Modeling Learning Behavior of Students of Mathematics

Budee U Zaman

August 2023

Abstract

This paper introduces an innovative approach to comprehending and modeling the collective behaviors of students within the context of mathematics education. The core objective is to present a comprehensive mathematical framework capable of addressing the entire spectrum of behaviors exhibited in mathematics classrooms. We introduce a novel SIR-based model, tailored to capture behaviors under the influence of individual students. Additionally, we propose that interactions among students across different classrooms can serve as a regulatory mechanism for these behaviors. To validate our approach, we conduct a series of simulations that demonstrate the practicality and significance of our model. This paper significantly contributes to the advancement of our understanding of student behaviors in the realm of mathematics education and their mathematical representation. By bridging the gap between mathematical modeling and the intricate dynamics of student conduct, this work provides valuable insights into the behaviors displayed in math classrooms.
Keywords Student behavior, Modeling,learning behavior

1 Introduction

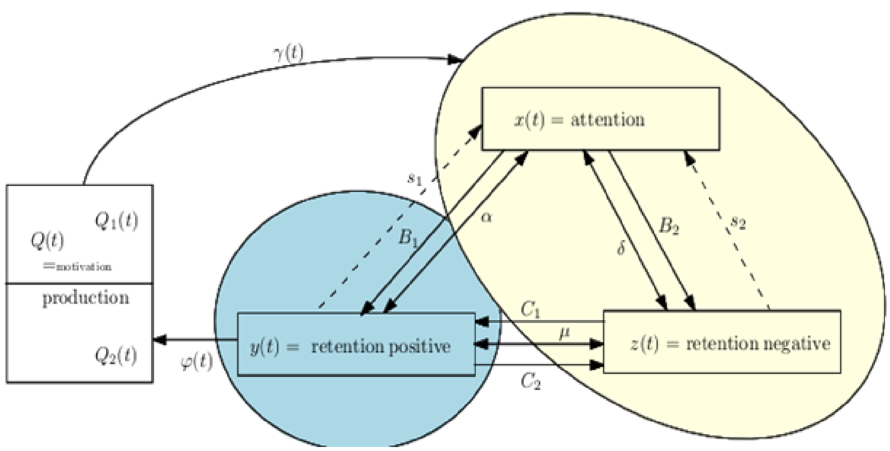
n the realm of education, the cultivation of an environment conducive to positive learning outcomes is a fundamental pursuit. The intricate interplay of various factors within this ecosystem, ranging from students' motivations and behaviors to the influences of educators and the learning environment, holds the key to shaping academic success. Understanding and harnessing these dynamics have far-reaching implications for educational advancement.

At the heart of this endeavor lies the quest to comprehend and model the collective behaviors exhibited by students in the context of mathematics education. The paramount objective is to construct a comprehensive mathematical framework capable of addressing the entire spectrum of behaviors manifesting within mathematics classrooms. This endeavor not only requires the formulation of novel theoretical approaches but also demands the integration of real-world interactions and observations.[1] [3] Central to our approach is the introduction of a pioneering SIR-based model, meticulously tailored to capture the intricacies of behaviors influenced by individual students. This model stands as a testament to our commitment to providing a robust foundation for understanding the complex web of learning behaviors that students exhibit. Additionally, we postulate that the interactions among students across different classrooms can function as a regulatory mechanism, further enriching our comprehension of these behaviors. To validate the effectiveness and significance of our proposed model, we conduct a series of rigorous simulations. These simulations serve as empirical evidence of the practical applicability of our framework and highlight its potential to revolutionize the way we perceive and address student behaviors in mathematics education.[4][5][6][8][10] The significance of this research extends beyond the realm of mathematical modeling. It presents a unique opportunity to advance our understanding of student behaviors within the educational landscape, bridging the gap between theoretical constructs and real-world manifestations. By unraveling the intricate dynamics that govern students' actions, we lay the groundwork for informed educational policies, fostering environments that nurture positive learning behaviors, motivate students, and harness the power of constructive social interactions.[2][11] As we embark on this journey at the crossroads of mathematics, psychology, and education, our aim is not only to deepen our scientific insights but also to provide tangible tools for educators, policymakers, and researchers to usher in a new era of enhanced educational experiences and outcomes.[13][12][7][9]

