a cubic ellipsoid geometric model of the atomic nucleus and its mass formula

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Abstract

This paper examines the hypothesis that the structure of the nucleus determines that of the atom and its properties and attempts to construct a geometric model of the nucleus that contributes to this hypothesis.
The model proposed here suggests that the structure of the nucleus is, in general, an ellipsoid with the nucleons connected by cubic bonds and the nucleus shells correlate with those of the atom.
In accordance with the model, a simplified theoretical mass formula was created to compare it with the experimental data; the tests included about 94 nuclei from Ar_{18}^{40} to Pu_{94}^{244}.
The mass formula depends on two terms:

- E_b: the total binding energy between the nucleons in the nucleus.
- E_c: the total electric energy of the nucleus.

and has two parameters:

- d_0: the minimum distance between two neighboring nucleons in the cubic structure of the nucleus.
- e_b: the binding energy between these neighboring nucleons.

The results for the calculation parameters were:

- e_b = 5.72±0.03 Mev
- d_0 = 1.62±0.03 fm

The results for the relative errors of the mass formula calculation were:

relative error	maximum	average	standard dev.
	1.9%	0.6%	0.5%

If we consider the nucleons for simplicity as rigid bodies, then we get a rough estimation for d_0 through the radii of the proton and neutron:

- $r_n \approx 0.80$ fm, $r_p \approx 0.84$ fm, $d_0 \approx (r_n + r_p) = 1.64$ fm
- *relative deviation for* d_0: $\left|\frac{d_0 - (r_n + r_p)}{(r_n + r_p)}\right| < 2\%$

These results for the mass calculation and d_0 strengthen the model assumption and the mass formula.

Remark: this paper shall serve as a starting point only for this model, and as such it doesn't cover many subjects and following research shall further develop and expand it.

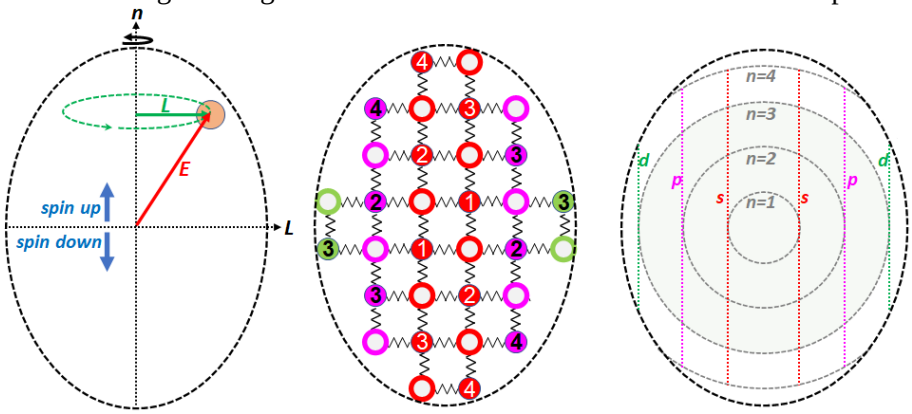
Content

The model at a glance

According to the model the following is the description of the nucleus:

- The nucleus has an ellipsoid shape.
- The nucleons are connected in a cubic form.
- Protons are connected to neutrons (**p**-**n**).
- Neutrons are connected mainly to protons.
- The protons are populated and organized in shells in the nucleus in a full analogy to those of the electrons in the atom.
- The energy layers (principal quantum number **n**) grow along the **z**-axis of the nucleus in its both directions (more precisely **n** grows with its distance from the origin).
- The perpendicular distance from the **z**-axis in the **x**-**y**-plane reflects the angular momentum (**L**) and so the orbitals.
- The upper half of the ellipsoid is referred to as spin-up and the lower part as spin-down.
- The nucleus possibly rotates around its **z**-axis.

The following drawings describe the idea via cross sections in the **x**-**z**-plane of the nucleus.



***1**: a nucleon in the nucleus*

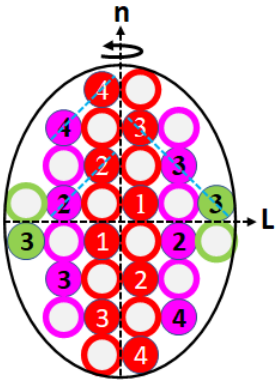
***2**: the bonds between the nucleons*

***3**: the energy levels of the nucleus*

1. One nucleon (**circle**) is observed inside the ellipsoid (dashed line) that encloses the nucleons and schematically defines the nucleus surface:
   - the distance from the origin represents its energy **E**.
   - the distance from the **z**-axis depicts it angular momentum **L**.
   - the nucleons in the upper half have spin up, and in the lower one spin down.
2. The bonds between the nucleons are shown for visibility as springs.
   - **Protons**: full circles of the **s**, **p** and **d** sub-orbitals. **Neutrons**: hollow circles.
3. The circles of equal energy states **n** in the ellipsoid.
   - the lines mark the development of the **s**, **p** and **d** sub-orbitals along the **z**-axis.
   - The **s** line crosses all **n** circles from 1 to 4 (**s1** to **s4**).
   - The **p** line begins by **n**=2 and reaches till **n**=4 (**p2** to **p4**).
   - The **d** line begins by **n**=3 and reaches the ellipsoid border, before it reaches the **n**=4 circle, and therefore there are no **d4** states at this stage (only **d3**).

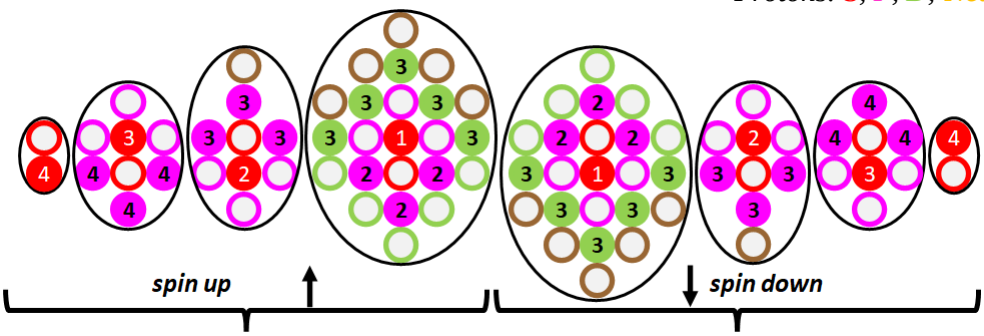
As an example the **Krypton** nucleus Kr_{36}^{84} is shown.

For ***n***=4 there are **s** and **p**, but no **d** nucleons, because the relevant part of the circle is outside its ellipsoid (as explained above).



x-z plane cross section

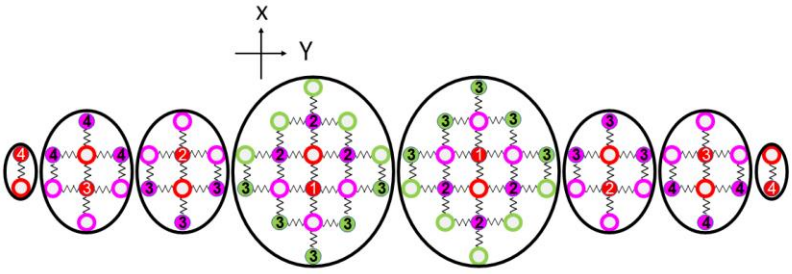
Kr_{36}^{84} in a 3D-illustration
Protons: S, P, D; Neutrons



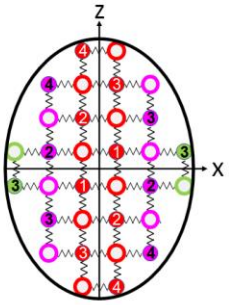
x-y planes cross sections along the z axis

***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**, beyond the number equal to the protons (unpaired neutrons).*

The ellipsoid coordinates are explained via the following drawings:



cross sections in the x-y plane along the z-axis



cross section in the x-z plane

Introduction

The nucleus and the atom are governed by different forces, have a size difference of about 5 orders of magnitude and according to current physics the order of the nucleus in shells is different than that of the atom [10] *(shell model)*.

The hypothesis, that the model of this paper offers, is that the structure of the nucleus determines the one of the atom; therefore an attempt was made to find a geometric model that could describe this and at the same time meet the requirements and constraints of the current theories of nuclear and atomic physics to justify this new perspective without contradictions.

The starting point was that the hypothesis holds and so, in the opposite direction, it is possible to learn and deduce from the atom about the structure of the nucleus.

Once such a model was obtained it was tested and compared with experimental data.

The methods used in this work to analyze the nucleus are essentially those of classical physics.

The steps of the research

In order to develop the model, the following steps were taken:

- Requirements were set regarding:
  - the shape of the nucleus.
  - the geometry of the bonds between the nucleons.
  - the correct number of neutrons for every isotope.
  - the structure and properties of the atom, that shall be reflected by the nucleus.
- A geometric model of the nucleus was designed.
- A theoretical mass formula to the model was created.
- The calculations were compared with the experimental data regarding the mass formula.
- The results were analyzed and discussed.

The requirements

The nucleus shape, the bonds of the nucleons and their consequences

- The structure of the nucleus shall "make sense" physically.
- The nuclear density is assumed to be (at least nearly) constant and the structure of the nuclear bonds is homogeneous and periodic.
- A proton is connected only to neutrons (**p-n** bond) because we assume that the **p-p** bond has a too strong electric repulsion; otherwise we could expect to observe a stable He_2^2 atom for instance.
- A neutron is preferably connected with protons (**p-n** bond) because it is assumed that the proton stabilizes the neutron and that the **n-n** bond alone (with no protons involved) is instable; otherwise we could expect to observe a stable **n-n** nucleus.

