The planets of the binary star Kepler-35

Herbert Weidner*
Corresponding author: herbertweidner@gmx.de
Am Stutz 3, D-63864 Glattbach, Germany

The two stars in *Kepler-35* emit gravitational waves with a frequency of 1116 nHz, which can be measured here on Earth. The decoding of the phase modulations of the GW shows *twelve* companions – one was discovered using electromagnetic waves. The orbital times fit well with the predictions of Dermott's law. The physical interpretation of the result of the phase modulation is difficult, but allows the masses of the planets to be estimated.

1 Introduction

It has been known since 2012 that one planet orbits the binary star system Kepler-35. It was discovered due to short-term fluctuations in the brightness of the stars [1]. Two stars $A1$ and $A2$ orbit each other every 20.733667 days and emit a gravitational wave (GW) of frequency 1.11645 μHz. The measurement method is described in [2,7] and not repeated here. The long-term analysis of the GW over a period of twenty years shows several results:

- Twelve planets orbit the binary system. They can be distinguished because each causes a periodic Doppler shift of the GW with a different frequency.
- The modulation index a of all phase modulations is remarkably large and can only be explained with the assumption $v_{GW} \ll c$ (in the examined frequency range).
- The frequency drift of the binary star system is measured precisely.

Electromagnetic wave observations allow the calculation of f_{GW} in 2012, but not the frequency drift. The analysis of historical data requires a reliable initial value for f_{GW} in the year 2000. At the expected frequency, the spectrum shows a clear maximum.

GWs are always phase modulated (PM) and each PM produces sidebands. The number of individual frequencies and their amplitudes cannot be predicted because the number of planets is initially unknown. Since each planet causes a PM with its orbital frequency, one has to reckon with a typical solar system that the total energy of the GW is distributed over about 50 spectral lines, each with a low amplitude. This can be desirable in technical applications because the bundle of many weak spectral lines is often mistaken for noise and remains undetected. Whether there is a signal at all can only be determined once the PM has been eliminated. The aim of this work is to reduce the energy content of the sidebands and to increase the amplitude of the central spectral line.

2 The order of measurements

As we know that the Kepler-35 binary emits a GW of frequency $f_{GW} = 2f_{orbit}$, the signal is filtered with a bandwidth of 0.3 nHz in order to minimize interference from neighboring GWs. This dispenses with all energy components that are transported by sidebands (a modulation produces sidebands). Initially, we have to accept an inaccurate result due to the poor S/N and the neglected sidebands. The aim of this first step is to assess the frequency stability of f_{GW} over a period of twenty years and to eliminate the *slow* modulations that may cause it.

- A curvature means a phase modulation (PM) with $f_{mod} > 10^{-9}$ Hz. From the curvature, we determine an approximate value for f_{mod}.
- A linear progression may be generated by a constant frequency drift or by a very low-frequency PM ($f_{mod} < 10^{-9}$ Hz) with a suitable phase. Then the solution requires a time-consuming iteration.

Before all other measurements, these possible modulations must be eliminated. To achieve this, one iterates the modulation index and phase of the suspected low-frequency PM until the intervals between two zero-crossings of the sine wave match (MSH-procedure).

Next, one compensates the inevitable PM with $f_{orbit} = 31.8$ nHz generated by the Earth's orbit. The signal amplitude and the accuracy of the frequency measurement increase because more energy is concentrated on f_{GW}. In addition, we are informed about the direction from where the GW arrives. As with all higher-frequency PM, the modulation index and phase are iterated until the amplitude of f_{ZF} reaches a maximum.

These two steps are repeated several times in order to increase the signal amplitude as much as possible.

After this preliminary work, the detective work begins: Planets force the GW source to orbit the common center of gravity and each planet modulates f_{GW} at a different frequency (see discussion in section 5). Since the corresponding sidebands have a very low amplitude and lie outside the narrow bandwidth of the signal processing (BW$<$ 0.4 nHz), the modulation frequencies have to be guessed at. It's pointless to look for the individual sideband frequencies in the surrounding noise since there are no clues for frequency and amplitude. And if anyone spots suspicious lines, it would be even more difficult to prove that the frequencies found are part of the GW in terms of amplitude and phase. The MSH method [5, 7] does these tasks in a completely different way.