2 The Mathematical Model

The research unequivocally underscores that teacher quality exerts a profound and undeniable influence on students' performance in the realm of science (Stevenson etal., 1990). The intricate tapestry of teacher quality comprises a myriad of pivotal factors, encompassing gender, age, experience, educational attainment, career stage, subject expertise, and underlying convictions about the subject matter itself (Beilock etal., 2010). It is crucial to note that instructional methodologies, including the art of questioning techniques and the artful dissemination of feedback, wield substantial power in shaping the outcomes of the students (House, 2002; Salili and Hau, 1994). At the collegiate level, the interplay of cultural forces assumes a momentous role in sculpting students' prowess in science. Linguistic nuances, curriculum design, and geographical setting collectively mold the educational landscape. Furthermore, variations among college types manifest in the form of teacher-student ratios, instructional time allocations, and an array of other determinants that decisively shape the final learning outcomes (Peverly, 2005). Drawing from a robust dataset comprising 10,959 elementary students, the meticulous utilization of a multi-stage sampling strategy is manifest. The selection process took into account college development stages, urban-rural dynamics, and economic factors, thereby yielding a representative and comprehensive sample. Rigorous analyses of college-level, class-level, and student-level factors ensued, revealing the significant sway of college development and parental occupational status on students' performance. Moreover, the dynamic of grade levels emerged as a pivotal influencer, where elevated grades demonstrated markedly superior science performance. An intriguing revelation emerges from the exploration of teacher attributes. The traditional determinants of teacher quality, namely gender and personal beliefs, exhibit a muted impact on the models. In stark contrast, the factor of teacher preparation looms large, serving as a decisive factor. Conclusively, formal teacher training emerges as a potent catalyst for elevated educational outcomes compared to non-formal alternatives. This unequivocally underscores the urgent necessity for a transformative overhaul in teacher training programs. Learning, an intricate cognitive metamorphosis, involves the acquisition of novel cognitive paradigms and behavioral patterns fueled by a fusion of information, skills, values, and aspirations. Its cadence is intricately interwoven with individual propensities, pedagogical processes, media dynamics, and the very materials that facilitate the journey. Contextual contours, overarching objectives, and the nature of instructional materials collectively orchestrate the tempo and culmination of learning in various organizational contexts and educational settings. This trans-formative process not only precipitates shifts in behavior but also catalyzes the re-calibration of routine practices, with the mode and pace of learning continually adapting to the contours of each unique situation.

Model of the Study



Development of the Model Parameter

$Q_1(t) =$ Class before address

$Q_2(t) =$ Understudies who receive the conduct after class

$x(t) =$Number of understudies who focus

$y(t) =$ Understudies who decidedly see the approach and emphatically held the training

$z(t)$ = Understudies who contrarily see the approach and adversely held the training

$\gamma(t)$ = Data scattering time

$\phi(t)$ = Reaction time

$s_1\ and\ s_2$ = Steady parameters speaking to the change

$B_1\ and\ B_2$ = When student focus

$C_1\ and\ C_2$ = Redundancy between student, positive and negative maintenance

$\alpha\ \mu\ \delta$ = transpose time

Furthermore, within the intricate tapestry of our study's framework, intricate interconnections and dynamic interactions flourish among diverse practices, vividly portrayed in the visual representation . As external forces exert their influence on the strategic pathways chosen for investigating a subject, this inquiry becomes bound by the commanding parameter known as Y. This dynamic parameter assumes a continuum of values, intricately entwined with the prevailing circumstances enveloping the subjects under meticulous scrutiny. For instance, a classroom setting is encapsulated by a distinctive discrete parameter, while an educational event embraces the fluidity of a continuous parameter, particularly when its announcement transpires in the proximity of its commencement. Within our distinct context, we confidently foresee aligning with the former scenario. Delving into the labyrinth of potential class persistence and its possible resurgence, we unveil directional cues etched as $s_1\ and\ s_2$, emblematic of steadfast constants. As the collective populace engages in a conscious choreography, pathways unfurl toward responsive behaviors, forging the landscape of affirmative or adverse retentive dynamics, symbolically captured as $B_1\ and\ B_2$correspondingly. Simultaneously, a subset of the population might traverse toward a realm of unfavorable responses, their journey guided by the precise coefficients $C_1\ and\ C_2$. Anchoring our exploration within the annals of prior scholarly endeavors, intricately woven into the literature, the fulcrum resides in classroom learning. In these instances, the intricate mechanisms are orchestrated by personal endorsements and systematic designs, elegantly portrayed through the symbolic imagery of $\alpha\ \mu\ \delta$. Graphically articulated,α plays the role of an academical catalyst, transmuting the essence of imitation between entities x and y, with a symphony of bidirectional mastery. This process draws parallels to an epidemiological dissemination, conforming to the rhythmic architecture: $\alpha f_1(x(t))y(t)$. Unveiling a tapestry where the orchestration of imitation finds its expression, predominantly from the realm of x to y. As our computational odyssey unfurls, we decree that the very bedrock of a minimum