Reflection of the properties of the atom by the structure of the nucleus

Based on the model hypothesis that the nucleus influences the atom, we assume that the structure of the nucleus can be deduced from the one of the atom.
We therefore demand that the nucleus will reflect the atom in terms of:

- the structure of the periodic table of elements:
    - the atomic energy levels or shells.
    - the orbitals and sub-orbitals and their population sequence.
- the correct number of neutrons for each isotope.
- the total nucleus spin.
- Pauli's exclusion principle.
- Hund's rules of electronic states population will possibly apply to the protons in a similar way.

Constraints from the experimental knowledge about the nucleus

- A mass formula shall be developed to fit the model and be compared with the experimental data.

Results

The model

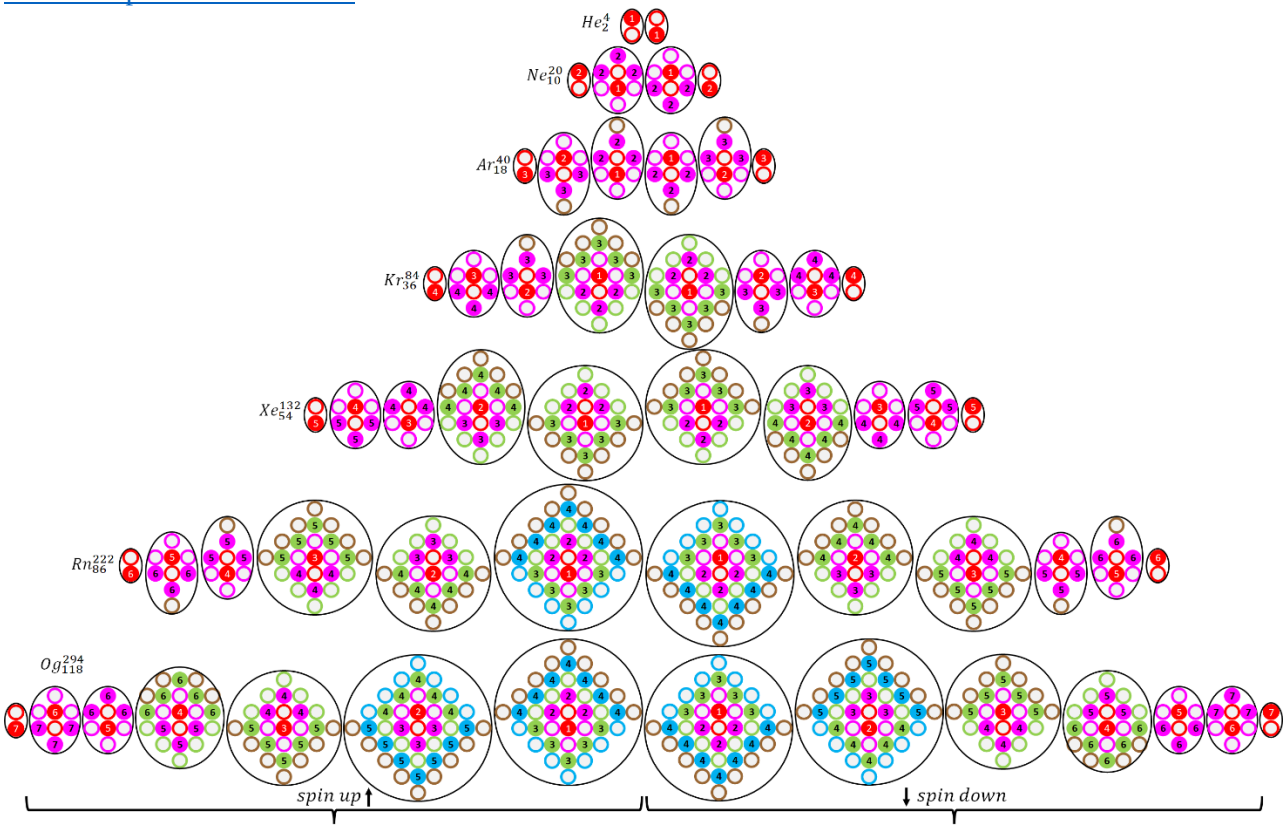
Based on the above requirement, we get the following model, which is developed and explained in detail below:

- The structure of the nucleus
  - The nucleus is in general an ellipsoid.
  - It is composed of nucleons connected to each other in a cubic shape.
  - A proton is connected to neutrons.
  - A neutron is preferably connected with protons.
  - The excess neutrons, beyond the number equal to that of the protons, are in the envelope of the ellipsoid.
- Properties
  - The energy levels grow along the **z**-axis in both directions (more precisely with their distance from the origin). *
  - The perpendicular distance from the **z**-axis (i.e. in the **x**-**y**-plane) depicts the angular momentum (and so the orbitals). *
  - The upper side of the ellipsoid was arbitrarily defined as spin-up and the lower part as spin-down. *
  - The model assumes that the nucleus possibly rotates around its main axis (the **z**-axis).*
- The model shows the following
  - The layers of the nucleus correlate with those of the atom.
  - The structure of the periodic table is justified by the model.
  - At the atomic level the model explains the electron shells, the energy levels, the orbitals and sub-orbitals.
  - The model delivers the right number of protons and neutrons in the nucleus and the correct nucleus spin.
  - The model doesn't contradict Pauli's exclusion principle.
  - Like in the atomic physics the population sequence of the protons is possibly according to Hund's rules in the range where the electronic states follow the L-S coupling.*
- Examining the model
  - The ellipsoid shape makes sense physically.
  - A new mass formula is suggested and tested on experimental data; the relative errors are in the range of several percent and strengthen the model assumption.
  - The mass formula also calculates the combined radii of the proton and neutron in good agreement with the experimental data.

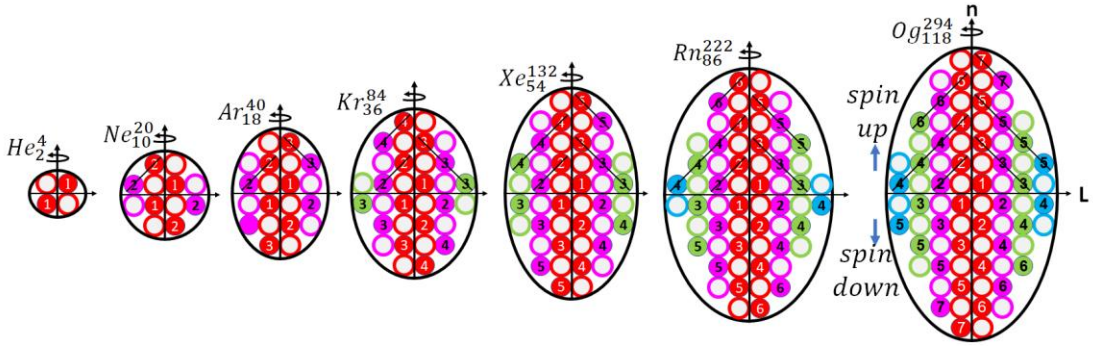
* Topics that are not essential to the first study and do not contradict the model, but help in its development and construction. They shall be developed in following studies in order to expand and establish the model.

The nuclei of the noble gases

The nuclei of the noble gases are shown next. The detailed explanation on how it is being constructed is described in the following sections of the mass formula and in the appendix: the development of the model.



***Nucleus layers**: cross sections in the **x-y** planes along the **z** axis.*
***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**: beyond a numb equal to the protons.*