It is helpful to know the orbital data of the two known planets, because then Dermott's empirical law [4] provides clues for the orbital periods of other planets of the binary system. Although these estimates are quite rough, the *capture range* of the iteration is sufficient to determine the exact value.

Previous studies have shown that binary stars often have many planets [5–11]. If Kepler-35 has twelve planets and each one causes a PM with the modulation index $a \approx 1$, the total energy of the GW is distributed over a total of about $12 \cdot 2 + 1 \approx 25$ spectral lines in the vicinity of f_{GW}, which disappear in the noise because of their rather small amplitudes. Since the signal power is spread over a large bandwidth, the signal PSD is low – often significantly lower than the noise PSD – so that it may not be possible to determine whether the signal is present at all. Therefore, at an early stage of the analysis, it is pointless to look for conspicuous lines in the spectrum. This changes in the course of the analysis because the amplitude of f_{GW} increases with each detected planet. The, the energy of many sidebands is accumulated in the central spectral line.

3 Results

Assuming that all phase modulations are generated by planets, the binary system $A1-A2$ of the GW source *Kepler-35* has twelve planets.

- Planet B with the orbital period $P_B = 72.34$ days ($f_B = 160.004$ nHz). The parameters $a_B = 2.1407$ and $\phi_B = 2.08805$ are discussed from section 5 onwards.
- Planet C with $P_C = 120.64$ days ($f_C = 95.93641$ nHz). This is the only planet from *Kepler-35* that was discovered with electromagnetic waves [1]. The measured orbital periods differ by 9%. $a_C = 1.89813$ and $\phi_C = 2.74825$.
- Planet D with $P_D = 196.73$ days ($f_D = 58.833$ nHz). $a_D = 2.6917$ and $\phi_D = 0.032$.
- Planet E with $P_E = 327.14$ days ($f_E = 35.379$ nHz). $a_E = 1.8921$ and $\phi_E = 6.149$.
- Planet F with $P_F = 513.5$ days ($f_F = 22.541$ nHz). $a_F = 1.8508$ and $\phi_F = 1.6018$.
- Planet G with $P_G = 2.518$ years ($f_G = 12.584$ nHz). $a_G = 0.3886$ and $\phi_G = 4.866$.
- Planet H with $P_H = 4.151$ years ($f_H = 7.634$ nHz). $a_H = 0.4445$ and $\phi_H = 2.7474$.
- Planet J with $P_J = 7.460$ years ($f_J = 4.2475$ nHz). $a_J = 0.2156$ and $\phi_J = 1.5575$.
- Planet K with $P_K = 14.51$ years ($f_K = 2.184$ nHz). $a_K = 0.4352$ and $\phi_K = 3.2612$.
- Planet L with $P_L = 35.65$ years ($f_L = 888.8$ pHz). $a_L = 1.8878$ and $\phi_L = 2.7704$.
- Planet M with $P_M = 91.37$ years ($f_M = 346.8$ pHz). $a_M = 3.2495$ and $\phi_M = 2.044$.
- Planet N with $P_N = 281.4$ years ($f_N = 112.6$ pHz). $a_N = 0.9875$ and $\phi_N = 3.31$.

All results are reproducible and have been calculated several times with different values of important parameters such as bandwidth.

After compensation of all PM with the frequencies $f_B ... f_N$ mentioned above, the residual ripple of f_{ZF} is so low that the existence of further planets with $P < 500$ years is improbable. The short database of only 20 years does not allow to determine even longer time constants.