55% threshold in conscious behaviors stands as an imperative edifice, a prerequisite for the dawn of affirmative retentive dynamics. Through the function $\delta f_2(x(t))z(t)$. Ultimately, the unchanging constant μ orchestrates the intricate interplay among governed individual behaviors, with a clear understanding that the student focus towards the realm of controlled individuals' classroom conduct. This is for the expression $\mu.g_(y(t))z_(t)$. Derived from the visual narrative portrayed earlier, the intricate web of graphical representation gives rise to a mathematical framework, from which the ensuing mathematical model takes shape.

$$\frac{dx}{dt} = \gamma(t)Q_1(t)(1-\frac{x(t)}{x_m})-(B_1+B_2)x(t)+\alpha f_1(x_(t))y_(t)+\delta f_2(x(t))z(t)+s_1y(t)+s_2z(t)$$

$$\frac{dy}{dt} = B_1x(t)-\alpha f_1(x_(t))y_(t)+C_1z(t)-s_1y(t)-C_1y(t)-\phi(t)y(t)(1-\frac{Q_2(t)}{Q_2(m)})+\mu g(y(t))z$$

$$\frac{dz}{dt} = B_2x(t) - s_2z(t) - \delta f_2(x(t))z(t) - C_1z(t) + C_2y(t) - \mu g(y)z(t)$$

$$\frac{dQ_1}{dt} = -\gamma(t)Q_1(t)(1 - \frac{x(t)}{x_m})$$

$$\frac{dQ_2}{dt} = \phi(t)y(t)(1 - \frac{Q_2(t)}{Q_2(m)})$$

Since the concerned number of student is supposed to be constant, that is the equality $Q_1(t) + Q_2(t) + x(t) + y(t) + z(t) = N\ for\ all\ t \in [0, T]$ is verified, system can be reduced to four equations and rewritten

$$\frac{dx}{dt} = \gamma(t)Q_1(t)(1-\frac{x(t)}{x_m})-(B_1+B_2)x(t)+\alpha f_1(x_(t))y_(t)+\delta f_2(x(t))z(t)+s_1y(t)+s_2z(t)$$

$$\frac{dy}{dt} = B_1x(t)-\alpha f_1(x_(t))y_(t)+C_1z(t)-s_1y(t)-C_1y(t)-\phi(t)y(t)(1-\frac{N - Q_1(t) - x(t) - y(t) - z(t)}{Q_2(m)})+\mu g(y(t))z(t$$

$$\frac{dz}{dt} = B_2x(t) - s_2z(t) - \delta f_2(x(t))z(t) - C_1z(t) + C_2y(t) - \mu g(y)z(t)$$

$$\frac{dQ_1}{dt} = -\gamma(t)Q_1(t)(1 - \frac{x(t)}{x_m})$$

3 Ensuring Accuracy through Model Calibration

Different Student Behavioral Patterns: Analyzing Response Rates The various behavioral patterns exhibited by students can be observed across different factors such as class participation, teaching methods, group dynamics, individual comprehension of new concepts, and overall classroom learning experiences. In a majority of instances, it has been found that approximately 35%

of individuals tend to display overtly responsive behaviors, while 25% maintain a composed demeanor. Meanwhile, 40% of students exhibit a visibly calm conduct, yet respond with varying degrees of emotional engagement and participation in class activities.

