

## Riemann Hypothesis :

### Abstract Physics Behind Mathematical Equations : New way of Looking at the Mathematics

15<sup>th</sup> August 2023

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**Abstract :** In this paper, I'm looking unconventionally at the underlying Physics behind the Mathematical Equation in context of Riemann Hypothesis to show that Riemann Hypothesis would be true ! It's about the new way of looking at Mathematics where one has to imagine and look at the governing abstract physicalities e.g. symmetry behind the arithmetical operations like addition, multiplication, 0, complex number etc.

Mathematics has its own abstract Physics behind it. Riemann Hypothesis is about imagining that rather trying to solve the equations endlessly in my humble view.

Long back Sir Richard Feynman quoted that. Next great era of Human Awakening would come – Today we don't see the content of the Equations. He was very right here in context of Riemann Hypothesis, 160 years old Mathematical puzzle, one of the most important problems in mathematics. Some of the greatest mathematical minds have unsuccessfully tried the

problem. So, let's think why is it so ? Albert Einstein said – No problem can be solved at the level that was formed. It seems to me contextually that previous attempts have been made at the same level at which Bernhard Riemann formulated the problem. There is somewhere need to look t the problem from a higher level. That's the point in my humble view our learned Mathematician's have not be able to do, may be probably due to lack of imagination. I don't prove conventionally rather show unconventionally. I also believe Mathematics needs to be broaden beyond Theorems to Laws like in Physics because there are certain physicalities governing the dynamics of structure of mathematical equations.

For confirmation, one can visualise wonderful computer generated graphics of Physicalities of Riemann Zeta function in picture . It resembles the hidden quantum beauty behind Numbers ! One can imagine the beauty of symmetry within the content of mathematical Arithmetical/Algebraic equations. I think Sir Michael Atiyah (Fields and Abel Laureate, Senior Respected Mathematical Authority) also meant at the development of Arithmetical Physics type new branch of mathematics while his address at Abel Lecture at ICM in 2018..

Infact it happens learned matured brained becomes too adapted often to the conventional approaches to imagine something beyond from higher level . That's the fundamental reason, some new brainpower is required to imagine at higher

level. One line of Imagination at higher level would be Equivalent to Hundreds of Lines at lower Levels... Like Coding Language.

So, the point is – How to look at the Riemann Hypothesis from higher level perspective?

For that, we have to imagine what actually is done when a mathematician's brain solves any problem. Infact what does it mean when a computer solves a problem. Infact that's related to algorithmic approach and David Hilbert once dreamed to formalize the whole mathematics.

As Kurt Godel wonderfully demonstrated : It starts with some axioms as the base and then manipulates the axioms to prove some results subsequently by the set of sequential statements based on arithmetical operators. That's what is called a Proof in Formal mathematics. But my humble question to the Learned Formal Mathematicians is where do those fundamental axioms come from ? Those axioms come from the day to day physical experiences of mathematicians. Those axioms are based on certain higher level physicalities.

But as David Hilbert once quoted – Advanced Mathematics is basically a Game of Symbols arbitrarily defined based on certain rules.

So, here would like to ask a very fundamental question – Is Mathematical System Self -Conscious like Human Brain which

can prove themselves and those axioms upon which they are based upon ? Can a mathematical system prove those fundamental rules/axioms upon which they are built upon ? It's like Self-referential Problem and that's the core principle behind Gödel's Results. Infact it's a deeper characteristic in the Universe and Nature not just Arithmetical system as demonstrated by Gödel. Infact I tried to show that it would hold true for any mathematical system including real numbers or any system because they are not self-conscious.

So, the point is – if a problem talks about those fundamental axioms and rules, how to prove that within that mathematical system.

For example :The rules of arithmetic operations , addition, multiplication etc. have been defined based upon certain physicalities and symmetries in the Euclidean Geometrical Space .

Now If a problem comes to prove something related to those physicalities on which these Arithmetical Operations were defined, Can it be proven by using those Arithmetic Operations themselves ? That means Problem about Arithmetical Operations can't be proven by using Arithmetical Operations only in that Mathematical system internally.