As expected, f_{GW} is also phase modulated with $f_{orbit} = 31.68754$ nHz. $a_{orbit} = 2.4977$ (see section 5.1). From the phase angle $\phi_{orbit} = 1.88426$ it follows that here on Earth, we receive maximum blueshift on every $365 \cdot \phi_{orbit}/2\pi = 110$th day of the year f_{GW}. According to [3], this should take place on the 123th day of the year (error $\approx 11\%$).

On January 1, 2000, the frequency of the GW source was 1.11645 μHz. The drift is $\dot{f}_{GW} = 111.06 \times 10^{-20}$ Hz/s and was never measured with electromagnetic waves.

In retrospect, it is confirmed that it is important to eliminate *all* PM: Compensating the PM with the MSH method allows the amplitude of the GW to rise back to 100%, improving the S/N significantly (figure 1).

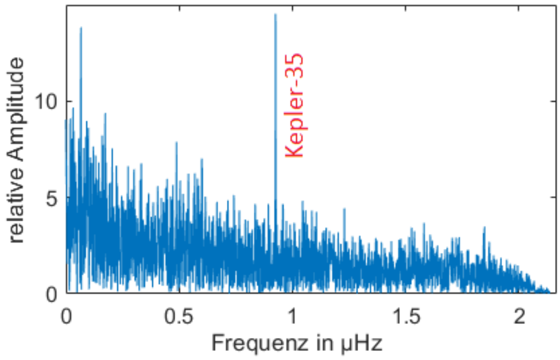

Figure 1): *Spectrum of the GW of Kepler-35 after changing the frequency to $f_{ZF} = 1/(300$ hours$)$ and compensating the phase modulations. This increases the amplitude of the carrier frequency of the GW significantly. The vicinity of f_{ZF} is filled with the distorted spread spectra of previously undiscovered GWs of similar frequency.*

4 Dermott's Law

For a long time people have been looking for reasons for obvious connections between the orbital periods P of planets. The ansatz (1) comes from Dermott [4]

$$P_n = P_0 \cdot c^n \tag{1}$$

with $n = 1, 2, 3, 4...$ Figure 2 shows the best approximation with $P_0 = 45.54$ days and $c = 1.635 \pm 0.001$. For the relation of Dermott and the older Titius-Bode series there is no deeper justification. Dermott's law reliably provides good initial values when searching for unknown planets.

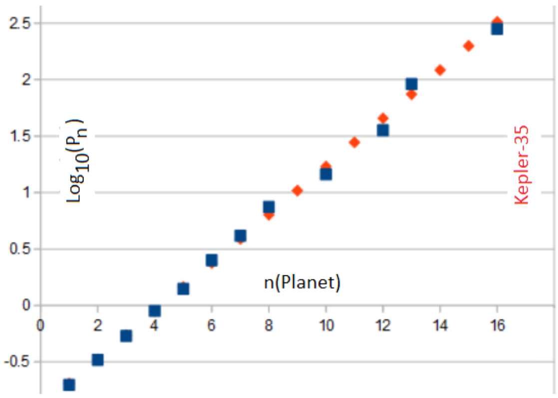

Figure 2): *The logarithm of the orbital period of the planets (in years) of Kepler-35 as a function of their order. The actual values (blue) hardly differ from Dermott's law (red). Despite an intensive search, the PM of the "missing" planets could not be detected. The resonance $P_L : P_M \approx 2 : 5$ could cause the deviation n=12, n=13.*

5 Notes from an astronomical point of view

A word of caution: The MSH method measures and removes phase modulations from f_{GW}. Now we assume that the Doppler effect caused by planets is the only reason for the PM of the GW source.

Translating the abstract results of the iteration (section 3) into astronomical terms, the following relationships apply: All time specifications refer to the beginning of the analyzed data chains on 2000-01-01 and apply under the condition that the corresponding celestial bodies describe circular orbits. The phase shift ϕ indicates at what later point in time the instantaneous frequency of the GW is blue-shifted to the maximum. Then one has to add the frequency deviation Δf produced by the Doppler effect to the average frequency f_{GW}. The results of the compilation given above may be evaluated independently of one another because all PM are linearly superimposed.