- x(t) = 30 to 75% of the student
- y(t) = 10 to 30% of the student
- z(t) = 08 to 25% of the student

To the best of our understanding, there is a lack of available data that quantifies the mechanisms through which transitions occur from one state to another.

The length of the Conduct.

The three distinct types of responses are characterized by varying durations (Vermeiren, 2007). The duration of both the cognitive consideration and responsive behaviors ranges from a few minutes to an hour, with the majority of instances falling within a 15-minute timeframe. However, the first type, particularly when associated with a delay, might extend further, leading to the gradual emergence of supportive and exploratory behaviors among students (Crocq, 1994). In the case of the second type, the resolution of overall engagement behaviors typically occurs swiftly. Yet, occasional external interventions enable the engagement population z(t) to revert to an automatic behavior x(t) briefly before transitioning into a positive engagement behavior y(t).Broadly, the duration of uncontrolled behavior x(t) + z(t) does not typically exceed 1 hour and 30 minutes. Within this model, it is assumed that an individual cannot sustain cognitive consideration behavior for one hour and then shift to maintenance behavior for another hour. The duration of maintenance behavior y(t) varies from a few minutes to fewer hours, contingent upon the intervention. The parameter selection process aims to uncover these underlying data patterns.

3.1 Numerical Cases

By integrating the impact of an alternative model via functions denoted as $f_1, f_2, and f_3$, our objective is to depict classroom simulations as precise indicators of students' learning capacity. Underlying this effort is the fundamental assumption that the student populace efficiently absorbs all classroom information in less than 1 minute, owing to their familiarity with the learning environment. This method's portrayal for the entire population is vividly illustrated in Figure 2.

Nevertheless, the attainment of the targeted and representative learning capacity, referred to as 'capacity', is not an instantaneous occurrence. Instead, we posit a deliberate and gradual progression that initiates roughly 5 minutes following a recreational interlude within the classroom. This methodical shift toward the 'capacity' state is graphically elucidated in Figure 2.

As previously emphasized, the nuanced characteristics of these curves must be meticulously adapted to align with the distinct attributes of the specific event under contemplation.

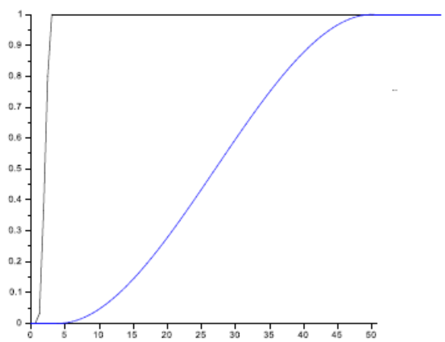


Figure 2: Functions φ and γ

The expressions $f_1(x(t))y(t)$ and $f_2(x(t))z(t)$ correspond to the interplay between x(t) and y(t), as well as x(t) and z(t), respectively. We extend this interplay from x(t) to y(t) with the premise that a minimum of 55% retentive practices is required for a meaningful emulation of constructive behaviors in students pursuing maintenance activities (as depicted in Figure 3).

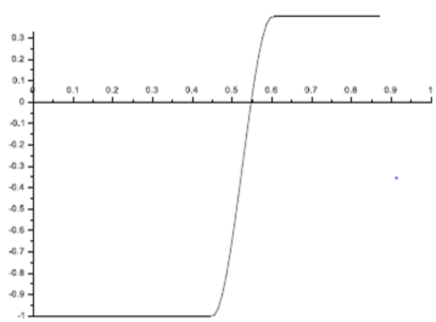


Figure 3: Function f_1 and f_2

Regarding the term $\mu g(y(t))z(t)$ for impersonation, our assumption is that the impersonation primarily operates from z(t) to y(t), as depicted in Figure 4.