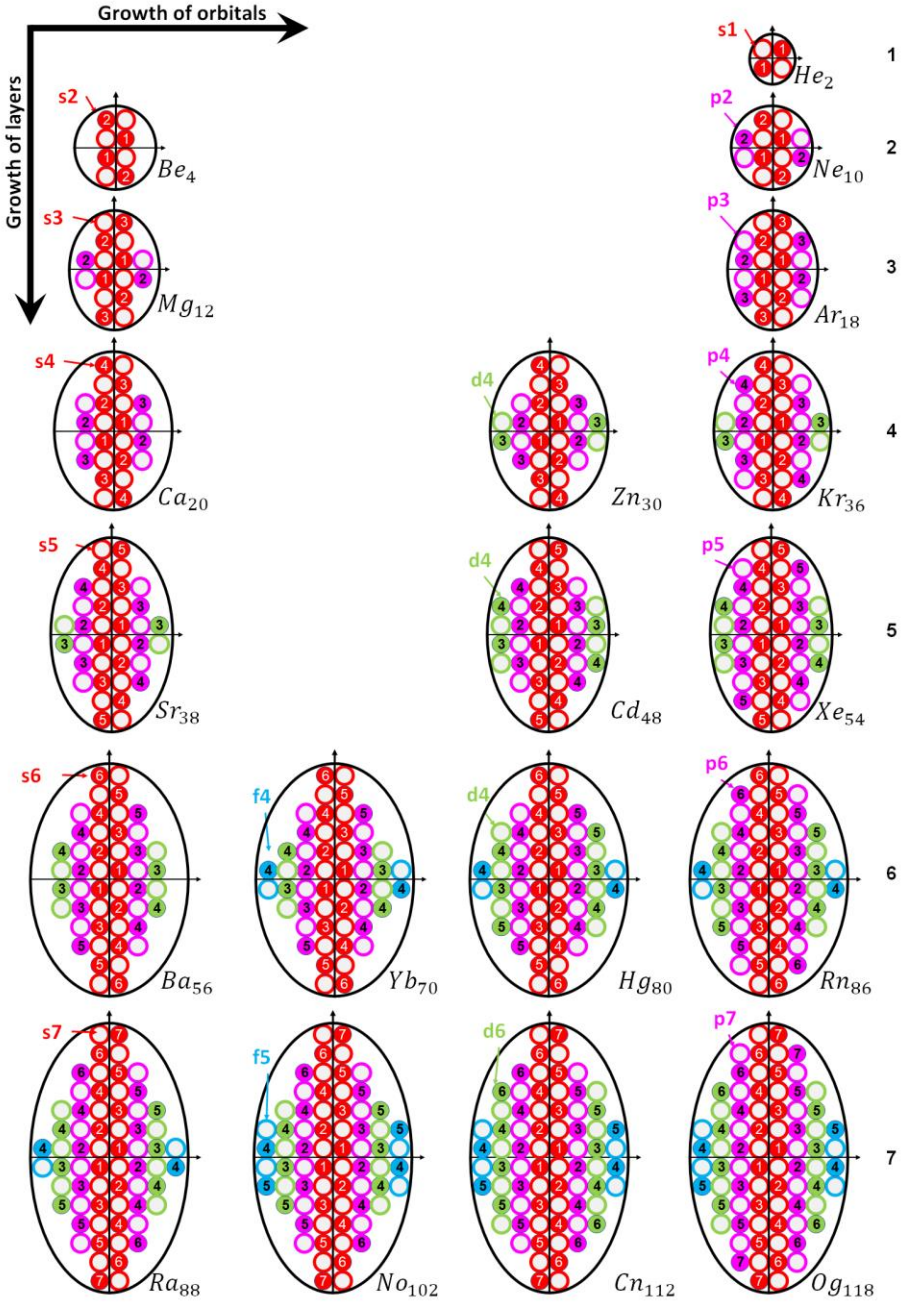


***The layers and energy levels of the nuclei:** a cross section in the **x-z** plane.*
***excess neutrons**, beyond a number equal to the protons, are not shown.*

The ellipsoids of the full sub-orbitals of the periodic table

In order to better understand the model, the ellipsoids of the full sub-orbitals are shown and ordered as they appear in the periodic table. The orbitals grow from left to right and the layers top to bottom; each colored arrow refers to the orbital that was filled last.

Cross sections in the x-z plane of the ellipsoids of the full sub-orbitals



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**, beyond a number equal to the protons, are not shown.*

The Mass formula

A new mass formula was developed to match the model and test its feasibility:
$m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$ experimental data from [1] (*JAEA*).

- m_{calc_x}: the calculated mass of the nucleus x.
- Z_x: the atomic number of the nucleus x (number of protons).
- m_p: the mass of the proton.
- N_x: the number of neutrons in the nucleus x (number of nucleons A_x minus Z_x).
- m_n: the mass of the neutron.
- E_{b_x}: the total energy of the nucleon bonds in the nucleus x.
- E_{c_x}: the total electric energy (between all protons) in the nucleus x.
- c: the speed of light.

The binding energy of the nucleus is:
$E_{b_x} = e_b \cdot n_{b_x}$

- e_b: the energy of a single nucleon-nucleon bond in the nucleus (assuming they are all equal for all bonds in all nuclei).
- n_{b_x}: the number of nucleon-nucleon bonds in the nucleus x.

The electric energy of the nucleus is:
$E_{c_x} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0} \left\{ \frac{1}{2} \sum_i^{Z_x} \sum_{j \neq i}^{Z_x} \frac{1}{d_{i,j}} \right\} = \frac{e^2}{4\pi\epsilon_0} \frac{1}{d_0} e_{c_x}$ where $e_{c_x} := \frac{1}{2} \sum_i^{Z_x} \sum_{j \neq i}^{Z_x} \frac{1}{d_{i,j}}$

- d_0: the minimum distance between two neighboring nucleons in femtometer (assuming all nuclei have the same cubic structure and distance between their nucleons).
- $d_{i,j}$: the unitless distance between the protons of the indices *i* and *j* measured in multiples of d_0: $d_{i,j} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$
- e_{c_x}: the unitless total electric energy of the nucleus (sum of the reciprocal distances).

The absolute relative error of the calculation for the nucleus is:
$rel\_err_x = \left| \frac{m_{calc_x} - m_{meas_x}}{Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}} \right| = \left| \frac{m_{calc_x} - m_{meas_x}}{mass\_defect_x} \right|$

- m_{meas_x}: the measured mass of the nucleus x.
- rel_err_x is represented here in percentage.
- $mass\_defect_x$: $Z_x \cdot m_p + N_x \cdot m_n - m_{meas_x}$ is the mass defect of the nucleus x.

The mass formula, in this simplified form, depends thus only on the two variables:

- e_b: the energy of a single nucleon-nucleon bond.
- d_0: the minimum distance between two neighboring nucleons.

The implementation requires two preparation calculation steps for all nuclei:

- Drawing the nucleus and counting the number of nucleon-nucleon bonds n_{b_x}.
- Calculating the relative total energy of the nucleus e_{c_x} (sum of reciprocal distances).

The implementation of the calculation

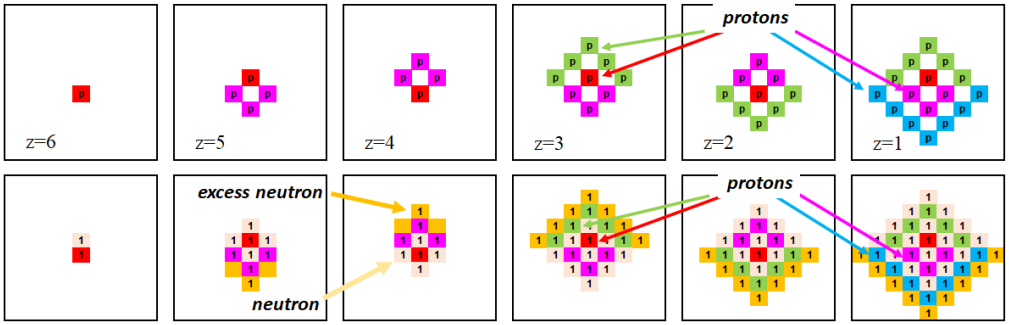
This section explains how the number of nucleon-nucleon bonds n_{b_x} and the relative total energy of the nucleus e_{c_x} were calculated.

Drawing the nucleus and counting the number of nucleon bonds n_{b_x}

The counting of the number of nucleon-nucleon bonds n_{b_x} of each nucleus was implemented by drawing all the nuclei in Excel sheets that automatically run this counting.
The sequence of the process was the following:

- First the Oganesson nucleus Og_{118}^{294} was built, because this is the nucleus that closes the noble gases, meaning it has closed orbital and as such is more probable to draw it correctly according to the model.
- The nuclei with closed sub orbitals were derived from Oganesson:
  - Og_{118}^{294} was copied and the number of protons and neutrons was adjusted to create the next full sub-orbital nucleus below it, which is Copernicium Cn_{112}^{282}.
  - In a similar manner Cn_{112}^{282} was copied and Nobelium No_{102}^{256} was drawn.
  - This process went on to create all the nuclei with closed sub orbitals (S, P, D, F).
- Beginning with every nucleus that closes a sub-orbital, the nuclei below it were built in a similar process to the above till the nucleus above the next closed sub-orbital was reached:
  - Oganesson was copied to the next below it, which is Tennessine.
  - Then Oganesson was copied to build Livermorium and so on till Nihonium was reached.
  - Then a similar process was done by using the Copernicium Cn_{112}^{282} nucleus for all nuclei below it and above Nobelium No_{102}^{256}.
  - The process went on till every nucleus with a closed sub-orbital created all the nuclei in its sub-orbital.

During this process the structure of the nuclei was analyzed and studied and ideas were developed on how to assess what configuration is more probable.
Next drawing shows the Excel sheet of Radon Ra_{86}^{222} (only the left half, spin-up, is shown due to its large size).



Legend:

- *upper raw: protons of the orbitals **S**, **P**, **D**, **F**. (only if marked with **p**).*
- **z***: the position on the* z*-axis (the layer from the center of the nucleus)*
- lower raw: nucleons *(only if marked with **1**).*
  - ***Protons** of the orbitals **S**, **P**, **D**, **F**.*
  - ***Neutrons** cream. **excess neutrons**: orange.*

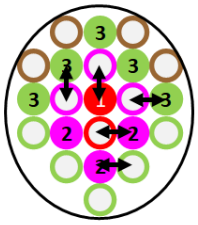
Calculating the relative total energy of the nucleus e_{c_x}

We first build an Excel sheet that calculates the sum of all proton-proton relative energies for Oganesson (the reciprocal distances):

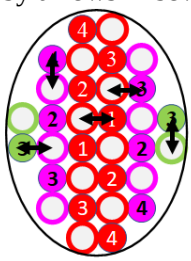
$e_{c_x} = \frac{1}{2}\sum_i^{Z_x} \sum_{j\neq i}^{Z_x} \frac{1}{d_{i,j}}$

then in a somewhat similar manner to the process of creating the Excel sheets for the counting of n_{b_x}, as described in the section above, the closed sub-orbitals are created and each of them creates the nuclei in its sub-orbital below it.

