta M$) due to partial cancellations. Either way, I expect that any system made of the above-tabulated (or any other known) particles will have ΔM/M very below 1.

5. Obviously, every normally-encountered macroscopic object has undetectably small inherent mass-uncertainty, $\Delta M \ll 10^{-20} M$.

In view of the above, it seems reasonable to **conjecture** that

A. No macroscopic physical system can ever have greater ΔM/M than two close-orbiting near-equal black holes.
B. And the only physical systems whose ΔM/M values can compete are some of the most-unstable particles (which, of course, are inherently *quantum* microscopic objects), the best one I know being the (currently poorly understood) "Roper resonance."
C. Two close-orbiting near-equal black holes are an inherently *quantum* macroscopic system, inherently requiring *quantum gravity* for precise treatment.

Now we must ask – to use a technical term – what the hell?!?!

"Macroscopic quantum phenomena" can be weird and mysterious. The two most familiar are *superconductivity* and *superfluid* liquid helium. I suspect that if these two phenomena had not been experimentally discovered by accident, then the theorists would never have been smart enough to predict them. As it was, H.K.Onnes discovered in 1911 that mercury superconducts below about 4°K. It took until about 1961 (50 years later) before Bardeen, Cooper, Schrieffer, and Eliashberg gained (what some contend to be) theoretical "understanding" of that – although those 4 people remained not smart enough to predict the Josephson effect. This understanding, however, even as of year 2023 remains rather pathetic. If we had real understanding, then we could have a supercomputer mentally search all possible ≤6-atom chemical compounds and tell us the predicted best (e.g. highest T_c) superconductor. But as of year 2023, that has never happened, and essentially all decent superconductors have been found by experimenters operating on hunches or by random trials, with essentially zero help from theoretical "understanding." And even the BCSE level of understanding is far greater than present understanding of the high-T_c cuprates.

Superfluidity in liquid helium below 2.17°K was first discovered in 1937 and was more-or-less explained within 10 years, but some questions remain disputed even today, such as the question of to what extent solid helium is "supersolid."

My point with this historical excursion is that despite 50-100 years of theoretical *and* experimental examination, neither superfluidity nor superconductivity are understood nearly as well as we would like. Given this historical proof of human incompetence about macroscopic quantum phenomena, plus our clear present incompetence about quantum gravity, I am unwilling to assert that I know what is going on about the present new (close black hole binaries) kind of macroscopic quantum phenomenon.

As far as I am aware, the LIGO team until now has operated under the belief that all their data has been 100% compatible with non-quantum general relativity. I now advise them to gather that data at higher accuracy and think about it more!

It might be that this whole macroscopic quantum phenomenon "does not matter." Regard the black hole binary as an unstable quantum system, *but* each time it radiates another graviton, we get a *new* such system. If all these systems were "independent" in some suitable sense their mass uncertainties might, if observed "blurred" over long time spans (say 10^{20} graviton-emissions) largely cancel out, e.g. effectively reducing ΔM by a factor 10^{10}. (And indeed, notice that the claimed experimental error bars on some of the masses tabulated above are considerably smaller than their inherent uncertainties, which is mainly due to averaging over many observed particles.) That's an interesting hypothesis, but I currently have almost no clue to what extent it is true and to what extent, and how, the largeness of ΔM/M "really matters." All I can say for now is: this certainly seems worthy of investigation.

References

Mariano Bauer & Pier A. Mello: On the Lifetime-Width Relation for a Decaying State and the Uncertainty Principle, Proc. National Academy of Sciences USA 73,2 (Feb.1976) 283-285.

Volker D. Burkert & Craig D. Roberts: Colloquium: Roper resonance: Toward a solution to the fifty year puzzle, Reviews in Modern Physics 91,1 (March 2019) #011003.

Arthur S. Eddington: On the spontaneous loss of energy of a spinning rod according to the relativity theory, Philosophical Magazine (vi) 46,276 (1923) 1112-1117; The Propagation of Gravitational Waves, Proc. Royal Soc. London A102 (1922) 268-282.

L.Fonda, G.C.Ghirardi, A.Rimini: Decay theory of unstable quantum systems, Reports of Progress in Physics 41,4 (1978) 587-631.

Eric A. Gislason, Nora H. Sabelli, John W. Wood: New form of the time-energy uncertainty relation, Phys. Rev. A 31,4 (April 1985) 2078-2081.

Leonid I. Mandelstam & Igor E. Tamm: The uncertainty relation between energy and time in non-relativistic quantum mechanics (in Russian), Izv. Akad. Nauk SSSR ser. fiz. 9 (1945) 122-128; English translation J.Phys.USSR 9 (1945) 249-254. Reprinted in English pp.115-123 in Tamm's Selected papers. QC21.2.T362513.

Warren D. Smith: Is Gravity Quantum?, March 2021, `http://vixra.org/abs/2109.0122`.

Ian N. Sneddon: The use of integral transforms, McGraw Hill 1972. QA432.S64.

Steven Weinberg: Gravitation and Cosmology, Principles and Application of the General Theory of Relativity, J.Wiley & Sons 1972. QC6.W47.530.1/1.