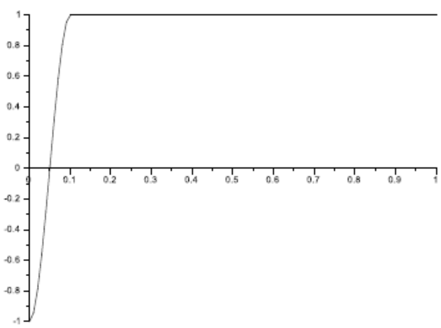


Figure 4: Function g

4 Simulation 1.

The initial aspect pertains to Figure 5. The selected parameter values facilitate the extraction of the presented alignment information. The regions situated between each curve and the horizontal axis provide an overview of the comprehensive level pertaining to the respective students. The global level of attention practices (x(t)) at 51.41% corresponds to values ranging between 30% and 75%. Similarly, the global proportions of positive learning and maintenance practices, denoted as 17.42% and 23.17% respectively, are encompassed within the range of 18% to 20%. Additionally, the model elucidates the evolution of these global practice distributions.

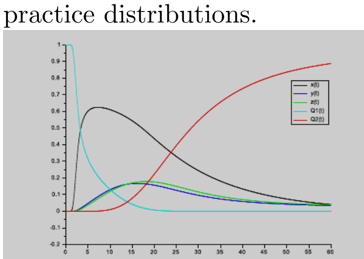


Figure 5 Estimates of the model $s_1 = .76, s_2 = .86, , \alpha = \delta = \mu = .09, x_m = .75, Q_2(m) = 1, B_1 = .05, B_2 = .22, C_1 = .80, C_2 = .11$

4.1 Simulation 2

In our current approach, we extend upon the foundation of previous research models by introducing a modification to parameter B_2. This adjustment is based on the fundamental concept that student learning is centered around retention and is likely to elicit positive outcomes. As illustrated in Figure 6, the lower parameter bounds of B_2 are intricately linked to the depth of learning inherent within the student strength. This depth of learning is resilient and remains retentive when subjected to interventions. However, when considering a higher value for this parameter, as exemplified in Figure 7, the depth of learning in the student strength experiences a swift decline. Furthermore, a prominent shift in the distributions of learning and controlled student strength becomes conspicuous when transitioning from Figure 6 to Figure 7.

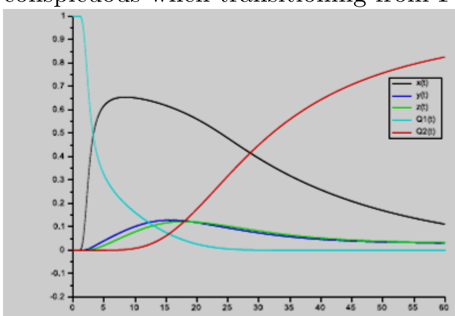


Figure 6 Estimates of the model,$s_1 = .70, s_2 = .65, , \alpha = \delta = \mu = .01, xm = .75, Q_2(m) = 1, B_1 = .60, B_2 = .19, C_1 = .86, C_2 = .14$

4.2 Simulation 3

Within this simulation model, we exert control over learning practices by manipulating key parameters, namely B1, B2, C1, and C2. As illustrated in Figure 8, the transition from controlled behavior back to daily conduct is rendered more fluid. On the other hand, Figure 9 introduces a scenario where a substantial increase in learning practices occurs, but achieving a return to the norm becomes more challenging, a sentiment echoed by Crocq (2013).

Figure 8 vividly demonstrates the eased progression from controlled actions to routine behaviors, signifying the potential for smoother transitions within the learning environment. Conversely, Figure 9 showcases a situation characterized by heightened learning practices, yet the process of reverting to the established norms becomes notably more intricate, aligning with insights shared by Crocq (2013).

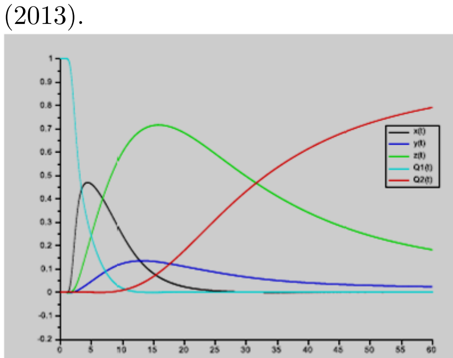


Figure 7 Estimates of the model $x_m = .75, Q_2(m) = 1, B_1 = .067, B_2 = .05, s_1 = .75, s_2 = .58, C_1 = .82, C_2 = .06, \alpha = \delta = \mu = .01$

5 Strengths and limitations

Our investigation makes a significant and distinctive contribution to the existing body of compelling evidence concerning the effectiveness of classroom learning within educational institutions. As the prevalence of this form of learning continues to grow, it becomes imperative to anchor its implementation in evidence-based practices. Although the utilization of a randomized controlled experimental design served to surmount the usual limitations that often impede the extrapolation of results, it was not exempt from its own set of constraints.