No. For that one will have to come at higher level and see the mathematical system from that perspective. That's quite common sense.

What I mean here is that let's say Prime Numbers are defined in our number system. But if it is asked why Prime Numbers distribution has this Physical Pattern inside their Plots. That can't be prove by using those games of operations involving prime numbers themselves.

The reason why I have explained all these is Riemann Hypothesis somewhere is similar case. It's related to those underlying physicalities and physical characteristics on which the rules of arithmetical operations like addition, multiple, Complex numbers etc..have been fundamentally defined. The reason in my humble view, many conventional Mathematicians are not able to solve is they are trying to solve the Self Conscious Statement of the Mathematical System by Standing within the System. As explained earlier about steps of proof, they try to do permutations and combinations of different operations internally (like playing with the piece of paper by folding it in different ways and writing on it inside) to come at the results about those external physicalities upon which those axioms about these operations were defined e.g. + multiples by + = +

(like why the Paper is a Square ?)

Like Prove that Plus multiplied by Plus = Plus using Plus & Minus themselves ? Or Prove that Circle is round ? Prove that Triangle is Triangular ? These are Self -Referential Problems.

Riemann Hypothesis is basically about looking at those Underlying Underlying Physicalities behind Mathematics itself upon which those Fundamental Symmetricity Physicalities of Arithmetical Operations, Complex Numbers, etc. were defined. If those basic Rules about Operations are changed, Riemann Hypothesis would definitely change. There is no mystery about it. The mystery is in the mind of Learned Mathematicians who are not able to imagine and look at it from higher level perspectives. First they created the system and then they are themselves finding it mysterious.

I am really worried if similar things happen during the age of AI/ML

First Mathematicians created and then they would say they find it mysterious and out of control .Infact Deep Learning etc. has become complex enough to be understood.

So, What I tried to show in my analysis to prove Riemann Hypothesis. I don't look at the problem internally rather try to

look at the Structural Symmetry & Physicalities behind the definition of those operations and variables on which Riemann Zeta Function and its Functional Equations have been created. That means Physicalities and Symmetrical Structures of Addition, Multiplication, 0, Numbers, Complex Numbers etc on which the Equations have been created !

For example, when Addition, Multiplication etc are defined on say Decimal Number System say for example

$$45.5 * 20$$

One can see how digits are arranged at different places (1st, 10<sup>th</sup>, 100<sup>th</sup> places etc like energy orbitals where digits transition from one level (place ) to higher level(place). This structure of Number systems and the Operations itself is borrowed from Quantum Energy Orbitals like for Electrons at different energy levels.

My point is that behind all these defined algebraic and arithmetic structures in Mathematics exist the Physicalities based on certain Symmetries.

What does "0" represent ? It lies on the midpoint line of symmetry on the Number line.

The Point I've been trying to convey that certain physicalities of symmetry lies behind the scene of these mathematical operations and numbers and functions.

So, I tried to look at the Symmetrical Physical Structure of the Equations and Relate Correspondingly to its Physical Graph and how in this Game of Symbols, such foundational underlying Physicalities and Symmetricity will have to remain Conserved.

Like if the Algebraic Equation of a Circle is Symmetric and Homogeneous, the Physicality behind its graph will also remain Symmetric and Homogeneous. If we tilt the Equation of Circle to form the Equation of Ellipse or something else, its physicalities would also change correspondingly.

On that basis as David Hilbert quoted mathematics being the game of symbols. Intrepid to play the game while conserving those symmetric physicalities. That's sufficient to prove Riemann Hypothesis for Riemann Zeta function.

I'll explain the proof here.

This is Riemann Zeta Function for  $\text{Re}(s) > 1$



$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots$$

And its analytic continuation elsewhere.