Following drawings show the minimum distance between two neighboring nucleons d_0 through cross sections of the nucleus (marked by arrows in several positions):

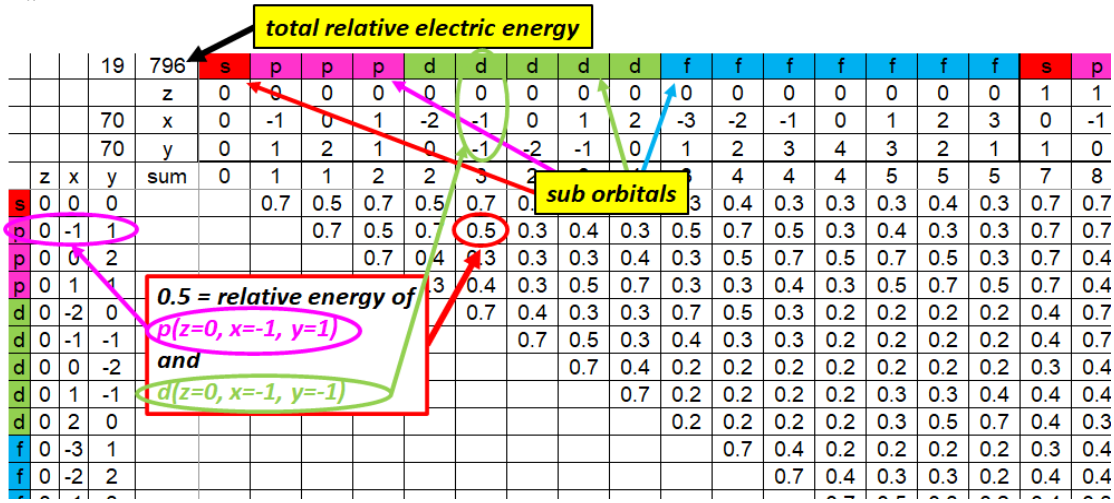


x-y plane



x-z plane

The following picture from the Excel sheet of Yb_{70} explains how the relative electric energy e_{c_x} is calculated (the reciprocal distances):



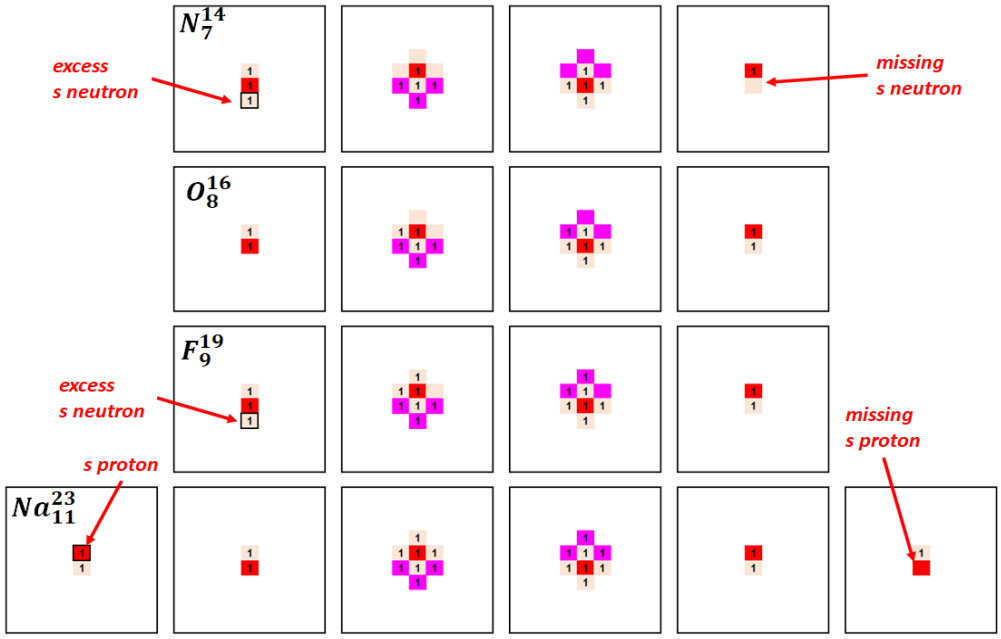
			19	796	s	p	p	p	d	d	d	d	d	f	f	f	f	f	f	f	s	p
				z	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
			70	x	0	-1	0	1	-2	-1	0	1	2	-3	-2	-1	0	1	2	3	0	-1
			70	y	0	1	2	1	0	-1	-2	-1	0	1	2	3	4	3	2	1	1	0
	z	x	y	sum	0	1	1	2	2	3	2				4	4	4	5	5	5	7	8
s	0	0	0			0.7	0.5	0.7	0.5	0.7	0.5			0.3	0.4	0.3	0.3	0.3	0.4	0.3	0.7	0.7
p	0	-1	1				0.7	0.5	0.7	0.5	0.3	0.4	0.3	0.5	0.7	0.5	0.3	0.4	0.3	0.3	0.7	0.7
p	0	0	2					0.7	0.4	0.3	0.3	0.3	0.4	0.3	0.5	0.7	0.5	0.7	0.5	0.3	0.7	0.4
p	0	1	1						0.4	0.4	0.3	0.5	0.7	0.3	0.3	0.4	0.3	0.5	0.7	0.5	0.7	0.4
d	0	-2	0							0.7	0.4	0.3	0.3	0.7	0.5	0.3	0.2	0.2	0.2	0.2	0.4	0.7
d	0	-1	-1								0.7	0.5	0.3	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.4	0.7
d	0	0	-2									0.7	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.4
d	0	1	-1										0.7	0.2	0.2	0.2	0.2	0.3	0.3	0.4	0.4	0.4
d	0	2	0											0.2	0.2	0.2	0.2	0.3	0.5	0.7	0.4	0.3
f	0	-3	1												0.7	0.4	0.2	0.2	0.2	0.2	0.3	0.4
f	0	-2	2													0.7	0.4	0.3	0.3	0.2	0.4	0.4

The coordinates of each proton appear once above and once from left; for each pair of (different) protons the relative electric energy $\frac{1}{d_{i,j}} = \frac{1}{\sqrt{(x_j-x_i)^2+(y_j-y_i)^2+(z_j-z_i)^2}}$ (their reciprocal distance) is calculated at the point of their intersection.

The sheet of every nucleus begins with the sheet of the full sub-orbital above it (the one that closes its sub-orbital sequence) and the excess protons are removed according to its scheme, as it was found or set in the section above (counting the number of nucleon bonds n_{b_x}).

The total spin

For the calculation of the total spin we assume maximum symmetry of the nucleus. Yet at least for small nuclei till about Ne_{10} also Hund's rules must be considered [9] (*Haken, Wolf; Atom- und Quantenphysik*).
For nuclei with even number of protons and neutrons there is no problem, because they are completely symmetric and their total spin is zero.
For other nuclei there is basically always a way to set the neutrons so, that the required spin is reached without harming the other properties and requirements of the nucleus.
Also the spin is calculated via Excel sheets. Some examples are shown below:



- $\boldsymbol{N_7^{14}}$: the nucleus is drawn with symmetry and under consideration of Hund's rule, so that all p protons are spin-up. The total spin is $\frac{1}{2}$ due to the excess s neutron on the left side and the missing one on the right, possibly due to the larger number of protons on the left.
- $\boldsymbol{O_8^{16}}$: the nucleus is drawn with symmetry and under consideration of Hund's rule. The total spin is zero.
- $\boldsymbol{F_9^{19}}$: the nucleus is drawn with symmetry and under consideration of Hund's rule. The total spin is $\frac{1}{2}$.
- $\boldsymbol{Na_{11}^{23}}$: the nucleus has one extra s proton and so a total spin $\frac{1}{2}$.

This topic was not investigated in depth in this paper and shall be further investigated, in the future work on the model. *Spin data* [1] *(JAEA)*.

Executing the mass formula calculation; analyzing and learning the model

The exact structure of the nucleus is crucial for the study of the model and for the correct execution of the mass formula. This structure determines:

- n_{b_x}: the number of bonds in the nucleus.
- e_x: the total electric energy of the nucleus.

The electric energy, e_x, is less sensible towards small changes of the nucleus structure, but for n_{b_x} small variations result in a large impact (of one percent or more) on the relative error of the mass formula calculation, so while running the tests the "correct" value of n_{b_x} for each nucleus was sought. "Correct" means logical in the physical sense and compared with other nuclei, so this isn't something that can be proved yet by this paper.
The process of filling up the nucleus with nucleon runs as follows:

- The following nuclei are selected to begin with:
    - nuclei of filled-up sub-orbitals (S, P, D, F) because their real shape is assumed to be more probable to be created correctly:
      Be_4, Ne_{10}, Mg_{12}, Ar_{18}, Ca_{20}, Zn_{30}, Kr_{36}, Sr_{38}, Cd_{48}, Xe_{54}, Ba_{56}, Yb_{70}, Hg_{80}, Rn_{86}, Ra_{88} (Hydrogen and Helium are not discussed in this paper due to their small number of bonds, that seem to deviate from the cubic structure; a separate future research shall deal with them).
- The nuclei are filled up with:
    - the protons: there is only one possible configuration.
    - the paired neutrons: also here there is only one possible configuration.
    - the excess neutrons are set in a way that reaches maximum symmetry and compared between various nuclei to assess the probable configuration.
- The mass formula is calculated and the best parameters are found.
- The nuclei that show larger deviation of their relative errors are checked again and their structure is being studied and possibly changed via comparison with other nuclei.
- At a second step all other nuclei are being created and the process repeats itself.

After creating all nuclei, trials are run in iterations with the construction of the nuclei.
This process is delicate, because we shall avoid the tendency to adapt the structure so that better results are achieved.
The results are not necessarily accurate as we cannot be sure of the correct structure of the nuclei, but a rough estimate seems possible.

Results of the mass formula calculations

The following tables show the calculations of the relative error of the mass formula (in percent) depending on the binding energy, e_b, and the distance between two neighboring nucleons, d_0, for 120 nuclei of the common isotopes of the elements from Li_3 to Pu_{94} (for several elements more than one isotope was taken).
The lighter nuclei till approximately Ar_{18} have larger relative errors than those of larger nuclei and are therefore shown in a different table.
The results of the mass formula calculation for 94 nuclei from Ar_{18}^{40} to Pu_{94}^{244} (detailed data in the appendix: Mass formula calculation: data):

maximum	average	st. dev.	≤ 2% *	≤ 1%	≤ 0.5%
1.9%	0.6%	0.5%	100%	82%	60%

* the amount of nuclei with relative error smaller than or equal to 2%.

- e_b = 5.72±0.03 Mev
- d_0 = 1.62±0.03 fm

these values are within a reasonable range [5] *(Local phenomenological nucleon–nucleon potentials)*.