Particularly noteworthy is the substantial rate of participant withdrawals, which introduces the potential for a bias towards positive confirmations. The impact of these withdrawals on the statistical robustness of the study likely curtailed the ability to discern alterations in attitudes and behaviors associated with the learning interventions. Furthermore, the potential for selection bias to influence the findings cannot be overlooked.

As we look ahead, the subsequent phase necessitates a comprehensive and meticulous examination of this model. Concurrently, a holistic strategy must be crafted for its dissemination. This will entail refining the methodology and approach to ensure an uncompromising analysis, while also enabling a broader application and more profound influence.

6 Conclusion

This paper introduces a groundbreaking advancement in the modeling of student dynamics within the classroom setting. Notably, it takes into account three concurrent behaviors and interweaves strategies for transitioning from one behavior to another seamlessly. Historically, prevailing models primarily focus on class behavior as a singular entity, which is not always representative of the nuanced reality. Furthermore, the learning process is not a constant phenomenon throughout all events; indeed, the overall student behavior can undergo fluctuations.

In this work, a novel approach has been adopted, encompassing two distinct behaviors in the model: the retentive behavior and the controlled behavior. Drawing inspiration from social sciences, our simulations convincingly demonstrate their potential to influence behavior and facilitate a return to effective learning states. As we move forward, the next critical phase entails a comprehensive scrutiny of this model. Simultaneously, a robust strategy must be devised for disseminating its implications effectively.

References

[1] T Akcadag. Problem behavior management. h. kiran (ed.), classroom management, 2005.

[2] Sevda Aslan. University students' opinions regarding school vandalism in turkey. 2017.

[3] Meral Atici. Comparison of teachers and counselors' ideas about co 1laboration in dealing with students' behaviour problems in primary schools. *Turkish Psychological Counseling and Guidance Journal*, 3(25):55–68, 2006.

[4] N Boz. Why is mathematics difficult. *Necatibey Faculty of Education Elektronic Journal of Science and Mathematics*, 2(2):52–65, 2008.

[5] Barış Çetin. Sınıfta istenmeyen öğrenci davranışlarıyla ilgili sınıf öğretmenlerinin karşılaştıkları sorunlar ve çözüm önerileri. *Journal of Kirsehir Education Faculty*, 14(1), 2013.

[6] B Donmez and M Comert. Competencies of the pre-service and in-service teachers, related to managing misbehavior in the classroom: pre-service teacher point of. *Journal of Kirsehir Education Faculty*, 10(2):47–55, 2009.

[7] N George and L Gamond. Premieres impressions. *L'essentiel Cerveaux et Psycho: Les émotions au pouvoir Science*, 7, 2011.

[8] I Gursimsek and G Saygili. Student'perception concerning with misbehaviors in classroom. *Journal of Buca Faculty of Education*, 23:152–159, 2008.

[9] Elaine Hatfield, Lisamarie Bensman, Paul D Thornton, and Richard L Rapson. New perspectives on emotional contagion: A review of classic and recent research on facial mimicry and contagion. *Interpersona: An International Journal on Personal Relationships*, 8(2), 2014.

[10] A Kaya and HH Tasar. Perception of primary school teachers' on effective classroom management. *Adıyaman University International Journal of Social Science*, 3(5):169–183, 2010.

[11] AP Memisoğlu. Teacher behaviors leading to undesirable behaviors in the classroom environment. *Contemporary education Journal*, 323:32–39, 2005.

[12] BAL Oğuz and Harun Şahin. Review of primary school english coursebooks in terms of creative writing activities. *International Journal of Contemporary Educational Research*, 8(3):152–164, 2021.

[13] Ünal Simsek, Erdal Yildirim, and Turgay Öntas. First-year experiences of social studies teachers starting their profession in the public sector. *International Journal of Educational Methodology*, 7(1):171–185, 2021.