The Functional Equation Satisfied is

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s),$$

We just look at the functional equation when the LHS term can be 0. Also one knows that many other power terms and Gamma function never attains the 0 value, so eventually it turns out to be a simple functional equation of the form

$$f(s) = \sin(\pi * s/2) * f(1-s) f(1/2-s)$$

With some transformation  $s$  replaced by  $\frac{1}{2} - s$ ,

It becomes

$$f(1/2 - s) = \sin[(\pi * 1/2 * (1/2 - s))] * f(1/2 + s)$$

So, It's the game of three terms

$f(1/2 - s)$  term A on LHS &  $\sin(\ )$  term B &  $f(1/2 + s)$  term C on RHS

Now I apply the Rules of Multiplication of 0 to find out when  $f(\ )$  can be 0. The Trajectory of Trivial 0s already come from the same equation. The Trajectory of Non-Trivial 0s would also come from the same equation.

Crux of the Game among A on LHS and B & C on RHS.

We have to find out the arrangement of 0s overall when A would be 0.

So the centre point is  $B \sin(\ )$

B would be 0 for certain set of point values say  $B(m)$  and non-0 for say  $B(n)$ .

Now if  $B = 0$ , A will have to be 0, C may or may not be 0

Because  $0 = 0 * 0$  or Non-0 ( as per the defined rule of arithmetic, multiplication of 0s)

If B is NOT 0, A will be 0 when C will also be 0

Because of the above rule of multiplication of 0s.

Or else both of them would be Non-0.

Also, already known there is no 0s for  $\text{Re}(s) > 1$ .

Now under these possible scenarios, we have to arrange the 0s in the plane by looking at the possibility of arranging the 0s on the plane.

Now here the symmetricity and homogeneity in the structure of the equation of  $f(s)$  i.e. Riemann Zeta function and Other Counter Examples to the Functional Equations would play the differentiating roles.

**For all the symmetric and homogeneous terms structure of the mathematical symbols form of  $f(s)$  in the simplest forms which are symmetric and homogeneous and satisfy the Riemann Zeta Functional Equations, all the values of  $s$  for  $B(m)$  and  $B(n)$  will play the game symmetrically and homogeneously.**

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For all the asymmetric and heterogeneous terms structure of the mathematical symbols, all of the  $B(m)$  and  $B(n)$  will not behave asymmetrically and heterogeneously. There would be likely further exploration of the sub groups of players within  $B(m)$  &  $B(n)$  say  $B(m,a)$  and  $B(m,b)$  &  $B(n,a)$  and  $B(n,b)$ . That's where the arrangement of 0s would become asymmetric for some specific values of  $s$  due to asymmetric and heterogeneous

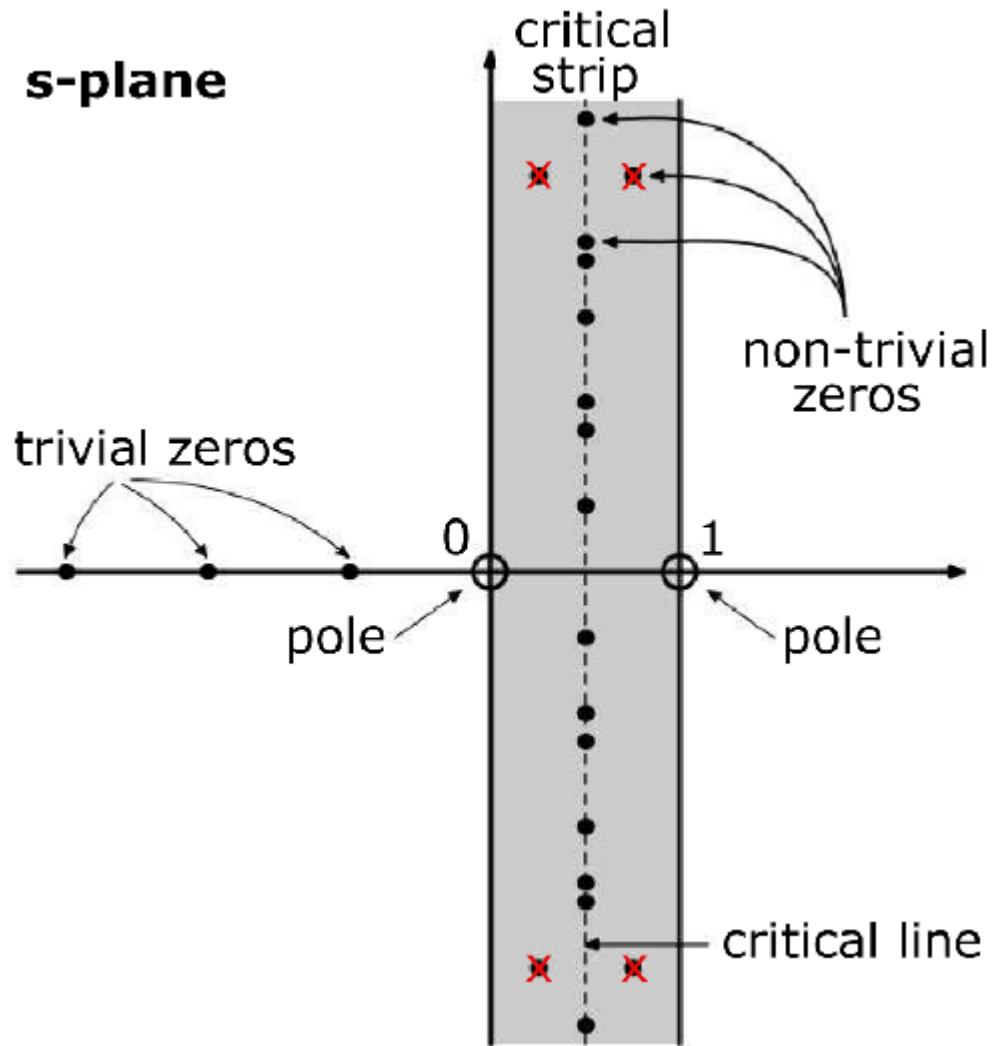
behavior by the set of sub groups of players as happens in the Game Theory.