If we consider the nucleons for simplicity as rigid bodies, then we get a rough estimation for d_0 through the radii of the proton and neutron: r_n [3] *(Neutron radius)*, r_p [4] *(Proton radius)*: $d_0 \approx (r_n + r_p)$.
Setting these values we get a result within a reasonable range:

- $r_n \approx 0.80$ fm, $r_p \approx 0.84$ fm, $d_0 \approx (r_n + r_p)$ =1.64 fm
- *relative deviation for* d_0: $\left|\frac{d_0 - (r_n + r_p)}{(r_n + r_p)}\right| = \left|\frac{1.62 - 1.64}{1.64}\right| \approx 1.3\%$

This estimation could strengthen the hypothesis of the model.

Following table shows the results of the mass formula calculation for 28 nuclei from Li_3^6 to Ar_{18}^{36} at e_b = 5.72MeV, d_0 = 1.62 fm (as found for the nuclei from Ar_{18}^{40}):

maximum	average	st. dev.	≤ 3%	≤ 2%	≤ 1%
7.1%	2.4%	1.9%	79%	54%	29%

The larger relative error of these lighter nuclei shall be analyzed in future research. The reasons may be a nuclear density that is not constant or a deviation from the cubic arrangement of the bond, but there may also be other reasons.

Discussion of the results and conclusion

This study offers a new direction and covers only a small part of it. Of course, further work is required to further develop, adapt and prove it.

What the model tries to explain or solve:

- The model suggests a tangible geometric shape for the nucleus and connects between the structure of the nucleus and that of the atom; it was demonstrated mainly for the nuclei of the noble gases and for the nuclei with closed sub-orbitals, but was tested on most of the elements.
- The model reflects the structure of the periodic table of the elements in terms of the shells, the protons and neutrons for each isotope, and qualitatively for the energy levels and orbitals. It was built that way from the beginning, but here it was shown to be possible.
- A theoretical mass formula was developed. It has the advantage that it relates directly to the suggested theory of the nucleus, as offered by this paper, rather than being semi-empirical like the current mass formula. [11] *(mass formula)*
- The distance d_0 between two neighboring nucleons agrees very well with the sum of the neutron and proton radii; this strengthens the model assumption and the concept of the mass formula.
- The excess neutrons in the nucleus are assumed to lay in the envelope of the nucleus, because otherwise, under the model assumption of a direct influence of the nucleus structure on the atom, a change of the chemical properties of the atom would be expected for different isotopes of the same element, due to the different positions of the protons while the number of neutrons changes. This was not developed quantitively or proved yet by the model.

Constraints that were considered during the development of the model, but not yet fully developed:

- The total spin of the nucleus depends only on the unpaired protons and neutrons and the symmetry of the nucleus. the model doesn't contradict this and delivers the correct total spin.
- Pauli's exclusion principle holds (especially if we assume a rotation of the ellipsoid). The reason is that position of each nucleon in the ellipsoid is unique with respect to the other nucleons if considering the whole configuration.
- The arrangement of the nucleus according to Hund does not contradict the symmetry requirement, nor does it change the number of bonds in the nucleus.

Future research shall expand the model and deal with the nuclei of:

- Hydrogen and Helium that were not included in the calculations.
- the light nuclei, below Ar_{18}^{40} that showed larger relative errors.

Sources and references

1. *Tables of Nuclear Data: Japan Atomic Energy Agency (JAEA)*
2. *Charge Radius: International Atomic Energy Agency (IAEA)*
3. *Neutron radius: Povh, B.; Rith, K.; Scholz, C.; Zetsche, F. (2002). Particles and Nuclei: An Introduction to the Physical Concepts. Berlin: Springer-Verlag. p. 73*
4. *Proton radius: Yong-Hui Lin, Hans-Werner Hammer and Ulf-G. Meißner: New insights into the nucleon's electromagnetic structure; Physical Review Letters, https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.128.052002*
5. *Reid, R. V. (1968). "Local phenomenological nucleon–nucleon potentials". Annals of Physics. **50** (3)*
6. *P. Roy Chowdhury; C. Samanta; D. N. Basu (January 26, 2006). "α decay half-lives of new superheavy elements". Physical Review C. **73** (1): 014612*
7. *C. Samanta; P. Roy Chowdhury; D. N. Basu (April 6, 2007). "Predictions of alpha decay half lives of heavy and superheavy elements". Nuclear Physics A. **789** (1–4): 142–154*
8. *G. Royer; K. Zbiri; C. Bonilla (2004). "Entrance channels and alpha decay half-lives of the heaviest elements". Nuclear Physics A. **730** (3–4): 355–376*
9. *Hermann Haken, Hans Christoph Wolf: Atom- und Quantenphysik – Einführung in die experimentellen und theoretischen Grundlagen. 5. Auflage. Springer, Berlin 1993, S. 329*
10. *LibreTexts physics: Nuclear Shell Model*
11. *LibreTexts physics: Binding energy and Semi-empirical mass formula*

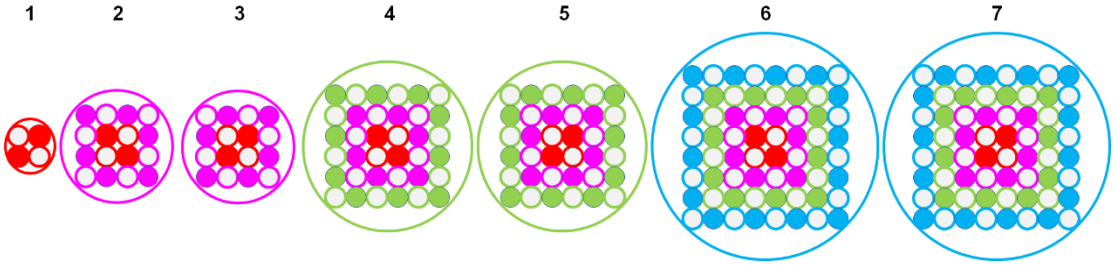
Appendix

The development of the model

The model development is described in two different approaches:

1. a visual way, trying to explain the logic behind its construction.
2. a physical approach.

constructing the nucleus - a visual approach
First we choose a visual way to clarify the idea, although this is obviously not the way the nucleus was formed, but it makes the idea more understandable.
We build the nucleus so, that the protons match the number of electrons in each layer of the periodic table: **2**, **8**, **8**, **18**, **18**, **32**, **32**.
Based on the assumption that there are proton-neutron pairs the layers consist of the following number of nucleons: **4**, **16**, **16**, **36**, **36**, **64**, **64**, or as square powers: $\mathbf{2^2}$, $\mathbf{4^2}$, $\mathbf{4^2}$, $\mathbf{6^2}$, $\mathbf{6^2}$, $\mathbf{8^2}$, $\mathbf{8^2}$.
We receive square layers in a cubic structure that are located above one another to form a square pyramid shape as shown in the following drawing:



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*
***neutrons**: hollow circles with colors according to their orbital.*

or as a table (**p**-protons, **n**-neurons, **A**=**p**+**n**: nucleons):

layer	p	n	A=p+n	p total	A total	S	P	D	F
1	2	2	4	2	4	2			
2	8	8	16	2+8=10	16+4=20	2	6		
3	8	8	16	10+8=18	20+16=36	2	6		
4	18	18	36	18+18=36	36+35=72	2	6	10	
5	18	18	36	36+18=54	72+36=108	2	6	10	
6	32	32	64	54+32=86	108+64=172	2	6	10	14
7	32	32	64	86+32=118	172+64=236	2	6	10	14

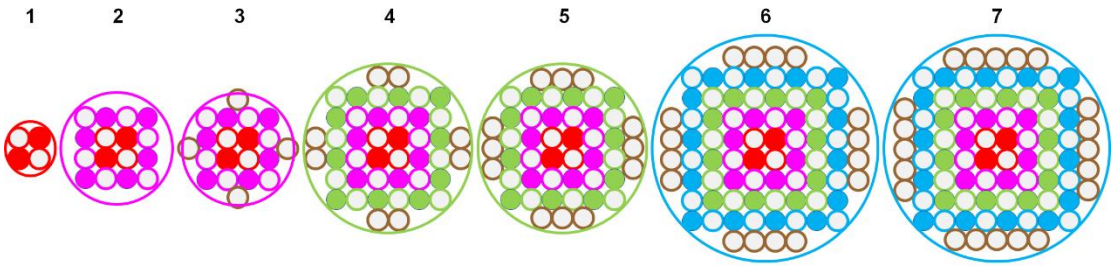
The interim results received for the noble gases are:
He_2^4, Ne_{10}^{20}, Ar_{18}^{36}, Kr_{36}^{72}, Xe_{54}^{108}, Rn_{86}^{172}, Og_{118}^{236}.

- The number of nucleons **A** (atomic mass) is correct for Helium and Neon.
- Argon has a stable isotope with 36 nucleons, but its most common isotope consists of 40 nucleons. [1] (*Tables of Nuclear Data, JAEA*).
- For nuclei larger than Argon the number of nucleons is larger than what we found.

Next assumption is that as the nucleus grows, excess neutrons are required to stabilize it. The model shall thus be expanded to explain the number of excess neutrons at each layer.