5.1 The Earth orbit causes a PM

From the modulation index $a_{orbit} = 2.4977 = \Delta f_{orbit}/f_{orbit}$ follows $\Delta f_{orbit} = 79.146$ nHz. This frequency deviation cannot be explained with the assumption that any GW travels at the speed of light. According to RT, we expect a maximum Doppler shift of

$$\Delta f_{orbit} = f_{GW} \cdot \left(\sqrt{\frac{c + v_{orbit}}{c - v_{orbit}}} - 1 \right) \approx f_{GW} \cdot \frac{v_{orbit}}{c} = f_{GW} \cdot 10^{-4}. \tag{2}$$

The position of *Kepler-35* is far north of the ecliptic plane (δ =+46° 41' 23"). Therefore, the Earth approaches this target with the maximum speed $v_{orbit} = 11900$ m/s. If you insert this value into equation (2), Δf_{orbit} should be smaller than 44 pHz. The actually measured value is about 1800 times larger! A measurement error of this magnitude can be ruled out after careful examination. What is causing the discrepancy? The equations of the PM and the Doppler effect are well founded and confirmed a million times. What remains is the correction of the assumption, that GWs propagate at the speed of light. The calculation of the instantaneous frequency uses the longitudinal Doppler effect, in which the frequency is corrected relativistically. For maximum blueshift applies

$$f_{GW} + \Delta f_{orbit} = f_{GW} \sqrt{1 - \left(\frac{v_{orbit}}{c}\right)^2} \cdot \frac{1}{1 - \frac{v_{orbit}}{v_{GW}}} \approx \frac{f_{GW}}{1 - \frac{v_{orbit}}{v_{GW}}} \tag{3}$$

If we transform the equation (3), we get

$$\frac{v_{orbit}}{v_{GW}} = 1 - \frac{f_{GW}}{f_{GW} + \Delta f_{orbit}} = 0.0662. \tag{4}$$

With this intermediate result, we calculate

$$v_{GW} = \frac{v_{orbit}}{0.0662} = 180 \times 10^3 \; \frac{m}{s} \approx \frac{1}{1670} c. \tag{5}$$

This result is much lower than the speed of light and is valid for $f_{GW} \approx 1116$ nHz. The comparison of the results of previous measurements of binary systems reveals a certain trend. This topic will be deepened in another paper.

5.2 The Planet *C*

The planet C was detected with electromagnetic waves [1] *and* with GW. Decoding the PM of the GW shows that C orbits the GW source $A1 - A2$ with the period $P_C = 120.64$ days. Assuming a circular orbit, it follows from the phase angle $\phi_C = 2.74825$ that this companion produces maximum blueshift of f_{GW} on $120.64 \cdot \phi_C / 2\pi = 53$th day after 2000-01-01. $P_C/2$ days later it caused maximum redshift. So, on the 83th day after 2000-01-01, planet C must have partially covered the GW source (transit of the planet across both stars A1 and A2).

From the modulation index $a_C = \Delta f_C / f_C = 1.89813$ of the PM, we calculate the maximum frequency deviation $\Delta f_C = 182.1$ nHz. This maximum value of the periodic frequency shift of f_{GW} should be the result of the Doppler effect, because the GW source rotates around the center of gravity of the system. It is difficult to explain this result with previous assumptions: The ratio $f_{GW}/\Delta f_C \approx 6$ contradicts the maximum value that results from previous assumptions of the theory of relativity (equation 2).

The calculation of the radial velocity is the task of classical astronomy. Considering the GW source $A1 - A2$ as a star and the planet C as a companion, Kepler's third law provides the orbital equation for the two-body system.

$$4\pi^2 (r_A + r_C)^3 = GT^2 (m_{A1} + m_{A2} + m_C) \tag{6}$$

The radii refer to the center of gravity of the trio and the center of gravity theorem $(m_{A1} + m_{A2}) r_A = m_C r_C$ applies (we ignore other planets). With the assumed masses [1] $m_{A1} = 0.8876 m_\odot$, $m_{A2} = 0.8094 m_\odot$ and $m_C \approx 40 m_{Earth}$ we calculate $r_A = 6.07 \times 10^6$ m and the orbital velocity $v_A = 3.66$ m/s (the orbital speed of C is much higher and not measurable because the planet C does not emit any GW).