One can look at the arrangement of numbers and arithmetical operations to imagine the same in the Riemann Zeta function Equation and also at different Counterexamples to the corresponding functional equation.

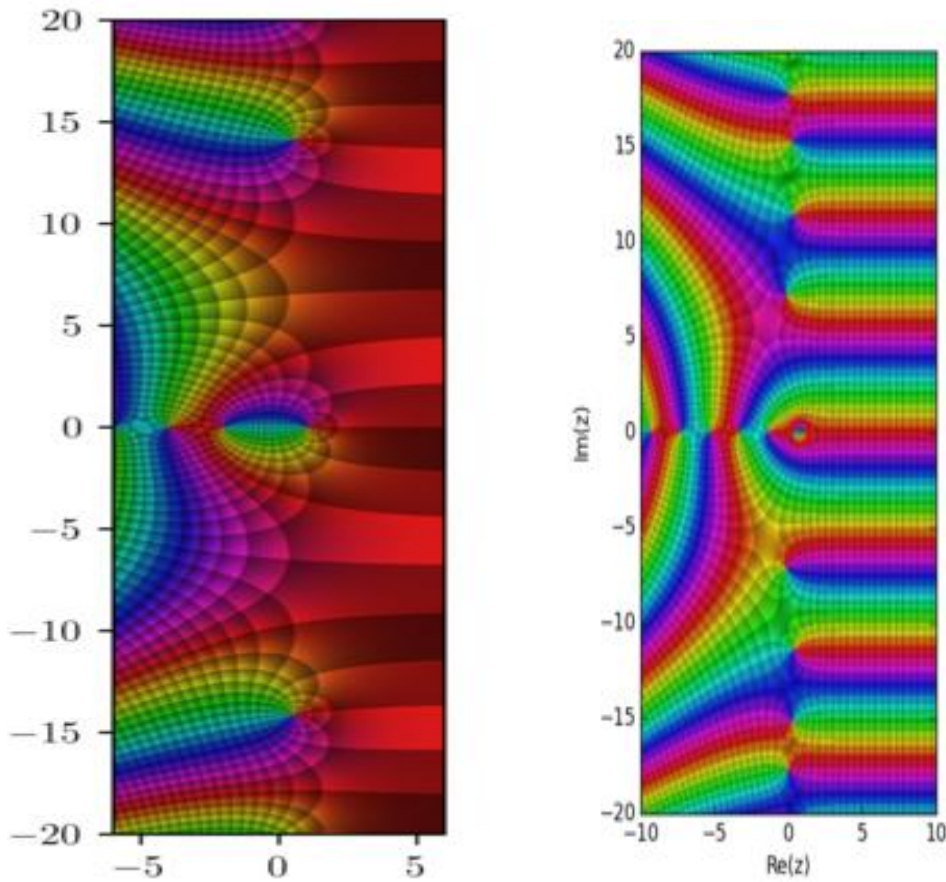
In fact whole Universe is a Game at different level try to bring the system to Equilibrium stage over the time.