Completing the nucleus layers

In a next step we assume that in order to stabilize the nucleus, excess neutrons are added in a way that tries to complete its layers from a square form to a more round one.
We ask how many neutrons shall be added to each layer in a cubic structure so that the surface of its surrounding circle is maximal used, and, in addition, we demand that each layer has more neutron than its predecessor under the assumption that this increases the nucleus stability due to a wider base. We receive the following pyramid shape:



***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**, **F**.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**: beyond the number of the protons.*

The number of excess neutrons for each layer is **0**, **0**, **4**, **8**, **12**, **16**, **20** respectively, meaning a total of **4**, **16**, **20**, **44**, **48**, **80**, **84** nucleons at each layer.

layer	p	n=p+add.	A=p+n	p tot.	A total	S	P	D	F
1	2	2	2+2=4	2	4	2			
2	8	8+**0**=8	8+8=16	10	4+16=20	2	6		
3	8	8+**4**=12	8+12=20	18	20+20=40	2	6		
4	18	18+**8**=26	18+26=44	36	40+44=84	2	6	10	
5	18	18+**12**=34	18+34=48	54	84+48=132	2	6	10	
6	32	32+**16**=48	32+48=80	86	132+80=212	2	6	10	14
7	32	32+**20**=52	32+52=84	118	212+84=296	2	6	10	14

The results received for the noble gases are:

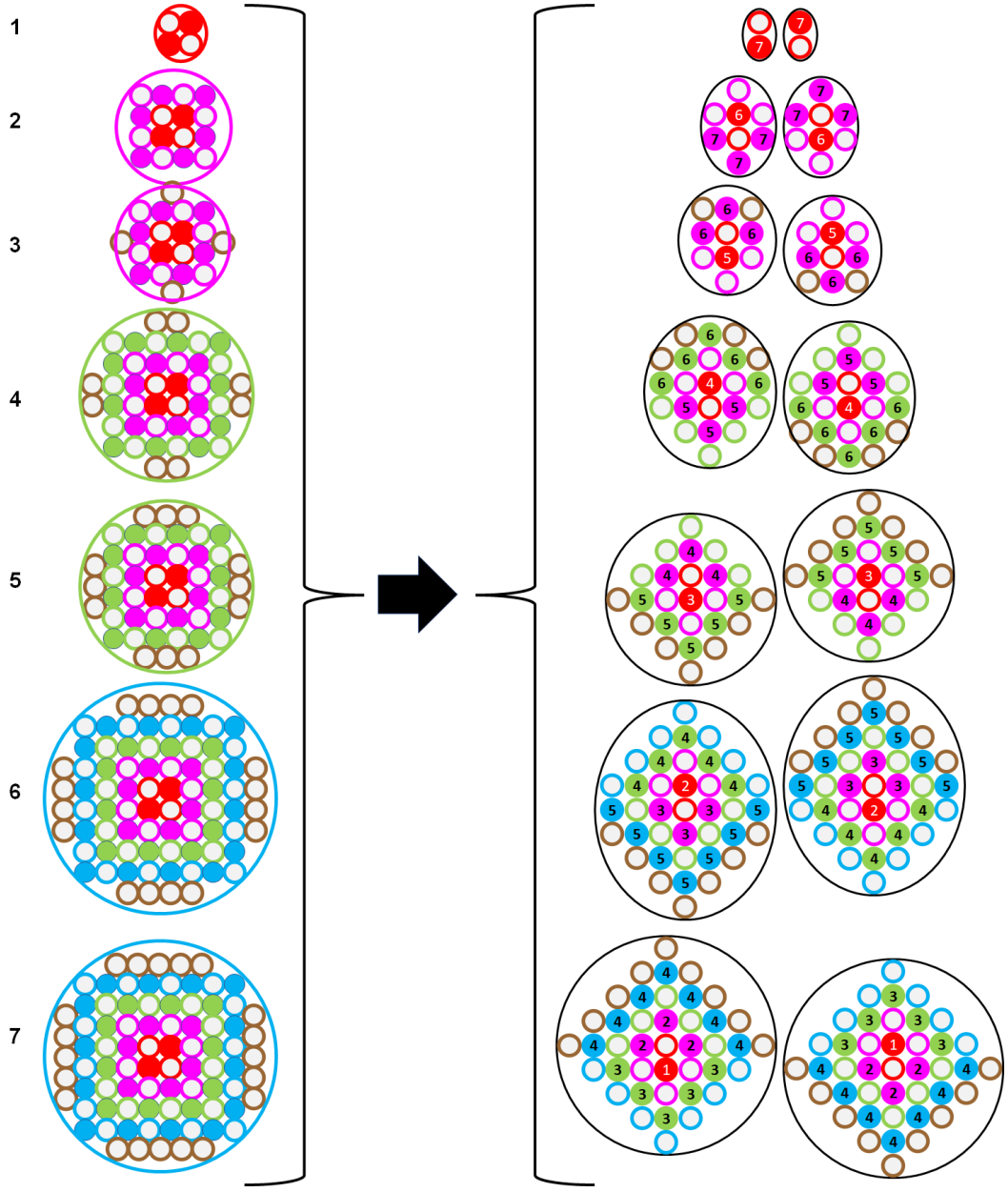
He_2^4, Ne_{10}^{20}, Ar_{18}^{40}, Kr_{36}^{84}, Xe_{54}^{132}, Rn_{86}^{212}, Og_{118}^{296}.

for the stable nuclei up to Xenon these results are correct [1] *(Tables of Nuclear Data, JAEA)*. Radon and Oganesson are radioactive and are large. For Radon Rn_{86}^{212} is an isotope with a half time in the range of minutes [1] *(Radon, Tables of Nuclear Data, JAEA)*. About Oganesson there is not much knowledge, but Og_{118}^{296} is expected to be one of its isotopes [6, 7, 8] ***(superheavy elements)***.

Remark: these results do not necessarily mean that the model is correct, yet it could strengthen the assumptions of the model and make the idea visual.

Splitting the layers to form an ellipsoid

The pyramid form is not symmetrical with respect to the nucleus center and does not fit to the energy levels nor to the liquid drop model.
Therefore, we split each layer in two, one at the positive side of the **z**-axis and the other at its negative side. We receive layers of **2**, **8**, **10**, **22**, **24**, **40**, **42** nucleons:

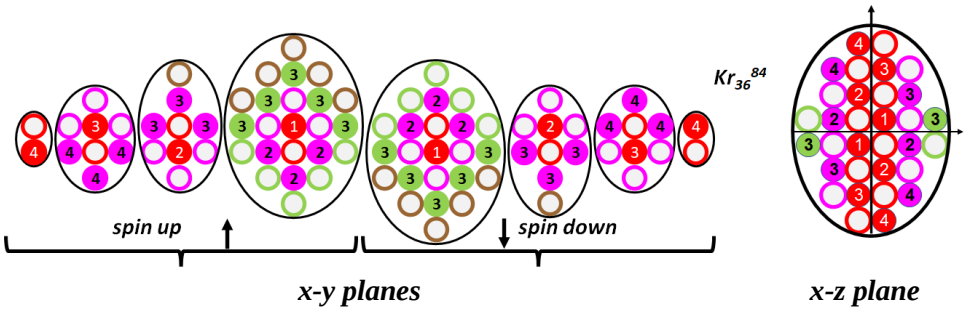


This is the shape of the nucleus, that the model suggests.

Example - the Krypton nucleus

The resulted nucleus is shown next for the Krypton. On the left the x-y cross sections along its z-axis are shown and on the right its x-z cross section.
The upper half of the ellipsoid is referred to as spin-up and the lower as spin down.

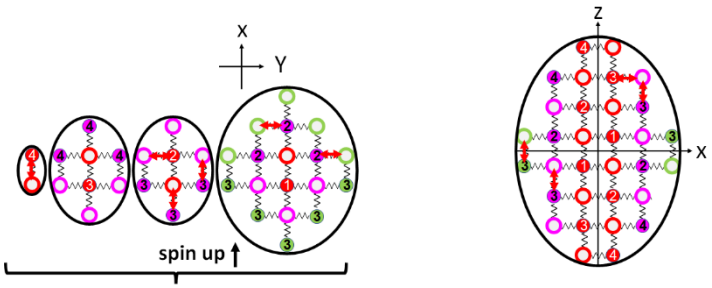
Kr_{36}^{84} cross sections



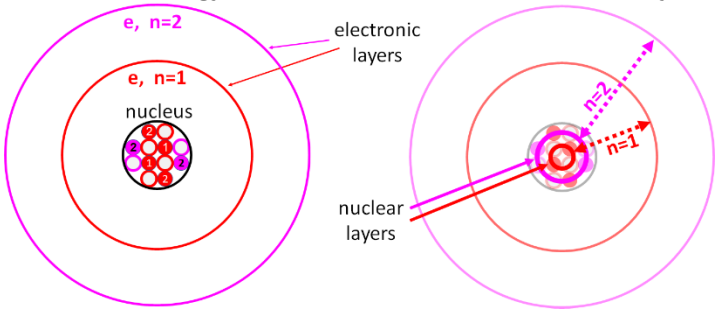
***Legend: protons**: full circles according to the orbitals **S**, **P**, **D**. **Numbers**: energy levels.*
***neutrons**: hollow circles with colors according to their orbital.*
***excess neutrons**, beyond the number equal to the protons.*

Below drawings show:

- the connections between the nucleons as springs (just to simulate a connection, not for calculation)
- the minimum distance d_0 between two neighboring nucleons in the cubic structure (marked by red arrows).
- the coordinates of the x-y sections and of the x-z section.



The model of the nucleus with the electronic layers is schematically shown for Helium. We assume an analogy between the nucleus structure and layers and those of the atom:



The correlation of the nucleus and atom layers is shown via dashed arrows.

constructing the nucleus - a physical approach

We begin with Hydrogen, then Helium and then the other noble gases, because their shells are complete, and so involve less complications than other nuclei because:

- the nucleus is symmetric.
- the total spin is zero.
- the exact location of each proton is unambiguous.

The Hydrogen H_1^2 nucleus looks so:

***x-y** plane*

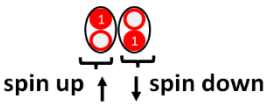
Its proton is in the **s**-1 state.

We know that its total spin is 1, so the spins of the proton and neutron are in the same direction.

Next is Helium He_2^4. The requirements are:

- both protons are in the **s**-1 state.
- the total spin is zero.
- the nucleons build a cubic bonding structure.
- the bonds are **p-n**.