Analogous to the equations (4) and (5) we get

$$v_{GW(C)} = \frac{3.66 \; m \; s^{-1}}{0.1402} = 26 \; \frac{m}{s} \approx \frac{1}{1.15 \times 10^7} c. \tag{7}$$

This result is ridiculous low and raises questions: Where is the source of the GW in a binary system? At the barycenter of the two stars $A1$ and $A2$ or closer to one of the massive stars? Does the Doppler effect correctly describe a GW close to the source? In the immediate vicinity of the GW source, is there a difference between the near field and the far field as with electromagnetic waves?

5.3 All Planets *B...N*

If you rely on the mass of the planet C [1], you get a result for $v_{GW(C)}$ that does not correspond to the usual expectations for the propagation speed of GW. Nevertheless, one can use it to classify the masses of the other planets of *Kepler-35*. Substituting the results for P_n and a_n from Section 3 into equations 6 and 7 gives the following estimates for m_n:

Planet	B	C	D	E	F	G	H	J	K	L	M	N
P_{orbit}	0.20	0.33	0.54	0.9	1.41	2.52	4.15	7.46	14.5	35.7	91.4	281
m_{planet}	57	40	42	23	17	2.5	2	0.67	0.86	2.1	1.9	0.27

Table 1): *The orbital periods of the planets (in years) and the estimated masses (as multiples of the Earth's mass) of the twelve Kepler-35 planets.*

6 Summary

From a communications point of view, decoding the phase modulations of f_{GW} is a standard task of digital signal processing. The signal has a good S/N (figure 1), the receiving antenna is insensitive to earthquakes. No assumptions are needed at any stage of decoding. We need no computationally intensive comparisons with pre-calculated patterns (search templates) based on model assumptions. PM demodulation is a standard task in digital communications engineering.

The opposite is true for the interpretation of the results from an astronomical point of view: The high values for the frequency deviation (Δf) can only be explained by the assumption that gravitational waves at low frequencies around 1.1 μHz propagate significantly more slowly than the speed of light. The processing of this question is not yet complete.

References

[1] Welch, W. et al., The Transiting Circumbinary Planets Kepler-34 and Kepler-35, https://arxiv.org/pdf/1204.3955.pdf, 2012

[2] Weidner, H., How to Receive Continuous Gravitational Waves, 2023, https://vixra.org/abs/2307.0112

[3] Calculate radial velocities, www.gb.nrao.edu/GBT/setups/radvelcalc.html

[4] Dermott, S., On the origin of commensurabilities in the solar system - II: The orbital period relation. Mon. Not. R. Astron. Soc. 141 (3): 363–376, 1968

[5] Weidner, H., Detection of the Companions Algol-D and Algol-E and the Speed of Gravitational Waves, (2023), https://vixra.org/abs/2306.0102

[6] Weidner, H., Measurement of a Continuous Gravitational Wave near 2619.9 µHz, https://vixra.org/pdf/2203.0130v1.pdf, (2022)

[7] Weidner, H., The Eight Planets of the Kepler-47 Star System, (2023), https://vixra.org/abs/2307.0152

[8] Weidner, H., The Planets of the Binary Star HD 75747, (2023), https://vixra.org/abs/2308.0032

[9] Weidner, H., The Planets of the Binary Star Kepler-16, 2023, https://vixra.org/abs/2308.0049

[10] Weidner, H. The planets of the binary star TOI-1338/BEBOP-1, 2023, https://vixra.org/abs/2308.0070

[11] Weidner, H., The Planets of the Binary Star Kepler-34, 2023, https://vixra.org/abs/2308.0072