Mathematical/Arithmetical system also have inherent Equilibrium stage like in Physics. Nash Equilibrium coming originally from that Equilibrium in the Mathematical System which itself has its own abstract Physics.

Equilibrium stage leads to the Solution of the Game i.e. the Best Possible arrangement of 0s where 0s represent the Symmetry. Any Equation becomes 0 for any value of the variable when there is symmetry in + terms and – terms on the ground. That where the fundamental hidden physicalities e.g. Symmetry come into the play.



The Key Point is : The Structure of the terms of Equations and their corresponding Physical Behavior in the Graph.



Infact that hidden Physicalities also bring Interconnectedness inside the mathematical space like one in the quantum physics.

One can visualise the ultimate Physical Beauty of Symmetricity in the Graph of Riemann Zeta Function & Riemann Hypothesis( in Computer Graphics) . It perfectly looks like a Quantum wave structure having physicalities etc as per the defined rules of the mathematical system.

So, I believe it's not mysterious rather it's just the hidden play of the same Physicalities on which foundations the rules of arithmetic operations like  $+$ ,  $-$ ,  $*$ ,  $/$ ,  $i$ (complex) etc. have been defined fundamentally.

So, what I did, I visualized this as the game of these three terms and showed that only way to arrange the Non-Trivial Os would be when they lie on the critical line or else the entire function would be 0. But this symmetry in the graph would be true for Riemann Zeta function only because of the symmetry in the structure of equations of RZ function. All other Counter examples like Finite Sum of Dirichlet L functions or many others will not be as symmetric and homogenous as the Riemann Zeta function in terms of the structure and arrangement of mathematical symbols. This is where one can imagine what I told that Equation of Circle being Symmetric and Homogenous is the reason why the Graph of Circle also has Symmetry and Homogeneity. If there is introduced some asymmetry and heterogeneity in the equation of circle, say like ellipse or something else, the graphical representation also gets similar asymmetry and heterogeneity.

One needs to look at the similar symmetry in the Structural Equation of RZ function as well and that's the reason why all

the values of  $s$  when  $\sin(\ ) = 0$  and when  $\sin(\ )$  is not  $0$ , they behave homogeneously and symmetrically for RZ function in the arrangement of  $0$ s game among those three functions. For other Counter examples, one can find out how different symbols lack the distortion in the symmetry and homogeneity leading to distortion in the graphs and hence possible violation of the Non-trivial Zeros being on the Line of Symmetry. Critical Line is basically the Line of Symmetry just like  $0$  lies on the line of symmetry of the Number line.

If someone says that Non trivial  $0$ s are not on critical line say they are on  $\text{Re}(s) = \frac{1}{2} - s$  and  $\frac{1}{2} + s$  for some specific value of  $s$ . then my question on the basis of symmetry and homogeneity would be when there was no asymmetry introduced in the game while defining them why it will be asymmetric for some specific value of  $s$  and not others ? Why this asymmetry would occur if the RZ function is symmetric and homogeneous in the structure of its equation? This is where the imagination is required to be able to look inside the structure of the equation and corresponding structure in the graph.