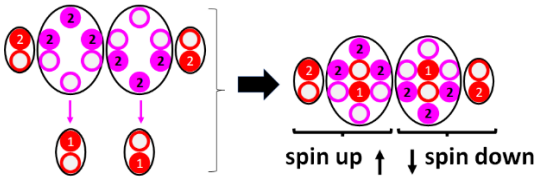
We get:



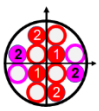
***x-y** planes*

***x-z** plane*

The next noble gas is Neon Ne_{10}^{20}, where the **s**-2 and **p**-2 states are added to the Helium:

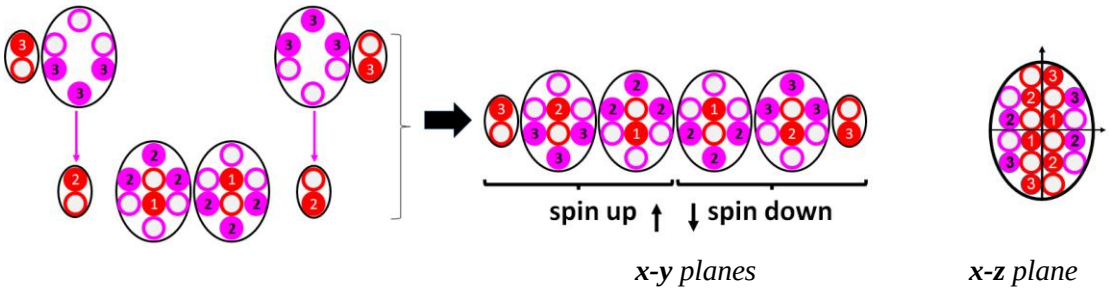


***x-y** planes*



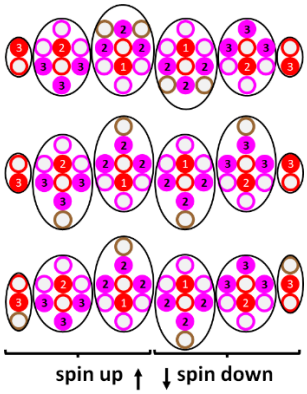
***x-z** plane*

Similarly by Argon the **s**-3 and **p**-3 states are added to the Neon resulted in Ar_{18}^{36}:



x-y planes *x-z plane*

The common isotope of Argon is the Ar_{18}^{40}, so four neutrons shall be added to the model.
The question is where exactly they are located.
As seen in the sections of the development of the model above it is easier and more elegant to assume that the excess neutrons are located in the envelope of the nucleus.
As a next step we observe the Ar_{18}^{40} that has the several possible configurations:



all three examples above are of Ar_{18}^{40} with four excess neutrons located in its envelope.
From the location of each nucleon we conclude and calculate the number of its bonds, so the main question is how many bonds each excess neutron has.
We assume first that the first or second drawing is the correct one, and assess it later through an iterative process of mass formula calculation and comparison with other nuclei.
The process goes on this way for all the other noble gas nuclei.
The excess neutrons (the unpaired ones, beyond the number equal to the protons) are located in the envelope, yet their exact location is not always clear by the model, but can still be predicted within a certain variation.

Mass formula calculation: data

This section shows the data according to which the mass of the nuclei was calculated via the mass formula.
The parameters with which the mass formula are shown here are those of the best results achieved while running the calculation:

- e_b = 5.72 Mev
- d_0 = 1.62 fm

The legend of the table below is given in the order of the columns from left to right:

- *nuc*: the nucleus (name)
- Z_x: atomic number of the nucleus of the element x. the number of protons.
- A_x: mass number of the nucleus of the element x. The number of nucleons.
- $N_x := A_x - Z_x$: the number of neutrons of the nucleus of the element x.
- n_k: the number of nucleon-nucleon bonds in the nucleus as it was calculated by the relevant Excel file. *
- e_c: total relative electric energy of the nucleus as it was calculated by the relevant Excel file. *
- *meas. [amu]*: measured mass of the nucleus in [amu].
- *Base:* $Z_x \cdot m_p + N_x \cdot m_n$ *[amu]*: base mass in [amu]:

  number of protons · proton mass + number of neutrons · neutron mass

- *calc. m [amu]*: calculated mass according to the mass formula in [amu]
  $m_{calc_x} = Z_x \cdot m_p + N_x \cdot m_n - \frac{(E_{b_x} - E_{c_x})}{c^2}$.
- Δ_1*: [amu] calc. - meas.*: calculated mass - measured mass in [amu]
- Δ_2*: [amu] base - meas.*: base mass - measured mass in [amu]
- *rel. err.* Δ_1*:* Δ_2: the relative error in percent:
  $rel.err. = \frac{calculated\ mass - measured\ mass}{base\ mass - measured\ mass}$

* see section: The mass formula

The results of the mass formula calculation for 94 nuclei from Ar_{18}^{40} to Pu_{94}^{244}:

e_b [MeV]		5.730	5.726	5.720	5.716	5.713
d_0 [fm]		1.610	1.615	1.620	1.625	1.630
relative error	max	2.0%	1.9%	1.9%	1.9%	1.9%
	mean	0.6%	0.6%	0.6%	0.6%	0.6%
	stdev	0.5%	0.5%	0.5%	0.5%	0.5%
	≤ 2% **	100%	100%	100%	100%	100%
	≤ 1%	81%	81%	82%	82%	82%
	≤ 0.5%	59%	60%	60%	59%	55%

** the amount of nuclei with relative error smaller than or equal to 2%.

nuc	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. err. Δ_1:Δ_2	base: $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: [amu] calc. - meas.	Δ_2: [amu] base - meas.
Li	3	6	6	1.9	6.0151	6.0128	7.0%	6.0478	-0.0023	0.0327
Li	3	7	7	1.9	7.0160	7.0154	1.6%	7.0565	-0.0006	0.0405
Be	4	9	11	3.4	9.0122	9.0082	6.6%	9.0724	-0.0040	0.0602
B	5	10	12	5.7	10.0129	10.0115	2.1%	10.0797	-0.0014	0.0668
B	5	11	14	5.7	11.0093	11.0079	1.8%	11.0884	-0.0014	0.0791
C	6	12	16	9.0	12.0000	12.0061	6.3%	12.0956	0.0061	0.0956
C	6	13	18	9.0	13.0034	13.0024	0.9%	13.1043	-0.0009	0.1010
N	7	14	19	12.7	14.0031	14.0071	3.7%	14.1116	0.0040	0.1085
N	7	15	21	12.7	15.0001	15.0035	2.8%	15.1203	0.0034	0.1201
O	8	16	24	16.7	15.9949	15.9962	1.0%	16.1275	0.0013	0.1326
O	8	17	25	16.7	16.9991	16.9987	0.3%	17.1362	-0.0004	0.1371
O	8	18	26	16.7	17.9992	18.0013	1.4%	18.1449	0.0021	0.1457
F	9	19	29	20.8	18.9984	18.9940	2.8%	19.1521	-0.0044	0.1537
Ne	10	20	32	26.1	19.9924	19.9879	2.7%	20.1594	-0.0045	0.1670
Ne	10	21	33	26.1	20.9938	20.9904	2.0%	21.1681	-0.0034	0.1742
Ne	10	22	34	26.1	21.9914	21.9930	0.9%	22.1767	0.0016	0.1854
Na	11	23	36	30.0	22.9898	22.9917	1.0%	23.1840	0.0019	0.1943
Mg	12	24	38	34.1	23.9850	23.9906	2.7%	24.1913	0.0055	0.2063
Mg	12	25	40	34.1	24.9858	24.9870	0.5%	25.2000	0.0011	0.2141
Mg	12	26	42	34.1	25.9826	25.9834	0.3%	26.2086	0.0008	0.2260
Al	13	27	45	39.3	26.9815	26.9772	1.8%	27.2159	-0.0043	0.2344
Si	14	28	46	44.8	27.9769	27.9836	2.7%	28.2232	0.0066	0.2463
Si	14	29	48	44.8	28.9765	28.9800	1.4%	29.2318	0.0035	0.2554
Si	14	30	50	44.8	29.9738	29.9764	1.0%	30.2405	0.0026	0.2667
P	15	31	52	53.0	30.9738	30.9791	2.0%	31.2478	0.0053	0.2740
S	16	32	56	59.7	31.9721	31.9682	1.4%	32.2551	-0.0038	0.2830
Cl	17	35	63	66.4	34.9689	34.9563	4.0%	35.2797	-0.0126	0.3108

nuc	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. err. Δ_1:Δ_2	base: $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: [amu] calc. - meas.	Δ_2: [amu] base - meas.
Ar	18	36	66	74.4	35.9675	35.9527	4.7%	36.2869	-0.0149	0.3194
Ar	18	40	70	74.4	39.9624	39.9628	0.1%	40.3216	0.0004	0.3592
K	19	39	70	79.9	38.9637	38.9580	1.6%	39.3116	-0.0057	0.3478
Ca	20	40	72	85.6	39.9626	39.9584	1.2%	40.3188	-0.0042	0.3562
Ca	20	42	74	85.6	41.9586	41.9635	1.3%	42.3362	0.0048	0.3775
Ca	20	44	78	85.6	43.9555	43.9563	0.2%	44.3535	0.0008	0.3980
Sc	21	45	81	92.8	44.9559	44.9520	1.0%	45.3608	-0.0039	0.4049
Ti	22	46	84	103.7	45.9526	45.9512	0.3%	46.3680	-0.0014	0.4154
Ti	22	48	88	103.7	47.9479	47.9440	0.9%	48.3854	-0.0039	0.4374
Ti	22	50	90	103.7	49.9448	49.9491	0.9%	50.4027	0.0043	0.4579
V	23	51	93	112.4	50.9440	50.9462	0.5%	51.4100	0.0022	0.4660
Cr	24	52	96	123.1	51.9405	51.9452	1.0%	52.4173	0.0047	0.4767
Cr	24	54	100	123.1	53.9389	53.9380	0.2%	54.4346	-0.0009	0.4957
Mn	25	55	103	132.2	54.9380	54.9355	0.5%	55.4419	-0.0025	0.5038
Fe	26	54	102	141.5	53.9396	53.9404	0.2%	54.4318	0.0008	0.4922
Fe	26	56	106	141.5	55.9349	55.9332	0.3%	56.4491	-0.0017	0.5142
Co	27	59	112	153.3	58.9332	58.9322	0.2%	59.4737	-0.0009	0.5405
Ni	28	58	112	164.5	57.9353	57.9328	0.5%	58.4637	-0.0026	0.5283
Ni	28	60	114	164.5	59.9308	59.9378	1.3%	60.4810	0.0070	0.5502
Ni	28	62	118	164.5	61.9283	61.9306	0.4%	62.4983	0.0023	0.5700
Cu	29	63	120	174.6	62.9296	62.9352	1.0%	63.5056	0.0056	0.5760
Zn	30	64	122	185.1	63.9291	63.9402	1.9%	64.5129	0.0111	0.5838
Zn	30	66	126	185.1	65.9260	65.9330	1.2%	66.5302	0.0070	0.6042
Zn	30	68	130	185.1	67.9248	67.9258	0.2%	68.5476	0.0010	0.6227
Ga	31	69	132	195.0	68.9256	68.9302	0.7%	69.5548	0.0046	0.6293
Ge	32	74	142	205.1	73.9212	73.9204	0.1%	74.5968	-0.0008	0.6756
As	33	75	144	218.5	74.9216	74.9281	1.0%	75.6041	0.0065	0.6825
Se	34	80	154	230.0	79.9165	79.9196	0.4%	80.6460	0.0030	0.7295
Br	35	79	154	241.7	78.9183	78.9206	0.3%	79.6359	0.0023	0.7176
Br	35	81	158	241.7	80.9163	80.9134	0.4%	81.6533	-0.0029	0.7370

nuc	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. err. Δ_1:Δ_2	base: $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: [amu] calc. - meas.	Δ_2: [amu] base - meas.
Kr	36	82	160	253.5	81.9135	81.9197	0.8%	82.6605	0.0062	0.7471
Kr	36	84	164	253.5	83.9115	83.9125	0.1%	84.6779	0.0010	0.7664
Kr	36	86	166	253.5	85.9106	85.9175	0.9%	86.6952	0.0069	0.7846
Rb	37	85	167	264.1	84.9118	84.9113	0.1%	85.6851	-0.0004	0.7734
Sr	38	84	164	271.1	83.9134	83.9264	1.7%	84.6751	0.0130	0.7617
Sr	38	86	168	271.1	85.9093	85.9192	1.3%	86.6924	0.0099	0.7832
Sr	38	88	172	271.1	87.9056	87.9120	0.8%	88.7098	0.0064	0.8041
Y	39	89	176	283.2	88.9058	88.9062	0.0%	89.7170	0.0004	0.8112
Zr	40	90	180	295.5	89.9047	89.9005	0.5%	90.7243	-0.0042	0.8196
Nb	41	93	186	313.4	92.9064	92.9054	0.1%	93.7489	-0.0010	0.8425
Mo	42	98	196	329.6	97.9054	97.9013	0.5%	98.7908	-0.0041	0.8854
Tc	43	98	197	343.5	97.9072	97.9070	0.0%	98.7895	-0.0003	0.8822
Ru	44	102	204	357.5	101.9043	101.9106	0.7%	102.8227	0.0063	0.9184
Rh	45	103	208	373.9	102.9055	102.9089	0.4%	103.8300	0.0034	0.9245
Pd	46	106	214	384.8	105.9035	105.9071	0.4%	106.8546	0.0036	0.9511
Ag	47	107	218	405.6	106.9051	106.9095	0.5%	107.8619	0.0044	0.9568
Cd	48	112	228	421.0	111.9028	111.9047	0.2%	112.9038	0.0020	1.0011
In	49	115	234	433.9	114.9039	114.9048	0.1%	115.9284	0.0009	1.0246
Sn	50	120	244	447.1	119.9022	119.8979	0.4%	120.9704	-0.0043	1.0682
Sb	51	121	247	463.9	120.9038	120.9027	0.1%	121.9776	-0.0011	1.0738
Te	52	126	256	478.4	125.9033	125.9032	0.0%	127.0196	-0.0001	1.1163
I	53	127	260	493.4	126.9045	126.9002	0.4%	128.0269	-0.0043	1.1224
Xe	54	132	268	508.0	131.9042	131.9069	0.2%	133.0688	0.0027	1.1646
Cs	55	133	270	519.5	132.9055	132.9128	0.6%	134.0761	0.0073	1.1706
Ba	56	138	278	531.0	137.9052	137.9166	0.9%	139.1180	0.0113	1.2128
La	57	139	280	545.4	138.9064	138.9253	1.6%	140.1253	0.0189	1.2189
Ce	58	140	284	566.6	139.9054	139.9281	1.8%	141.1326	0.0226	1.2271
Pr	59	141	288	582.7	140.9077	140.9261	1.5%	142.1398	0.0184	1.2322
Nd	60	144	298	603.2	143.9101	143.9088	0.1%	145.1644	-0.0013	1.2543
Pm	61	145	298	621.6	144.9128	144.9336	1.7%	146.1717	0.0208	1.2590

nuc	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. err. Δ_1:Δ_2	base: $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: [amu] calc. - meas.	Δ_2: [amu] base - meas.
Sm	62	152	310	636.1	151.9197	151.9330	1.0%	153.2310	0.0133	1.3112
Eu	63	153	316	659.5	152.9212	152.9257	0.3%	154.2383	0.0044	1.3170
Gd	64	158	326	683.3	157.9241	157.9288	0.3%	159.2802	0.0047	1.3561
Tb	65	159	330	700.8	158.9254	158.9282	0.2%	160.2875	0.0028	1.3621
Dy	66	164	340	718.4	163.9292	163.9255	0.3%	165.3294	-0.0037	1.4002
Ho	67	163	340	735.8	162.9287	162.9319	0.2%	164.3194	0.0031	1.3906
Er	68	166	348	756.6	165.9303	165.9271	0.2%	167.3440	-0.0032	1.4137
Tm	69	169	354	777.9	168.9342	168.9352	0.1%	170.3686	0.0010	1.4344
Yb	70	172	362	796.0	171.9364	171.9279	0.6%	173.3932	-0.0085	1.4568
Lu	71	175	368	815.1	174.9408	174.9338	0.5%	176.4178	-0.0070	1.4770
Hf	72	178	374	834.0	177.9437	177.9395	0.3%	179.4424	-0.0042	1.4987
Ta	73	181	383	859.3	180.9480	180.9329	1.0%	182.4670	-0.0150	1.5190
W	74	184	390	882.6	183.9509	183.9368	0.9%	185.4916	-0.0142	1.5407
Re	75	185	393	903.1	184.9530	184.9450	0.5%	186.4989	-0.0080	1.5459
Os	76	192	404	923.6	191.9615	191.9563	0.3%	193.5581	-0.0052	1.5967
Ir	77	193	410	947.2	192.9629	192.9491	0.9%	194.5654	-0.0138	1.6025
Pt	78	194	412	971.0	193.9627	193.9667	0.3%	195.5727	0.0040	1.6100
Au	79	197	420	992.4	196.9666	196.9626	0.2%	198.5973	-0.0040	1.6307
Hg	80	198	426	1,014.0	197.9668	197.9535	0.8%	199.6046	-0.0133	1.6378
Hg	80	200	430	1,014.0	199.9683	199.9463	1.3%	201.6219	-0.0220	1.6536
Hg	80	202	432	1,014.0	201.9706	201.9514	1.2%	203.6392	-0.0193	1.6686

nuc	Z_x	A_x	n_k	e_c	meas. [amu]	calc. m [amu]	rel. err. Δ_1:Δ_2	base: $Z_x \cdot m_p + N_x \cdot m_n$ [amu]	Δ_1: [amu] calc. - meas.	Δ_2: [amu] base - meas.
Tl	81	205	436	1,032.2	204.9744	204.9687	0.3%	206.6638	-0.0057	1.6894
Pb	82	208	442	1,050.5	207.9767	207.9739	0.2%	209.6885	-0.0027	1.7118
Bi	83	208	444	1,072.4	207.9797	207.9810	0.1%	209.6871	0.0013	1.7073
Po	84	208	448	1,092.0	207.9812	207.9737	0.4%	209.6857	-0.0075	1.7044
At	85	209	451	1,111.1	208.9862	208.9807	0.3%	210.6930	-0.0055	1.7068
Rn	86	222	470	1,132.0	222.0176	221.9952	1.3%	223.8042	-0.0224	1.7866
Fr	87	223	471	1,146.8	223.0197	223.0104	0.5%	224.8115	-0.0094	1.7917
Ra	88	226	476	1,162.0	226.0254	226.0187	0.4%	227.8361	-0.0067	1.8107
Ac	89	227	481	1,183	227.0278	227.0155	0.7%	228.8434	-0.0122	1.8156
Th	90	232	488	1,205	232.0381	232.0350	0.2%	233.8853	-0.0031	1.8472
Pa	91	231	493	1,235	231.0359	231.0230	0.7%	232.8753	-0.0129	1.8394
U	92	238	502	1,263	238.0508	238.0539	0.2%	239.9345	0.0032	1.8837
Np	93	237	503	1,290	237.0482	237.0625	0.8%	238.9245	0.0143	1.8763
Pu	94	244	520	1,310	244.0642	244.0368	1.4%	245.9837	-0.0274	1.9195