**So, my So Called Law(in place of Theorem is) : Physicalities of Symmetry and Asymmetry assumed at the foundation of axioms and rules behind the definition of mathematical/arithmetical/algebraic system/game definitions remains conserved in the graphical form as well !**



**Like Emmy Noether 's Theorems based on Symmetry and Conservation Laws , My point is the Conservation of Physicalities behind Mathematics.**

**Even Mathematics has its own abstract Physics like the Physics of Bodies in Real World. So, new branch of Mathematics like Arithmetic Physics or similar should study this abstract Physics of Mathematics itself !**

**Hence rather than making mathematics as a mechanical system of theorems and axioms, we should further study it as a discipline like Laws of Mathematics where the Underlying abstract Physics of Structure of Equations, Graphs etc are deeply studied !!**

**Infact this will lead to new branch of mathematics at the boundary of mathematics and physics where symmetrical rules behind the mathematical system's definition needs to be studied in detail for further advancement !**

It also paves the way forward to broaden the new branch of mathematics called Arithmetical Physics or Some other Physics where those Physicalities behind the Basic Rules of Mathematical Systems are studied deeply. I had talked about these things and hidden concepts in my amateur public paper around 2011 and then subsequently as well as an amateur Number Theorist . Sir Michael Atiyah approach to the Problem

(2018) talks about similar Physicalities( Arithmetic Physics) to some extent .

What is the need of the hour that our learned Contemporary Mathematicians need to broaden their views of Mathematics rather than just playing Permutations and Combinations of those game rules like a machine which even Computer can play to some extent. But Mathematics is beyond that.. It's not Self-conscious to prove the results about those Physicalities behind the Rules/Axioms upon which they are formulated. Here is the Role of Human Conscious Understanding of the Mathematicians' brains. Simply by making a mathematical tough and tough by maintaining the inertia that they don't have to solve every problem by residing at the same level on which they are formed as Albert Einstein wonderfully quoted , would let them go nowhere except maintaining and satisfying their ego for centuries at the cost of future development of true beauty of mathematics as the creative subject rather a mechanical subject !! I also believe that there would be the role of Human in the Mathematics rather than an automated mechanical system. Somewhere Proof theory based on Theorems believes foundationally in Completely Deterministic aspects of Mathematics in the Universe....Automated way. That's probably not true and the way its axioms are derived from based on physical characteristics of spaces, it has to be

treated as physics type science subject having human role too as the observer.

In fact, I have also written a paper on the role of observer and the time as the dimension to mathematics at deeper level is crucial for evolving traditional mathematics and resolving long standing paradoxes and truth of mathematics at the foundation of Formalists theorems based mathematics.

At deeper level, Conventional Mathematics needs to evolve more for Real World applications. That would be a different topic for different paper.

Note : Author has discussed this with world renowned mathematician and authority Prof .Ken Ono in 2019 (while he was Vice President, American Mathematical Society). He agreed with his views partially and then regarding counter examples, he was allegedly not able to grasp and hot dubious, may be due to lack of communication over email exchange or some other Reasons).

The Author doesn't claim to have proven Riemann Hypothesis conventionally rather shown his raw ideas how Riemann Hypothesis should be looked at why it would be true , using unconventional approaches and how it could have vast unforeseen changes in how Mathematics should be imagined in future at higher level by contemporary learned conventional Mathematicians

Note : I had discussion with Prof Ken Ono, a giant internationally renowned expert in the field in 2019, when he was Vice President at American Mathematical Society (AMS) . He kindly agreed with my views partially but couldn't grasp further related to explanation of counterexamples way back then and advised me to put the views publicly.

I further know as it requires fundamental crude natural imaginative views, it could take enough time but then I could be wrong too !!

References :

## 1. Sir Michael Atiyah Paper on Riemann Hypothesis

[https://www.google.com/url?sa=t&source=web&rct=j&opi=89978449&url=https://ep00.epimg.net/descargables/2018/09/25/b133e2bf9a3e7bb55f5fae26dcf9b8c0.pdf&ved=2ahUKEwickO7fqtyAAxWj4DgGHY4CBCsQFnoECBUQBg&usg=AOvVaw3\\_5J9J0ray47Zs6NBBNpzE](https://www.google.com/url?sa=t&source=web&rct=j&opi=89978449&url=https://ep00.epimg.net/descargables/2018/09/25/b133e2bf9a3e7bb55f5fae26dcf9b8c0.pdf&ved=2ahUKEwickO7fqtyAAxWj4DgGHY4CBCsQFnoECBUQBg&usg=AOvVaw3_5J9J0ray47Zs6NBBNpzE)

## 2. Please Google Riemann Hypothesis